Correlation Complexity Under Variant Descending and Dead-End Avoiding Heuristics

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Example Planning Task (SAS$^+$): Blocksworld

Variables for each $A \in \mathcal{B}$:

$$\mathrm{dom}\left(\boxed{A} \right) = \mathcal{B} \cup \{\mathrm{Table}\}$$

$$\mathrm{dom}\left(\boxed{A} \right) = \{ \boxed{} , \boxed{} \}$$

Example Planning Task (SAS$^+$): Blocksworld

Variables for each $A \in \mathcal{B}$:

$$\text{dom}\left(\boxed{A} \right) = \mathcal{B} \cup \{\text{Table}\}$$

$$\text{dom}\left(\boxed{A} \right) = \{\boxed{}, \boxed{}\}$$

Operators:
e.g. *move-A-from-B-to-C*:

$$\textit{pre}\left(\boxed{A \atop B} \;\boxed{C} \right) = \left\{ \boxed{A \atop B}, \boxed{A}, \boxed{C} \right\}$$

$$\textit{eff}\left(\boxed{A \atop B} \;\boxed{C} \right) = \left\{ \boxed{A \atop C}, \boxed{B}, \boxed{C} \right\}$$

Potential Heuristic

A **potential heuristic** estimates the distance to the goal of a state by summing the weights of all features it contains:

$$h_{\text{pot}}(s) = \sum_{F \in \mathcal{F}} w(F)[F \subseteq s]$$

The **dimension** of h_{pot} is the size of the most complex feature (i.e., how many variables it involves).

Descending and Dead-End Avoiding (DDA) Heuristics

Goal: Guide search towards the goal while avoiding dead ends.

Descending: For every alive non-goal state s, there exists a successor s' with:

$$h(s') < h(s)$$

Dead-End Avoiding: Improving successors are always solvable:

$$s \text{ is alive and } h(s') < h(s) \implies s' \text{ solvable}$$

Correlation Complexity

The **correlation complexity** is defined as the smallest dimension d such that there exists a DDA potential heuristic of dimension d for the planning task.

Correlation Complexity

The **correlation complexity** is defined as the smallest dimension d such that there exists a DDA potential heuristic of dimension d for the planning task.

Unsolvable task $\Rightarrow$ Every potential heuristic fullfills DDA property.

DDA Variants

DDA

Every **alive** non-goal state s has at least one improving successor s'.

DDA Variants

> **DDA**
>
> Every **alive** non-goal state s has at least one improving successor s'.

> **Unrestricted DDA (UDDA)**
>
> **Every** ~~alive~~ non-goal state s has at least one improving successor s'.

DDA Variants

DDA

Every **alive** non-goal state s has at least one improving successor s'.

Unrestricted DDA (UDDA)

Every ~~alive~~ non-goal state s has at least one improving successor s'.

∞-DDA

i) Every ~~alive~~ non-goal state s with a **finite heuristic value** has at least one improving successor s'.

ii) The initial state must be finite.

DDA Variants

DDA

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Predicate-Based Pruning DDA (PDDA)

$$h_{\text{pot}}(s) = \begin{cases} \infty & \text{if } h_{\text{pot2}}(s) > 0 \\ h_{\text{pot1}}(s) & \text{otherwise} \end{cases}$$

plus ∞-DDA conditions on h.

Correlation Complexity

The **correlation complexity** is defined as the smallest dimension d such that there exists a DDA potential heuristic of dimension d for the planning task.

We can extend this definition to each DDA variant (UDDA, ∞-DDA, and PDDA).

