

# Post-hoc Optimization for the Sliding Tile Puzzle

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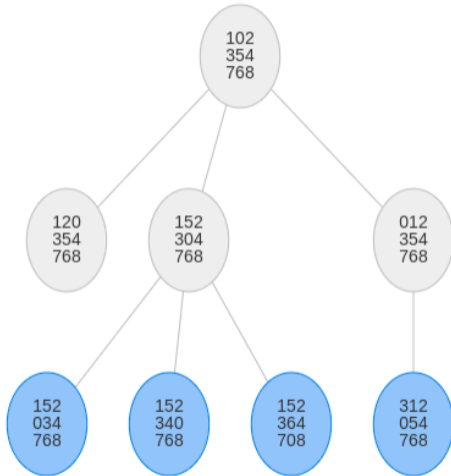
# Outline

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1. Background
2. Sliding Tile Puzzle
3. Heuristics
4. Post-hoc Optimization Heuristic
5. Experimental Evaluation

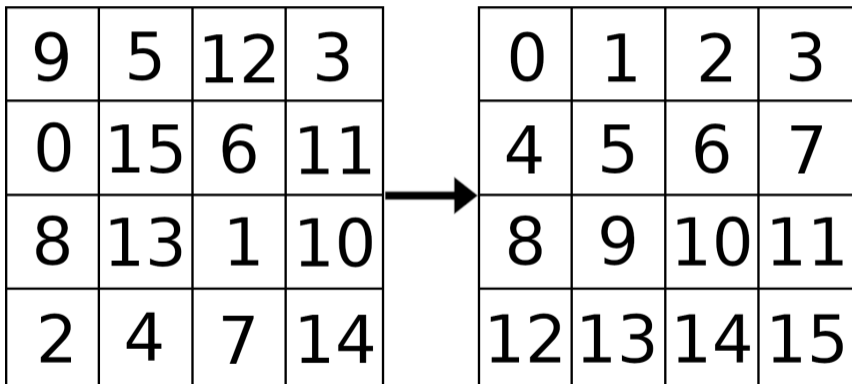
# Classical Planning, State Spaces and Heuristics

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## Sliding Tile Puzzle

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Move

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9	5	12	3
0	15	6	11
8	13	1	10
2	4	7	14

## Move

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9	5	12	3
0	15	6	11
8	13	1	10
2	4	7	14

## Move

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0	5	12	3
9	15	6	11
8	13	1	10
2	4	7	14

# Heuristics

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1	0	2
3	5	4
7	6	8



## Manhattan Distance

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1	0	2
3	5	4
7	6	8

# Manhattan Distance

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1. Manhattan Distance: 1

1	0	2
3	5	4
7	6	8

# Manhattan Distance

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1. Manhattan Distance: 1

1	0	2
3	5	4
7	6	8

# Manhattan Distance

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1. Manhattan Distance: 1

1	0	2
3	5	4
7	6	8

# Manhattan Distance

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1. Manhattan Distance: 2

1	0	2
3	5	4
7	6	8

# Manhattan Distance

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1. Manhattan Distance: 3

1	0	2
3	5	4
7	6	8

# Manhattan Distance

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1. Manhattan Distance: 4

1	0	2
3	5	4
7	6	8

# Manhattan Distance

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1. Manhattan Distance: 5

1	0	2
3	5	4
7	6	8



# Manhattan Distance

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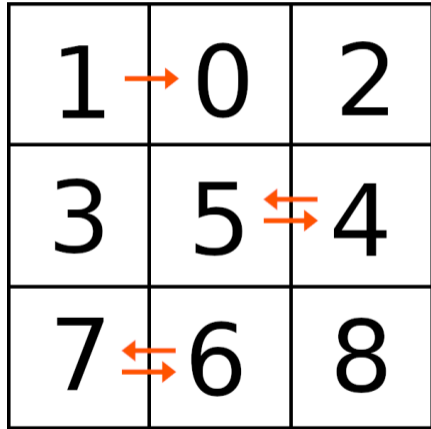
1. Manhattan Distance: 5

1	0	2
3	5	4
7	6	8

# Manhattan Distance

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1. Manhattan Distance: 5



# Linear Conflict

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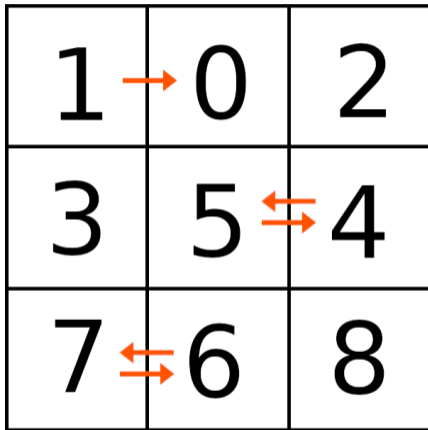
1. Manhattan Distance: 5
2. Linear Conflicts:  $5 + x * 2$

