Integrating Partial Order Reduction and Symmetry Elimination for Cost-Optimal Classical Planning

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Abstract

Pruning techniques based on partial order reduction and symmetry elimination have recently found increasing attention for optimal planning. Although these techniques appear to be rather different, they base their pruning decisions on similar ideas from a high level perspective. In this paper, we propose safe integrations of partial order reduction and symmetry elimination for cost-optimal classical planning. We show that previously proposed symmetry-based search algorithms can safely be applied with strong stubborn sets. In addition, we derive the notion of *symmetrical strong stubborn sets* as a more tightly integrated concept. Our experiments show the potential of our approaches.

1 Introduction

Heuristic search is a leading approach to optimally solving classical planning problems. However, for optimal planning, pure heuristic search based on A^* is limited in the sense that even almost perfect heuristics can lead to exponentially large state spaces in typical planning problems [Helmert and Röger, 2008]. Hence, additional (and orthogonal) *pruning techniques* are desired to be able to efficiently scale to larger problems. Pruning techniques allow for considering only a subset of successors in every state (and thus potentially reduce the branching factor of the given planning task), while preserving completeness and optimality of optimal search algorithms. Recent approaches for optimal planning include strong stubborn sets [Alkhazraji *et al.*, 2012; Wehrle *et al.*, 2013; Wehrle and Helmert, 2014], symmetry elimination [Pochter *et al.*, 2011; Domshlak *et al.*, 2012; 2013], partition-based path pruning [Nissim *et al.*, 2012], tunneling [Coles and Coles, 2010; Nissim *et al.*, 2012], sleep sets [Wehrle and Helmert, 2012; Holte *et al.*, 2015], move pruning [Holte and Burch, 2014], and commutativity pruning [Haslum and Geffner, 2000].

Recently, techniques based on partial order reduction and symmetry elimination have found particular attention in the planning community. From a technical point of view, these techniques appear to be rather different: While search-based planning algorithms with partial order reduction work on the original state space and prune "permutations" of plans in every state, algorithms based on symmetry elimination work on a modified state space (called the *orbit space*) that consists of equivalence classes of symmetrical states. Despite these differences, both approaches exploit the "equivalence" of permutations (variables and values vs. sequences of operators, respectively) for their pruning decisions. Originally, both partial order reduction and symmetry elimination stem from the area of model checking, where also combinations of them have been studied [Emerson *et al.*, 1997; Bosnacki and Scheffer, 2015]. Overall, it naturally arises the question whether these techniques can be integrated safely and efficiently for the purpose of optimal planning as well.

In this paper, we propose safe integrations of partial order reduction based on strong stubborn sets and symmetry elimination for optimal planning. Our first approach applies strong stubborn sets "directly" as the basis for the orbit space computation obtained by symmetry elimination. We show that this combination is safe and optimality preserving when applied for planning. Our second approach, called *symmetrical strong stubborn sets*, provides a more tightly integrated concept based on restricting the original definition of strong stubborn sets to canonical ("symmetrical") operators. We prove that symmetrical strong stubborn sets yield a safe state-based successor pruning function as introduced by Wehrle and Helmert [2014], which have the property that the pruning decision in a state s is based solely on s. Such techniques can safely be applied in graph search algorithms like A^* and are hence particularly attractive. We empirically investigate the performance of our approaches on the benchmarks from the international planning competitions.

2 Background

We consider SAS$^+$ planning with a finite set of finite-domain state variables $\mathcal{V}$. Every variable $v \in \mathcal{V}$ has a finite domain $dom(v)$. A variable/value pair $\langle v,d \rangle$ for $v \in \mathcal{V}$ and $d \in dom(v)$ is called a *fact*. A *partial state* s is defined as a function from variables $vars(s) \subseteq \mathcal{V}$ to values in the domains of $vars(s)$, whereas all variables in $\mathcal{V} \setminus vars(s)$ have an undefined value $undef$. We denote the value of v in s with $s[v]$ (including $s[v] = undef$ in case $v \in \mathcal{V} \setminus vars(s)$). A *state* is a partial state where all values are defined, i. e., with $vars(s) = \mathcal{V}$.

An *operator* o is a tuple $\langle pre(o), \mathit{eff}(o) \rangle$, where $pre(o)$ and $\mathit{eff}(o)$ are partial states and denote the *precondition* and the *effect* of o, respectively. An operator o is *applicable* in a state s iff $s[v] = pre(o)[v]$ for all $v \in vars(pre(o))$. If o is applicable in s, the *successor state* $o(s)$ of s is obtained from s by setting the values of variables in $vars(\mathit{eff}(o))$ to their values in $\mathit{eff}(o)$, and leaving the remaining variable values unchanged. We denote the set of applicable operators in s with $app(s)$. Furthermore, we say that an operator o is an *achiever* of a fact $\langle v,d \rangle$ if $\mathit{eff}(o)[v] = d$.