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Motivation

> **Question:** Do correlation complexity variants form a hierarchy?
> **Answer: Sometimes.**
>   > For some pairs of variants, a clear hierarchy exists
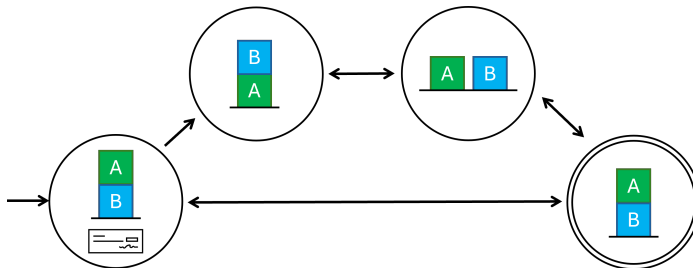>   > For other pairs, no hierarchy is possible

Notation: $A < B$ means that there exist problems for which the correlation complexity of A is strictly smaller than that of B.

DDA $<$ ∞-DDA

Proof idea: Will be shown in Bocksworld.

DDA > ∞-DDA

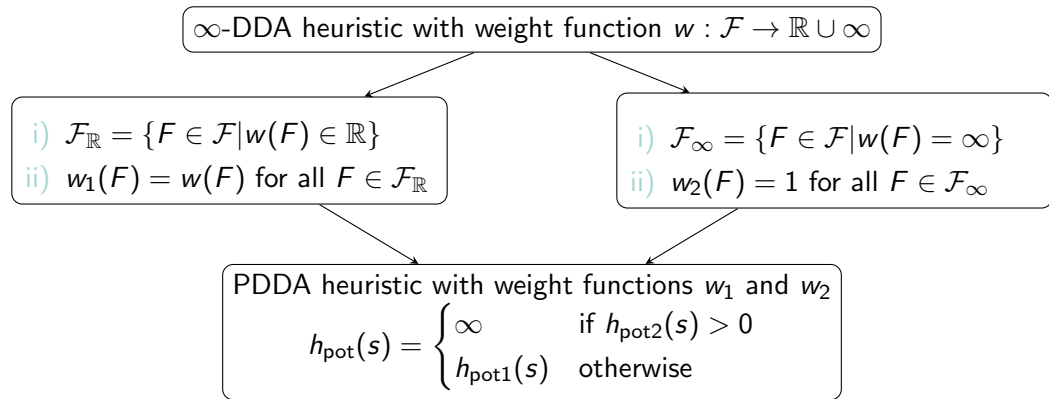*Proof idea:* In the Blocksworld domain, a shortcut allows ∞-DDA to bypass the longer plan required by DDA.

∞-DDA > PDDA

Proof idea: Will be shown in Bocksworld.

∞-DDA $\not\leq$ PDDA

Proof idea: Every ∞-DDA heuristic can be transformed into a PDDA heuristic:

$$\boxed{\infty\text{-DDA heuristic with weight function } w : \mathcal{F} \to \mathbb{R} \cup \infty}$$

i) $\mathcal{F}_{\mathbb{R}} = \{F \in \mathcal{F} | w(F) \in \mathbb{R}\}$
ii) $w_1(F) = w(F)$ for all $F \in \mathcal{F}_{\mathbb{R}}$

i) $\mathcal{F}_{\infty} = \{F \in \mathcal{F} | w(F) = \infty\}$
ii) $w_2(F) = 1$ for all $F \in \mathcal{F}_{\infty}$

PDDA heuristic with weight functions w_1 and w_2
$$h_{\mathrm{pot}}(s) = \begin{cases} \infty & \text{if } h_{\mathrm{pot2}}(s) > 0 \\ h_{\mathrm{pot1}}(s) & \text{otherwise} \end{cases}$$

Correlation Complexity Hierarchy
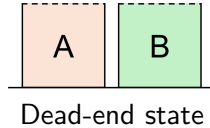
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Blocksworld

From previous work: DDA correlation complexity is 2.