1	0	2
3	5	4
7	6	8

# Linear Conflict

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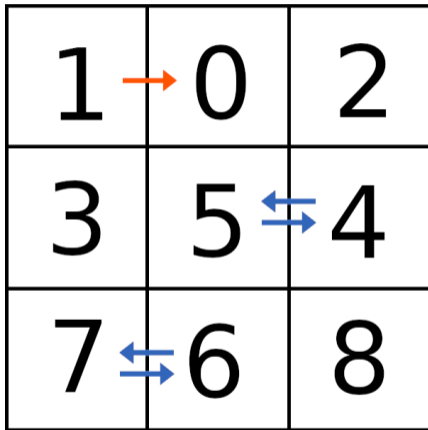
1. Manhattan Distance: 5
2. Linear Conflicts:  $5 + x * 2$



# Linear Conflict

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1. Manhattan Distance: 5
2. Linear Conflicts:  $5 + 2 * 2 = 9$



## Statically-Partitioned Additive Pattern Database Heuristic

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1. Manhattan Distance: 5
2. Linear Conflicts: 9
3. Statically:

1	0	2
3	5	4
7	6	8

# Statically-Partitioned Additive Pattern Database Heuristic

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<https://sliding-puzzle-solver.herokuapp.com/>

## Statically-Partitioned Additive Pattern Database Heuristic

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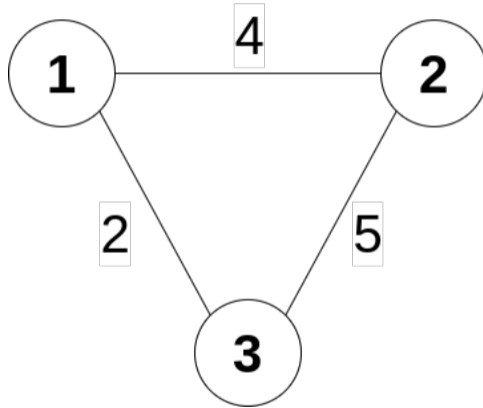
1. Manhattan Distance: 5
2. Linear Conflicts: 9
3. Statically:  $1 + 4 + 6 = 11$

1	0	2
3	5	4
7	6	8

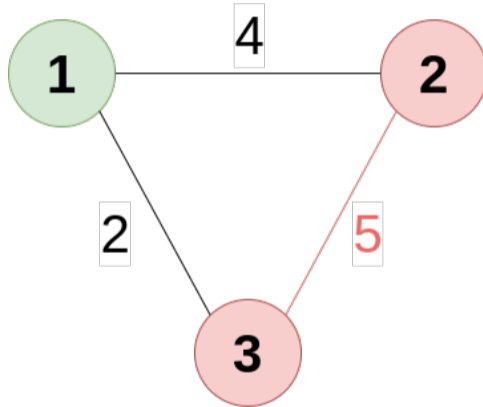


# Dynamically-Partitioned Additive Pattern Database Heuristic

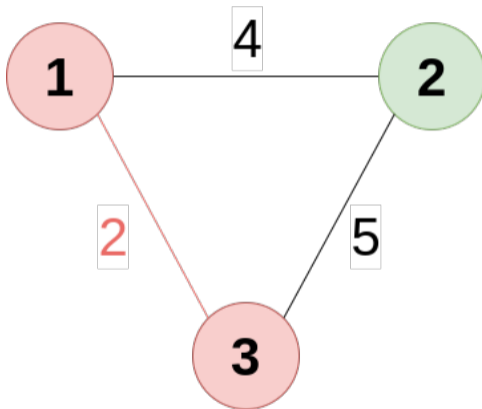
# Dynamically-Partitioned Additive Pattern Database Heuristic



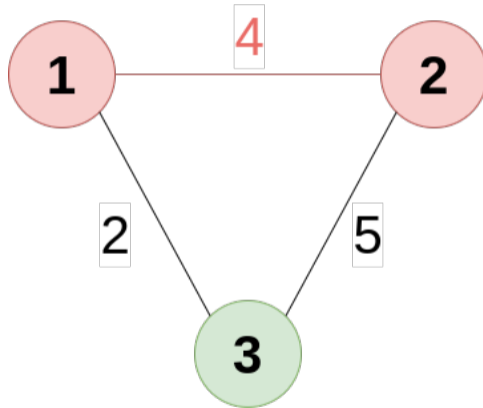
# Dynamically-Partitioned Additive Pattern Database Heuristic