A $\textsc{sas}^+$ planning task is a tuple $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star, C \rangle$, consisting of a finite set of finite-domain state variables $\mathcal{V}$, a finite set of operators $\mathcal{O}$, an initial state s_0, and a partial goal state $s_\star$. In addition, we are given a cost function C that assigns a non-negative cost value $C(o)$ to each operator $o \in \mathcal{O}$. We denote the set of all states of Π with $\mathcal{S}$. The *state transition graph* $\mathcal{T}_\Pi = \langle \mathcal{S}, E \rangle$ of Π is a directed graph whose set of vertices is the set of states $\mathcal{S}$, and there is an edge $\langle s, s', o \rangle \in E$ from s to s' labeled with o iff o is applicable in s and $s' = o(s)$. A *plan* in state s is a sequence of operators $\sigma = o_1 \dots o_n$ such that σ is sequentially applicable in s and leads to a state that complies with $s_\star$. A state s is called *solvable* if there is a plan in s. A plan π is *optimal* if the sum of operator costs in π is minimal among all plans. A plan π is *strongly optimal* if π is optimal and contains a minimal number of zero-cost operators. In the following, we focus on finding optimal plans based on A^* search [Hart *et al.*, 1968].

2.1 Strong Stubborn Sets

Strong stubborn sets yield *safe successor pruning functions* as defined by Wehrle and Helmert [2014]: A successor pruning function for a planning task Π is a function $f : \mathcal{S} \to 2^{\mathcal{O}}$ that maps states s to subsets $ops(s)$ of applicable operators in s, i. e., $ops(s) \subseteq app(s)$. A successor pruning function is *safe* iff in a modified state transition graph where only the operators in $ops(s)$ may be applied in all states s, the original solution costs remain the same. As a sufficient criterion, Wehrle and Helmert show that if for all solvable non-goal states s, $ops(s)$ contains at least one operator that starts a strongly optimal plan, then the corresponding successor pruning function is safe. In the following, we call pruning functions that satisfy this criterion *strongly safe*.

Recently, strong stubborn sets have been considered in different generalities. In the following, we introduce a variant that is general enough for the purpose of this paper, based on a state-dependent notion of interference [Wehrle and Helmert, 2014]. For this, we need more terminology. Firstly, we say that two operators o, o' *interfere* in a state s iff both o and o' are applicable in s, and at least one of the following conditions holds: o disables o' (i. e., o' is not applicable in $o(s)$), or vice versa, or $o(o'(s))$ and $o'(o(s))$ are both defined, but $o(o'(s)) \neq o'(o(s))$. Secondly, for a planning task $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star, C \rangle$, for a state s and an operator $o \in \mathcal{O}$, a *necessary enabling set* for o and s is a set of operators N such that for all plans π that start in s and include o, some $o' \in N$ occurs in π before the first occurrence of o. Furthermore, a *disjunctive action landmark* L in a state s is a set of operators such that every plan in s includes at least one operator of L.

Definition 1 (strong stubborn set) *Let s be a solvable non-goal state in planning task $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star, C \rangle$. A set of operators $T(s)$ is a* strong stubborn set *in s if*

1. *$T(s)$ contains a disjunctive action landmark in s,*
2. *for every $o \in T(s)$ not applicable in s, $T(s)$ contains a necessary enabling set for o and s, and*
3. *for every $o \in T(s)$ applicable in s, $T(s)$ contains all operators that interfere with o in s.*

2.2 Symmetry Elimination

In contrast to strong stubborn sets, symmetry elimination considers equivalence classes of symmetrical states, and allows for using representative states of each equivalence class. Recently, Shleyfman et al. [2015] introduced the notion of *structural symmetries*, which capture previously proposed concepts of symmetry for classical planning. In a nutshell, structural symmetries directly work on the factored representation of a given planning task Π. They map operators to operators, and variable/value pairs to variable/value pairs in such way that the induced mapping on the state transition graph $\mathcal{T}_\Pi$ is an automorphism of $\mathcal{T}_\Pi$. More formally, structural symmetries for $\textsc{sas}^+$ planning tasks are defined as follows.

Definition 2 (Structural symmetry) *For a planning task $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star, C \rangle$, let F be the set of Π's facts, i. e., pairs $\langle v, d \rangle$ with $v \in \mathcal{V}$ and $d \in dom(v)$. A* structural symmetry *for Π is a permutation $\sigma : F \cup \mathcal{O} \to F \cup \mathcal{O}$ where*

1. *$\sigma(F_V) = F_V$, where $F_V := \{\{\langle v, d \rangle \mid d \in dom(v)\} \mid v \in V\}$,*
2. *$\sigma(\mathcal{O}) = \mathcal{O}$ such that for $o \in \mathcal{O}$, $\sigma(pre(o)) = pre(\sigma(o))$, $\sigma(\mathit{eff}(o)) = \mathit{eff}(\sigma(o))$, and $C(\sigma(o)) = C(o)$.*
3. *$\sigma(s_\star) = s_\star$,*

where $\sigma(\{x_1, \dots, x_n\}) := \{\sigma(x_1), \dots, \sigma(x_n)\}$, and for a partial state s, $s' := \sigma(s)$ is the partial state obtained from s such that for all $\langle v, d \rangle$ with $v \in vars(s)$ and $d \in dom(v)$, $\sigma(\langle v, d \rangle) = \langle v', d' \rangle$ and $s'[v'] = d'$.

