The Relative Pruning Power of Strong Stubborn Sets and Expansion Core: Additional Proofs

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Abstract

This technical report provides additional proofs and examples for "The Relative Pruning Power of Strong Stubborn Sets and Expansion Core" (Wehrle et al. 2013).

Proposition 1. *There is a family of planning tasks of size $\Theta(n)$ where $\Theta(4^n)$ states are reachable from the initial state when pruning based on EC, but only $\Theta(2^n)$ states are reachable when pruning based on $OBEC$.*

Proof: For $n > 0$, let $\Pi_n = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star \rangle$ be a planning task with the following components:

- $\mathcal{V} = \{a, b_1, \dots, b_n, c_1, \dots, c_n\}$ with $\mathcal{D}_v = \{0, 1, 2\}$ for all $v \in \mathcal{V}$
- $\mathcal{O} = B \cup C$ with $B = \{o, o_1, \dots, o_n, \overline{o_1}, \dots, \overline{o_n}\}$ and $C = \{o', o'_1, \dots, o'_n, \overline{o'_1}, \dots, \overline{o'_n}\}$
- $pre(o) = \{a \mapsto 0\}$, $eff(o) = \{b_1 \mapsto 1, \dots, b_n \mapsto 1\}$
- $pre(o_i) = \{b_i \mapsto 1\}$, $eff(o_i) = \{b_i \mapsto 2\}$ for $1 \leq i \leq n$
- $pre(\overline{o_i}) = \{b_i \mapsto 2\}$, $eff(\overline{o_i}) = \{b_i \mapsto 1\}$ for $1 \leq i \leq n$
- $pre(o') = \{a \mapsto 0\}$, $eff(o') = \{c_1 \mapsto 1, \dots, c_n \mapsto 1\}$
- $pre(o'_i) = \{c_i \mapsto 1\}$, $eff(o'_i) = \{c_i \mapsto 2\}$ for $1 \leq i \leq n$
- $pre(\overline{o'_i}) = \{c_i \mapsto 2\}$, $eff(\overline{o'_i}) = \{c_i \mapsto 1\}$ for $1 \leq i \leq n$
- $s_0 = \{a \mapsto 0, b_1 \mapsto 0, \dots, b_n \mapsto 0, c_1 \mapsto 0, \dots, c_n \mapsto 0\}$
- $s_\star = \{b_1 \mapsto 2, \dots, b_n \mapsto 2\}$

We observe that variable a is trivial (i.e., has no active operators modifying it) in all reachable states. Consequently, $s[a] = 0$ in all reachable states s.

In a reachable state, the variables b_i either all hold the value 0 (if o has never been applied) or can take on an arbitrary combination of 1s and 2s (after applying o and then some subset of o_i operators). This yields $2^n + 1$ possible assignments to the b_i variables. Similarly, noting the symmetry between operators modifying b_i and operators modifying c_i, there are $2^n + 1$ possible assignments to the c_i variables. It is easy to see that Π_n has $(2^n + 1)^2 = 4^n + 2^{n+1} + 1$ reachable states in total. In the following, we compare EC and OBEC on this planning task.

1. Expansion core performs no pruning, leading to a total number of reachable states equal to $4^n + 2^{n+1} + 1$. To see this, we first observe that all operators are active in all reachable states. (This follows because the precondition of each action and the goal are reachable from every reachable state, which is easy to verify.)

   Let s be an arbitrary reachable state. Then a is a potential precondition of $b_1, \dots, b_n$ in s because of o and of $c_1, \dots, c_n$ because of o'. Furthermore, the same operators show that $b_1, \dots, b_n, c_1, \dots, c_n$ are potential dependents of a in s. Therefore, the potential dependency graph in s is strongly connected: every variable is connected to a in both directions. It follows that $dc(s)$ must contain all variables and hence all applicable operators are included in $EC(s)$: there is no pruning by the EC method.

2. Now consider the OBEC algorithm. Let s be an arbitrary reachable non-goal state. All goals are of the form $b_i \mapsto 2$, so that the first operator added to $OBEC(s)$ must be some operator o_i. Then $OBEC(s)$ must include the corresponding operator $\overline{o_i}$ and o because they share an effect variable with o_i (rule OBEC4). After adding o, all other operators of the form o_j and $\overline{o_j}$ must be added because of OBEC4. At this point, no further operator can be added to $OBEC(s)$, so a fixed point has been reached. We see that OBEC never considers operators that modify c_i variables, while including all operators that modify b_i variables. Hence, exactly $2^n + 1$ states are reachable when pruning with OBEC (those where $s[c_1] = \dots = s[c_n] = 0$).

■

Proposition 2. *There exist planning tasks $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star \rangle$ for which neither $OBEC(s_0)$ nor any subset of it is a strong stubborn set, no matter how the choices of disjunctive action landmarks and necessary enabling sets are resolved.*

Proof: We show that the statement is already true when considering $EC(s_0)$ instead of $OBEC(s_0)$, which is a stronger statement because $OBEC(s_0) \subseteq EC(s_0)$ as shown in the paper.

Let $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star \rangle$ be a planning task with the following components:

- $\mathcal{V} = \{v, w, x\}$ with $\mathcal{D}_v = \mathcal{D}_w = \{0, 1, 2\}$ and $\mathcal{D}_x = \{0, 1\}$

- $\mathcal{O} = \{o_1, o_2, o_3, o_4\}$
- $pre(o_1) = \emptyset$, $\mathit{eff}(o_1) = \{v \mapsto 1, x \mapsto 1\}$
- $pre(o_2) = \{v \mapsto 1\}$, $\mathit{eff}(o_2) = \{v \mapsto 2\}$
- $pre(o_3) = \{w \mapsto 0\}$, $\mathit{eff}(o_3) = \{w \mapsto 1\}$
- $pre(o_4) = \{v \mapsto 2, w \mapsto 1\}$, $\mathit{eff}(o_4) = \{w \mapsto 2\}$
- $s_0 = \{v \mapsto 0, w \mapsto 0, x \mapsto 0\}$
- $s_\star = \{v \mapsto 2, w \mapsto 2, x \mapsto 1\}$

All operators are active in s_0 because $\langle o_1, o_2, o_3, o_4 \rangle$ is a plan in s_0. Assume that x is the initialization variable that is chosen for rule EC1, so $x \in dc(s)$. By rule EC4, this leads to $v \in dc(s)$ because of o_1. At this point, no rule application could extend $dc(s)$ further: in particular, the missing variable w will not be added. (The only critical case here is EC2 with $v' = w$ and $o = o_4$; however, the rule is not applicable because o_4 is not v-applicable in s_0.) We conclude that $dc(s_0) = \{v, x\}$ and hence $EC(s_0) = OBEC(s_0) = \{o_1, o_2\}$.

In contrast, *every* strong stubborn set T_{s_0} must necessarily contain o_3 or o_4 and hence cannot be a subset of $EC(s_0)$. To see this, note that T_{s_0} must contain at least one applicable operator (by the completeness of the stubborn set method), and hence it contains o_1 or o_3. If $o_3 \in T_{s_0}$, we are done. Otherwise we have $o_1 \in T_{s_0}$ and therefore also $o_4 \in T_{s_0}$ because o_1 disables o_4. ∎

References

Wehrle, M.; Helmert, M.; Alkhazraji, Y.; and Mattmüller, R. 2013. The relative pruning power of strong stubborn sets and expansion core. ICAPS 2013.