Our contribution:

> There exists no UDDA heuristic
> ∞-DDA correlation complexity is 3
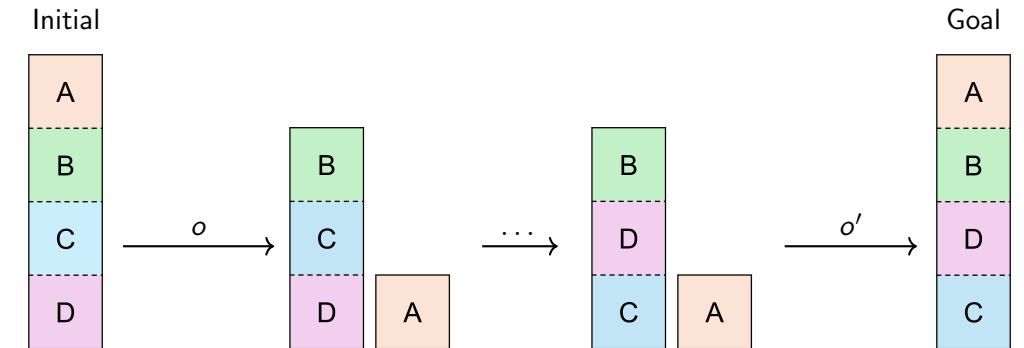> PDDA correlation complexity is 2

∄ UDDA Heuristic

Proof idea: Show that a dead-end state exists (i.e., no operator is applicable).


Dead-end state

$2 \leq$ PDDA Correlation Complexity

Proof idea: The operators $o = \textit{move-to-T}(A, B)$ and $o' = \textit{move-to-B}(A, T)$ are both action landmarks, yet they cannot both decrease the heuristic value.

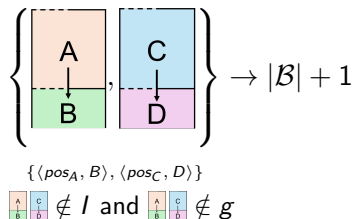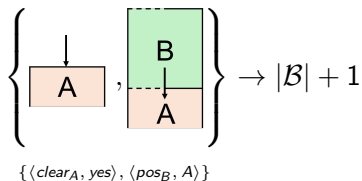# $3 \leq \infty$-DDA Correlation Complexity

Proof idea: To avoid the unsolvable state, at least one singleton or pair of atoms must be assigned **infinity**. However, no feature can be chosen as the infinite one, because then the ∞-DDA property would be violated.
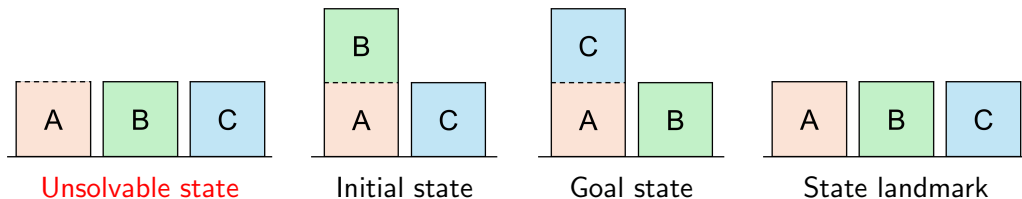
PDDA Correlation Complexity ≤ 2

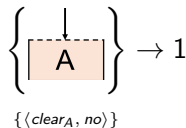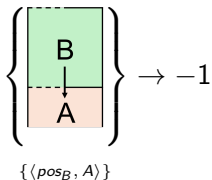**Proof idea:** Construct a PDDA heuristic of dimension 2 with w_2:

> Weights that directly assign an infinite value:

$\{\langle clear_A, yes \rangle, \langle pos_B, A \rangle\}$

$\{\langle pos_A, B \rangle, \langle pos_C, D \rangle\}$

 $\notin I$ and $\notin g$

PDDA Correlation Complexity ≤ 2

Proof idea: Construct a PDDA heuristic of dimension 2 with w_2:

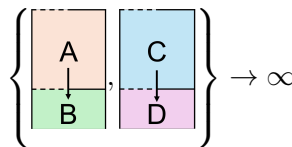> Weights for blocks marked as *not clear* but actually clear:

$$\left\{ \begin{array}{|c|} \hline B \\ \downarrow \\ \hline A \\ \hline \end{array} \right\} \to -1 \qquad\qquad \left\{ \begin{array}{|c|} \downarrow \\ \hline A \\ \hline \end{array} \right\} \to 1$$

$\{\langle pos_B, A \rangle\}$ $\{\langle clear_A, no \rangle\}$

∞-DDA Correlation Complexity ≤ 3

Proof idea: Construct an ∞-DDA heuristic of dimension 3 with w:

> Same features as for PDDA heuristic:
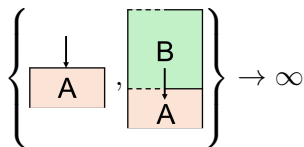
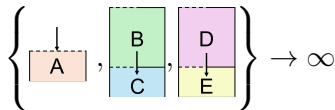
$\{\langle clear_A, yes\rangle, \langle pos_B, A\rangle\}$

$\{\langle pos_A, B\rangle, \langle pos_C, D\rangle\}$

 $\notin I$ and $\notin g$

∞-DDA Correlation Complexity ≤ 3

Proof idea: Construct an ∞-DDA heuristic of dimension 3 with w:

$\rangle$ Changes in contrast to the PDDA heuristic:

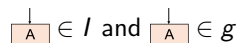$\{\langle clear_A, no \rangle, \langle pos_B, C \rangle, \langle pos_D, E \rangle\}$

$\boxed{\begin{smallmatrix}B\\A\end{smallmatrix}} \in l$ and $\boxed{\begin{smallmatrix}D\\A\end{smallmatrix}} \in g$

$\{\langle clear_A, no \rangle, \langle pos_B, C \rangle\}$

$\boxed{A} \in l$ and $\boxed{\begin{smallmatrix}B\\A\end{smallmatrix}} \in g$

or

$\boxed{A} \in g$ and $\boxed{\begin{smallmatrix}B\\A\end{smallmatrix}} \in l$

$\{\langle clear_A, no \rangle\}$

$\boxed{A} \in l$ and $\boxed{A} \in g$
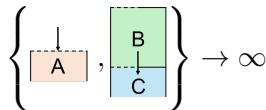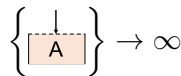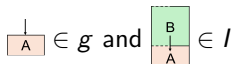
Blocksworld

We have shown:

> There exists no UDDA heuristic

> ∞-DDA correlation complexity is 3

> PDDA correlation complexity is 2

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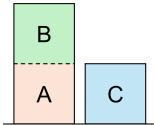
Domains

Blocksworld

Spanner

Logistics
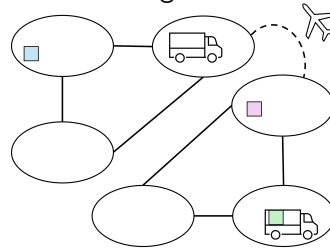
(Linear) Termes

Results

Domain	DDA	UDDA	∞-DDA	PDDA
Blocksworld	2	-	3	2
Spanner	2	-	2	2
Logistics	2	2	2	2
Linear Termes	?	-	2 or 3	2 or 3

Experiments – Empirical Validation

Goal

> Confirm UDDA, ∞-DDA, PDDA heuristics behave as predicted

Setup

> Modified *Fast Downward* with multi-dimensional domain-specific heuristics

Validation Strategy

1. **Search behaviour:** Expanded states $=$ plan length $+1$
2. **State-space verification:** Exhaustively check UDDA / ∞-DDA property

Conclusion

> Compared **DDA variants** (DDA, UDDA, ∞-DDA, PDDA)
> Analyzed their relationships and showed when a hierarchy exists
> Proved **lower and upper bounds** in Blocksworld and other domains
> Confirmed results through **empirical validation** with Fast Downward

Questions?