For a planning task Π with states $\mathcal{S}$, a set of structural symmetries Σ induces a group Γ and an equivalence relation $\sim_\Gamma$ on $\mathcal{S}$, where $s \sim_\Gamma s'$ iff there is $\sigma \in \Gamma$ such that $\sigma(s) = s'$. For a state s, pruning algorithms based on symmetry elimination only consider the equivalence classes of the successor states of s instead of all successor states, and only keep one representative element of these classes. In this sense, A^* with symmetry elimination applies all operators in s, but prunes some of the resulting successor states. The resulting reduced state transition graph is guaranteed to still contain an optimal plan in s. To achieve this, Domshlak et al. [2012] have introduced a variant of the A^* algorithm (called DKS) that performs duplicate elimination based on canonical states. For that, the DKS algorithm maintains an additional (i. e., the canonical) state for each search node, leading to an increased memory consumption. To overcome this problem, a further variant of A^* called *orbit space search* (OSS) has been introduced [Domshlak *et al.*, 2015]. Orbit space search directly performs the search in the *orbit space* which is induced by canonical representatives. In other words, orbit space search searches the state transition graph induced by canonical representatives for each encountered state, i. e., every state is mapped to a corresponding representative.

3 Strong Stubborn Sets + Orbit Space Search

Strong stubborn sets and symmetries are orthogonal concepts in terms of pruning power. (For space reasons, we refer to a technical report showing examples where strong stubborn sets can prune more than symmetries, and vice versa [Wehrle *et al.*, 2015]). Following previous work in model checking [Emerson *et al.*, 1997; Bosnacki and Scheffer, 2015], our first approach "directly" integrates strong stubborn sets and symmetry elimination for optimal planning: In every state s, instead of computing canonical states for *all* successor states of s, the computation of canonical states is restricted to those successors that result from a strong stubborn set in s. The proposed modification is both completeness and optimality preserving. In the following, we prove the more general result that both DKS and OSS can be safely combined with strongly safe successor pruning functions.

Theorem 1 *Restricting the successor generation of the DKS and OSS algorithms with a strongly safe successor pruning function yields complete and optimal planning algorithms.*

Proof: Let f be some strongly safe successor pruning function. For each expanded state s, we have that if there exists a plan for s, then there exists a strongly optimal plan $\pi_s = o_1, \ldots, o_n$ for s such that $o_1 \in f(s)$. Then $s_1 = o_1(s)$ is one of the successors generated using f. DKS will prune s_1 only if some other state s'_1 with the same canonical representative was already generated by DKS. OSS will generate the canonical representative s^c_1 of s_1 instead of s_1. In both cases, at least one representative state in each equivalence class remains eligible for expansion at each time. For each plan from s_1, there exists a plan of the same cost from s^c_1 and there exists one from s'_1 as well. Thus, as DKS and OSS are complete and optimal planning algorithms, the claim follows with a simple induction over the length of the strongly optimal plan. □

3.1 Experimental Evaluation

We have implemented the resulting search algorithms in the Fast Downward planner [Helmert, 2006] on top of the strong stubborn set implementation using full envelopes, mutex-based inference, and static FD ordering [Wehrle and Helmert, 2014], and on top of the orbit space search implementation [Domshlak *et al.*, 2015]. Our experiments were performed on machines with Intel Xeon E5-2660 CPUs running at 2.2 GHz. The time and memory bounds used per run were 30 minutes and 2 GB, respectively. All configurations used the LM-Cut heuristic [Helmert and Domshlak, 2009], the state-of-the-art heuristic for optimal planning, on all optimal STRIPS IPC benchmarks up to 2011 that are supported by LM-Cut (44 domains, 1396 instances). The results are given in Figure 1.

	no	sss	oss	o/s	no	sss	oss	o/s
airport	**28**	**28**	**28**	**28**	**16879**	**16879**	17103	17103
barman-11	4	4	**8**	**8**	4.8e+6	4.8e+6	**1.1e+6**	**1.1e+6**
blocks	**28**	**28**	**28**	**28**	**816728**	**816728**	**816728**	**816728**
depot	7	7	**9**	**9**	674671	674516	321352	**321279**
driverlog	13	**14**	13	**14**	265448	250655	191816	**179217**
elev.-08	**22**	**22**	**22**	**22**	3.3e+6	2.7e+6	2.6e+6	**1.9e+6**
elev.-11	**18**	**18**	**18**	**18**	2.9e+6	2.7e+6	2.0e+6	**1.9e+6**
floortile-11	7	7	**8**	**8**	3.7e+6	3.7e+6	2.3e+6	**2.3e+6**
freecell	**15**	14	**15**	14	935600	935600	**935003**	**935003**
grid	**2**	**2**	**2**	**2**	76778	76778	**71953**	**71953**
gripper	7	7	**20**	**20**	1.3e+7	1.3e+7	**322**	**322**
logistics00	20	**21**	20	**21**	859645	271971	687504	**173516**
logistics98	**6**	**6**	**6**	**6**	107126	46413	15072	**6904**
miconic	141	141	**142**	**142**	125703	125703	**64570**	**64570**
mprime	22	22	**23**	**23**	83047	82544	32431	**32303**
mystery	17	17	17	**18**	6.4e+6	6.1e+6	855601	**733538**
nomys.-11	14	14	**15**	14	**65815**	**65815**	66042	66042
openst.-08	19	20	**24**	**24**	1.8e+6	1.1e+6	449913	**319856**
openst.-11	14	15	**19**	**19**	1.8e+6	1.1e+6	448874	**318857**
openst.	**7**	**7**	**7**	**7**	914091	914091	**907304**	**907304**
parcpr.-08	18	**30**	18	**30**	299130	**208**	299130	**208**
parcpr.-11	13	**20**	13	**20**	299125	**203**	299125	**203**
parking-11	2	2	**3**	2	43167	43167	**41917**	**41917**
pathw.-nn	**5**	**5**	**5**	**5**	46689	6889	44442	**6484**
pegsol-08	27	27	**28**	**28**	2.0e+6	2.0e+6	**869397**	**869397**
pegsol-11	17	17	**18**	**18**	2.2e+6	2.2e+6	**937137**	**937137**
pipesw.-nt	17	17	**21**	**21**	1.2e+6	1.2e+6	272760	**272748**
pipesw.-t	12	12	**16**	**16**	1.5e+6	1.5e+6	**629390**	**629390**
psr-small	49	49	**50**	**50**	6.3e+6	5.5e+6	2.7e+6	**2.4e+6**
rovers	7	9	8	**10**	110002	32663	108770	**31816**
satellite	7	12	13	**14**	100571	3916	27019	**1281**
scan.-08	15	14	**17**	16	706977	706977	**12359**	**12359**
scan.-11	12	11	**14**	13	706973	706973	**12355**	**12355**
soko.-08	**30**	29	**30**	**30**	1.5e+7	1.5e+7	**7.9e+6**	**7.9e+6**
soko.-11	**20**	**20**	**20**	**20**	3.1e+6	3.1e+6	**1.4e+6**	**1.4e+6**
tidybot-11	**14**	**14**	**14**	13	64499	**43619**	64299	52190
tpp	6	6	**8**	**8**	27810	27810	**4069**	**4069**
trans.-08	**11**	**11**	**11**	**11**	56208	56208	**44620**	**44620**
trans.-11	6	6	**7**	**7**	55681	55681	**44103**	**44103**
trucks	10	10	**12**	**12**	610852	610852	**172920**	**172920**
visitall-11	**11**	**11**	**11**	**11**	6.6e+6	6.6e+6	**6.5e+6**	**6.5e+6**
wood.-08	17	**27**	20	**27**	232835	2165	109694	**2018**
wood.-11	12	**19**	14	**19**	232811	2152	109670	**2005**
zenotravel	**13**	**13**	**13**	**13**	224467	210874	127141	**121795**
Sum	762	805	828	**859**	8.4e+7	7.8e+7	3.7e+7	**3.4e+7**

Figure 1: Results overview for LM-Cut, coverage (left) and expansions without last f layer (right). Abbreviations: no: pure A^*, sss: A^* with strong stubborn sets, oss: orbit space search, o/s: orbit state space with strong stubborn sets.

We observe that the strengths of strong stubborn sets and symmetry elimination are rather orthogonal: Our integration, called o/s in Fig. 1, mostly achieves at least the same coverage (i. e., number of solved problems) as the maximum coverage of the previous approaches. In particular, this is the case in domains where the coverage of the previous approaches is very different (e. g., in Gripper and Parcprinter, respectively). Overall, our approach solves 859 out of 1396 problems, which is particularly remarkable due to the usual exponential complexity increase in the size of the problems. Considering the number of expanded states, we observe a more fine grained picture: While still in most domains the number of expansions with o/s is at most as high as the minimum of the previous approaches, in 19 out of these the number of expansions is *strictly* lower than the minimum. Although the difference is sometimes moderate, it shows that there exist synergy effects which could further be exploited by future benchmark problems. In addition, in the Mystery, Rovers and Satellite domains, this synergy effect is already strong enough to yield the uniquely highest coverage.

4 Symmetrical Strong Stubborn Sets

The concept of *symmetrical strong stubborn sets* integrates strong stubborn sets and symmetries more tightly. It is based on restricting strong stubborn sets to *canonical representatives* of equivalence classes of symmetrical operators.

4.1 Canonical Operators

We will derive canonical operators by applying symmetry-based reasoning to operators. This concept will provide a strongly safe successor pruning function in its own right, and will form the basis for symmetrical strong stubborn sets. While it might appear obvious to achieve these objectives for a given structural symmetry group (i. e., by just considering the representative operators of each equivalence class of operators induced by the given symmetries), care must be taken with the details: structural symmetries σ that do *not stabilize* the current state s (i. e., $\sigma(s) \neq s$) are not guaranteed to yield safe successor pruning functions. For brevity, we again refer to a technical report for an example showing that non-stabilizing symmetries do not yield strongly safe successor pruning functions in general [Wehrle *et al.*, 2015].

In the following, we make these ideas more precise. Let $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star, C \rangle$ be a planning task, Σ be a set of structural symmetries of Π, and s be a state of Π. Let $\Sigma_s \subseteq \Sigma$ be the set of structural symmetries that stabilize s (that is, $\sigma(s) = s$ for all $\sigma \in \Sigma_s$). Let Γ_s be a group induced by Σ_s and let $\sim_s$ be the equivalence relation over the operator set $\mathcal{O}$ induced by Γ_s. The relation $\sim_s$ defines a partitioning of the operator set $\mathcal{O}$ into equivalence classes. Each equivalence class is identified with one of the operators from the class, which is chosen to be the canonical operator for that equivalence class. Slightly abusing the notation, the canonical operator for the operator $o \in \mathcal{O}$ is denoted by $[o]_s$. The mapping $CL_s : \mathcal{O} \mapsto \mathcal{O}$ defined by $CL_s(o) := [o]_s$ is called the *canonical operator labeling in s*. We denote the induced successor pruning function $sop(s) := \{CL_s(o) \mid o \in app(s)\}$ as *symmetrical operator pruning*. Symmetrical operator pruning is strongly safe, and more generally, it can safely be combined with any strongly safe successor pruning function.

Theorem 2 *Let f be a strongly safe successor pruning function, and let ops be the successor pruning function defined as $ops(s) := \{CL_s(o) \mid o \in f(s)\}$. Then ops is strongly safe.*

Proof: Let $\pi = o_1, \ldots, o_n$ be a strongly optimal plan for s such that $o_1 \in f(s)$. Such a plan exists, since f is strongly safe. Let σ be some structural symmetry stabilizing s such that $\sigma(o_1) = CL_s(o_1)$. Then $\pi' = \sigma(\pi) = \sigma(o_1), \ldots, \sigma(o_n)$ is a strongly optimal plan for $s = \sigma(s)$, starting with an operator in $ops(s)$. Thus ops is strongly safe. □

Computing canonical operator labelings for a given state is polynomial time. Let $o_1, \ldots, o_k$ be the operators of a planning task, with i being the index of o_i. For a state s, a canonical operator labeling CL_s for s is computed as follows.

1. In a pre-search phase, we compute for each generator σ a canonical operator labeling CL_σ for σ as follows: Starting with $CL_\sigma(o_i) = i$, we iteratively set $CL_\sigma(o) := \min(CL_\sigma(o), CL_\sigma(\sigma(o)))$ for each operator o until a fixed-point is reached.
2. During search, for a set of generators stabilizing s, we compute CL_s for s by starting with an identity labeling, and continue the following procedure until a fixed-point in CL_s is reached: For each stabilizing generator σ, for each operator o, $CL_s(o) := \min(CL_\sigma(o), CL_s(o))$.

Since the indices only reduce, the number of iterations is bounded by $O(k^2)$, with each iteration being linear in k and the number of symmetry group generators.

Overall, Theorem 2 allows us to safely use symmetrical operator pruning in polynomial time within A^*, and more generally, it allows us to use it on top of any strongly safe successor pruning functions. In addition, Theorem 2 will serve as an ingredient for integrating strong stubborn sets and symmetries.

4.2 Symmetrical Strong Stubborn Sets

As outlined, symmetrical strong stubborn sets restrict the definition of strong stubborn sets to canonical operators.

Definition 3 (symmetrical strong stubborn set) *Let $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\star, C \rangle$ be a planning task, let s be a state of Π, and CL_s be the canonical operator labeling in s. A* symmetrical strong stubborn set (SSSS) *in s is a set of operators $H \subseteq \mathcal{O}$ with the following properties. If s is an unsolvable or goal state, every set $H \subseteq \mathcal{O}$ is a SSSS. If s is a solvable non-goal state, then H satisfies the following constraints:*

1. *H contains the canonical operators of a disjunctive action landmark L in s, i. e., $\{CL_s(o) \mid o \in L\} \subseteq H$,*
2. *for every $o \in H$ not applicable in s, H contains the canonical operator labeling of a necessary enabling set N for o and s, i. e., $\{CL_s(o) \mid o \in N\} \subseteq H$, and*
3. *for every $o \in H$ applicable in s, H contains the canonical operator labeling of all operators o' that interfere with o, i. e., $\{CL_s(o') \mid o$ interferes with o' in $s\} \subseteq H$.*

Symmetrical strong stubborn sets yield a safe successor pruning function.

Theorem 3 *Let ops be a successor pruning function defined as $ops(s) = H(s) \cap app(s)$, where $H(s)$ is a symmetrical strong stubborn set in s. Then ops is strongly safe.*

Proof: Our proof is based on the proof of Theorem 1 by Wehrle and Helmert [2014]. Let s be a state and H be a SSSS in s. We show that if s is a solvable non-goal state, then H contains an operator which is the first operator in a strongly optimal plan for s. The claim then follows with Proposition 1 of Wehrle and Helmert [2014]. In the following, we refer to the three conditions of Def. 3 as C1–C3.

Let $\pi = o_1, \ldots, o_n$ be a strongly optimal plan for s such that $CL_s(o_i) \in H$ for some $i \in \{1, \ldots, n\}$. Such a plan must exist because of C1. Let $k \in \{1, \ldots, n\}$ be the minimal index for which $o_k^c := CL_s(o_k) \in H$ and let σ_k be a structural symmetry that stabilizes s such that $o_k^c = \sigma_k(o_k)$.

We show by contradiction that o_k^c is applicable in s. Assume it is not applicable. Since $o_k^c \in H$, C2 guarantees that H contains the canonical operator labeling of a necessary enabling set for o_k^c. Let $\pi_k = \sigma_k(\pi) = \sigma_k(o_1), \ldots, \sigma_k(o_n)$. π_k is a strongly optimal plan for s, since it is a mapping of π with a structural symmetry that stabilizes s. Therefore, a necessary enabling set for o_k^c will include the canonical operator labeling of some operator $\sigma_k(o_i)$ for $i < k$. Since $CL_s(\sigma_k(o_i)) = CL_s(o_i)$, according to C2, H must contain the canonical operator labeling of o_i, contradicting the minimality of k. It follows that o_k^c is applicable in s.

Let $s^0, \ldots, s^n$ be the sequence of states visited by π_k: $s^0 = s$ and $s^i = \sigma_k(o_i)(s^{i-1})$ for all $i \in \{1, \ldots, n\}$. It follows that o^c_k does not interfere with any of the operators $\sigma_k(o_1), \ldots, \sigma_k(o_{k-1})$ in any of the states s^j: if it did, then from C3 (with $o = o^c_k$), the canonical operator labeling of the interfering operators would be contained in H, together with $CL_s(\sigma_k(o_i)) = CL_s(o_i)$ contradicting the minimality of k.

The remainder of the proof, showing that if o^c_k is not already the first operator in π_k, it can be shifted to the front of π_k, is exactly as in Wehrle and Helmert (2014). □

Symmetrical strong stubborn sets generalize symmetrical operator pruning and stubborn sets in the following sense.

Theorem 4 *Let G be a strong stubborn set in s. Then $H = \{CL_s(o) \mid o \in G\}$ is a symmetrical strong stubborn set in s.*

Proof: It is clear that C1 will hold for H. To see that C2 holds, let $o \in G$ be some non-applicable operator and let $N \subseteq G$ be its necessary enabling set. Let σ be some structural symmetry stabilizing s such that $\sigma(o) = CL_s(o)$. Then $N^\sigma = \{\sigma(o') \mid o' \in N\}$ is a necessary enabling set for $CL_s(o)$. Note that for all actions $o' \in N$ we have $CL_s(o') = CL_s(\sigma(o'))$ and thus $N^c = \{CL_s(o') \mid o' \in N\}$ is the canonical operator labeling of a necessary enabling set for $CL_s(o)$. To see that C3 holds as well, let $o \in G$ be some applicable operator and let o' interfere with o in s. Let σ be some structural symmetry that stabilizes s such that $\sigma(o) = CL_s(o)$. Then $\sigma(o)$ interferes with $\sigma(o')$ in $s = \sigma(s)$. Thus, $CL_s(\sigma(o')) = CL_s(o') \in H$. □

We remark that although Theorem 4 shows a theoretical dominance result, the choice of the algorithms in practical implementations does not necessarily guarantee the dominance in terms of state explorations. However, in our experiments, the latter is established in almost all domains (we will come back to this point in the next section).

In addition, symmetrical strong stubborn sets offer the potential to prune more than the combination of strong stubborn sets with orbit space search according to Theorem 1, and than the combination of strong stubborn sets and symmetrical operator pruning according to Theorem 2. Intuitively, this is the case because symmetrical strong stubborn sets recognize (and can exploit) symmetries also for inapplicable operators.

Example 1 *Let Π_1 be a planning task with binary variables $\mathcal{V} = \{a, b, c, d, g\}$ and uniform-cost operators $\mathcal{O} = \{o_1, o_2, o_3, o_4, o_5\}$ with*

- $pre(o_1) = \{\langle a,1\rangle, \langle b,1\rangle\}$, $\mathit{eff}(o_1) = \{\langle g,1\rangle\}$
- $pre(o_2) = \{\langle b,1\rangle, \langle c,1\rangle\}$, $\mathit{eff}(o_2) = \{\langle g,1\rangle\}$
- $pre(o_3) = \emptyset$, $\mathit{eff}(o_3) = \{\langle a,1\rangle\}$
- $pre(o_4) = \emptyset$, $\mathit{eff}(o_4) = \{\langle c,1\rangle\}$
- $pre(o_5) = \{\langle d,1\rangle\}$, $\mathit{eff}(o_5) = \{\langle b,1\rangle\}$.

Let $s_0 = \{\langle a,0\rangle, \langle b,0\rangle, \langle c,0\rangle, \langle d,1\rangle, \langle g,0\rangle\}$ and $s_\star = \{\langle g,1\rangle\}$.

We observe that there is a structural symmetry σ that maps operator o_1 to o_2 and o_3 to o_4, and vice versa, and stabilizes the initial state (by mapping $a = 1$ and $c = 1$ to each other). Without loss of generality, assume that the canonical operator for both o_1 and o_2 is o_1 and for both o_3 and o_4 is o_3.

- *Consider the combination of strong stubborn sets with orbit space search (o/s) based on Theorem 1. Strong stubborn sets in s_0 can be obtained according to the following procedure: $\{o_1, o_2\}$ is a disjunctive action landmark (consisting of the operators that set the goal variable). A necessary enabling set for o_1 is $\{o_3\}$, a corresponding set for o_2 is $\{o_5\}$, resulting in the strong stubborn set $\{o_1, o_2, o_3, o_5\}$. Out of this set, o_3 and o_5 are applicable. Furthermore, as previously proposed algorithms for symmetry detection base their computation on a syntactic planning task description [Pochter* et al.*, 2011], the successor of s_0 under o_5 will not be recognized as symmetrical to s_0's other successors (like the successor under o_3 in particular). Thus orbit space search with strong stubborn sets will classify s_0's successors under o_3 and o_5 in different orbits.*
- *Consider the successor pruning function obtained by the combination of strong stubborn sets and symmetrical operator pruning based on Theorem 2: Starting with the strong stubborn set according to the description in the bullet above, o_3 and o_5 out of this set are applicable, but not symmetrical, hence no further reductions are obtained. Thus again, two successor states are generated.*

In contrast, symmetrical strong stubborn sets only produce one successor state in s_0 because of the restriction to canonical operators: Only o_1 is included due to C1 (compared to o_1 and *o_2 for the other methods), and o_3 is included due to C2, resulting in a set that only contains one applicable operator o_3. Intuitively, symmetrical strong stubborn sets can achieve more pruning as they already recognize symmetries in "intermediate" steps during the fixed-point computation.*

The example shows that symmetrical strong stubborn sets can further increase the pruning power under reasonable practical design choices w.r.t. their computation (i. e., using the achievers of an unsatisfied goal fact as disjunctive action landmark, and using the achievers of the first unsatisfied precondition fact as necessary enabling sets). We remark that selecting the achievers of a first unsatisfied precondition fact according to a particular ordering for computing necessary enabling sets (e. g., selecting the achievers of $\langle b,1\rangle$ for o_2 in the initial state) can be viewed as tie-breaking. However, this is a way strong stubborn sets have recently been successfully implemented, e. g., by Alkhazraji et al. [2012].

4.3 Experimental Evaluation

We have implemented and evaluated symmetrical strong stubborn sets in the same setting as in the previous experimental section. The coverage results are given in Fig. 2. We compare symmetrical operator pruning (called sop), symmetrical strong stubborn sets (ssss), and the combination of standard strong stubborn sets and symmetrical operator pruning according to Theorem 2 (s/s) with standard strong stubborn sets (sss) within A^* and orbit space search.

We observe that symmetrical strong stubborn sets yield a slightly higher coverage than strong stubborn sets within A^*, and lowers the coverage within OSS. One reason for the latter is the overhead for computing canonical operators. The relative coverage difference of A^* and OSS (slightly improved

Domain	A^*					OSS				
	no	sss	sop	s/s	ssss	no	sss	sop	s/s	ssss
barman-11	4	4	4	4	4	**8**	**8**	**8**	**8**	**8**
depot	7	7	7	7	7	**9**	**9**	8	8	8
driverlog	13	**14**	13	**14**	**14**	13	**14**	13	**14**	**14**
floortile-11	7	7	7	7	7	**8**	**8**	**8**	**8**	**8**
freecell	**15**	14	**15**	14	13	**15**	14	**15**	14	14
gripper	7	7	11	11	11	**20**	**20**	**20**	**20**	**20**
logistics00	20	**21**	20	**21**	**21**	20	**21**	20	**21**	**21**
miconic	141	141	140	140	140	**142**	**142**	141	141	141
mprime	22	22	22	22	22	**23**	**23**	22	22	22
mystery	17	17	17	17	17	17	**18**	16	16	16
nomys.-11	14	14	**15**	14	14	**15**	14	**15**	14	14
openst.-08	19	20	19	20	20	**24**	**24**	20	21	21
openst.-11	14	15	14	15	15	**19**	**19**	15	16	16
parcpr.-08	18	**30**	18	**30**	**30**	18	**30**	18	**30**	**30**
parcpr.-11	13	**20**	13	**20**	**20**	13	**20**	13	**20**	**20**
parking-11	2	2	**3**	2	2	**3**	2	**3**	2	2
pegsol-08	27	27	**28**	**28**	**28**	**28**	**28**	**28**	**28**	**28**
pegsol-11	17	17	**18**	**18**	**18**	**18**	**18**	**18**	**18**	**18**
pipesw.-nt	17	17	18	18	18	**21**	**21**	20	20	20
pipesw.-t	12	12	12	12	12	**16**	**16**	13	13	13
psr-small	49	49	49	49	49	**50**	**50**	**50**	**50**	**50**
rovers	7	9	7	**10**	**10**	8	**10**	8	**10**	**10**
satellite	7	12	7	11	11	13	**14**	7	12	12
scan.-08	15	14	14	13	13	**17**	16	16	15	15
scan.-11	12	11	11	10	10	**14**	13	13	12	12
soko.-08	**30**	29	**30**	29	29	**30**	**30**	**30**	**30**	**30**
tidybot-11	**14**	**14**	**14**	13	13	**14**	13	**14**	13	13
tpp	6	6	7	7	7	**8**	**8**	**8**	**8**	**8**
trans.-11	6	6	6	6	6	**7**	**7**	**7**	**7**	**7**
trucks	10	10	10	10	10	**12**	**12**	10	10	10
visitall-11	**11**	**11**	**11**	**11**	10	**11**	**11**	**11**	**11**	**11**
wood.-08	17	**27**	19	**27**	**27**	20	**27**	19	**27**	**27**
wood.-11	12	**19**	13	**19**	**19**	14	**19**	13	**19**	**19**
others	160	160	160	160	160	160	160	160	160	160
Sum	762	805	772	809	807	828	**859**	800	838	838

Figure 2: Coverage results overview for LM-Cut. Domains with equal coverage in all configurations are summarized in "others". Abbreviations: no additional pruning, sss: strong stubborn sets, sop: symmetrical operator pruning, s/s: combination of strong stubborn sets and symmetrical operator pruning, ssss: symmetrical strong stubborn sets.

for A^* vs. considerably reduced for OSS) is presumably due to the different number of expanded states (see below for more details). Furthermore, symmetrical strong stubborn sets perform favorably compared to symmetrical operator pruning and similar to the combination s/s. The overall best configuration remains the previously introduced integration of strong stubborn sets and orbit space search, where no additional computational overhead for canonical operators occurs.

In the following, let us discuss the number of expanded states for the various configurations. For space reasons, we will only provide one scatterplot, and shortly explain the tendencies for the other comparisons in the running text.

Firstly, we compare symmetrical strong stubborn sets to strong stubborn sets. For A^*, Fig. 3 shows that symmetrical strong stubborn sets can further reduce the number of expansions (by several orders of magnitude in some cases). In contrast, for OSS, only slightly fewer states are expanded by symmetrical strong stubborn sets across all domains. This difference in state expansions in turn explains the relative coverage difference of these configurations as discussed above. Secondly, symmetrical strong stubborn sets show promising search behavior compared to symmetrical operator pruning: for both A^* and OSS, symmetrical strong stubborn sets expand (sometimes significantly) fewer states than symmetrical operator pruning. The resulting scatterplots (which are not shown again for space reasons) look similar to the

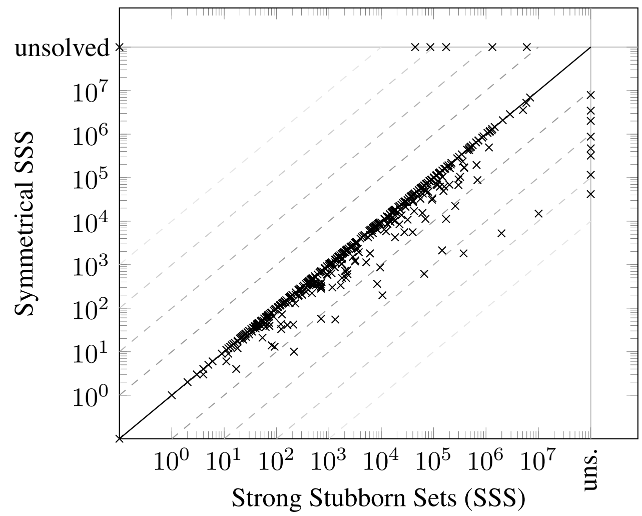

Figure 3: Expansions (without last f layer) for A^* + LM-Cut: strong stubborn sets vs. symmetrical strong stubborn sets

plot in Fig. 3. Thirdly, symmetrical strong stubborn sets compared to the combination of strong stubborn sets and symmetrical operator pruning yields almost no additional pruning in almost all domains (for both A^* and OSS). Apparently, in practice, OSS already mostly captures the additional pruning power offered by symmetrical strong stubborn sets.

Overall, the experiments show that the generalization result of Theorem 4 often carries over to practice. However, the additional pruning offered by symmetrical strong stubborn sets is exploited in terms of coverage by actual implementations only partly. Nevertheless, for A^*, there are problems where the additional pruning power is significant and pays off.

5 Related Work

Combinations of partial order reduction and symmetry elimination have already been (and still are) studied in the area of computer aided verification [Emerson *et al.*, 1997; Bosnacki and Scheffer, 2015]. Both Emerson et al. and Bosnacki and Scheffer consider partial order reduction based on *ample sets*. While Emerson et al. require considering unique canonical states within orbit space search, Bosnacki and Scheffer extend this theory by also allowing multiple representatives. Like the latter, we allow using multiple representatives: the approach by Bosnacki and Scheffer corresponds to our integration of strong stubborn sets and orbit space search for goal reachability, which we have additionally shown to be optimality preserving. Symmetrical strong stubborn sets further extend these concepts for goal reachability. In addition, we have empirically shown these approaches to be useful on a large number of planning benchmarks.

6 Conclusions

We have proposed two integration approaches of partial order reduction and symmetry elimination for planning, and proved them to be completeness and optimality preserving.

Our experiments show that the most direct integration is already most powerful in terms of coverage: restricting the orbit space search to states generated by strong stubborn sets often inherits the strengths of both partial order reduction and symmetry elimination, and significantly increases the number of solved problems in the standard benchmark suite from the international planning competitions. Furthermore, our concept of symmetrical strong stubborn sets offers additional pruning power compared to previous approaches, which is partly exploited by our current implementations. For the future, it will be interesting to further investigate the questions if more powerful integrations of partial order reduction and symmetry elimination can be derived, and to which extent the pruning power can be carried over to practice.

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