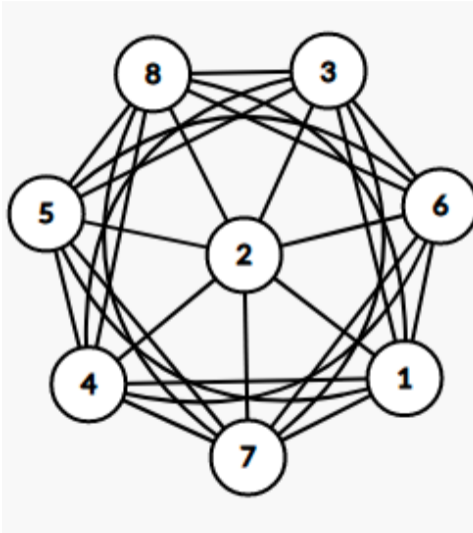
## Dynamically-Partitioned Additive Pattern Database Heuristic



# Dynamically-Partitioned Additive Pattern Database Heuristic



# Dynamically-Partitioned Additive Pattern Database Heuristic



# Dynamically-Partitioned Additive Pattern Database Heuristic

1. Manhattan Distance: 5
2. Linear Conflicts: 9
3. Statically: 11
4. Dynamically:

1	0	2
3	5	4
7	6	8

# Dynamically-Partitioned Additive Pattern Database Heuristic

1. Manhattan Distance: 5
2. Linear Conflicts: 9
3. Statically: 11
4. Dynamically:  $1 + 6 + 6 + 0 = 13$

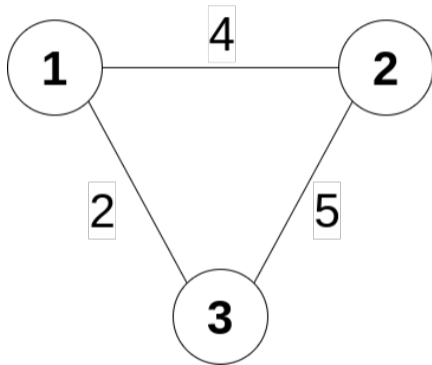
1	0	2
3	5	4
7	6	8



# Post-hoc Optimization Heuristic

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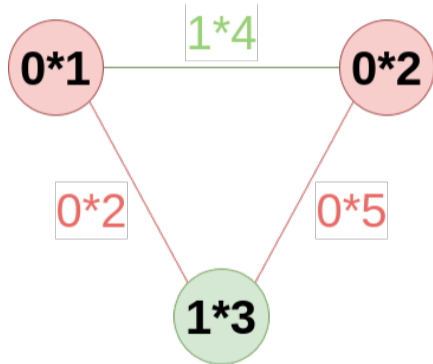
1. Manhattan Distance: 5
2. Linear Conflicts: 9
3. Statically: 11
4. Dynamically: 13
5. PhO:



# Post-hoc Optimization Heuristic

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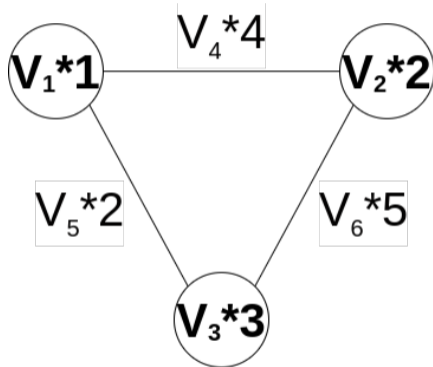
1. Manhattan Distance: 5
2. Linear Conflicts: 9
3. Statically: 11
4. Dynamically: 13
5. PhO:



# Post-hoc Optimization Heuristic

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1. Manhattan Distance: 5
2. Linear Conflicts: 9
3. Statically: 11
4. Dynamically: 13
5. PhO:



## Post-hoc Optimization Heuristic

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*Maximize  $V_1 * 1 + V_2 * 2 + V_3 * 3 + V_4 * 4 + V_5 * 2 + V_6 * 5$  subject to*

$$V_1 + V_4 + V_5 \leq 1$$

$$V_2 + V_4 + V_6 \leq 1$$

$$V_3 + V_5 + V_6 \leq 1$$

*$V_i \geq 0$  for all  $i \in \{1, 2, 3, 4, 5, 6\}$ .*

# Post-hoc Optimization Heuristic

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Linear program for 8-Puzzle with patterns up to the size of 3 variables:

92 Variables (Heuristics)

8 Constraints (Tiles)

29 Variables/Constraint

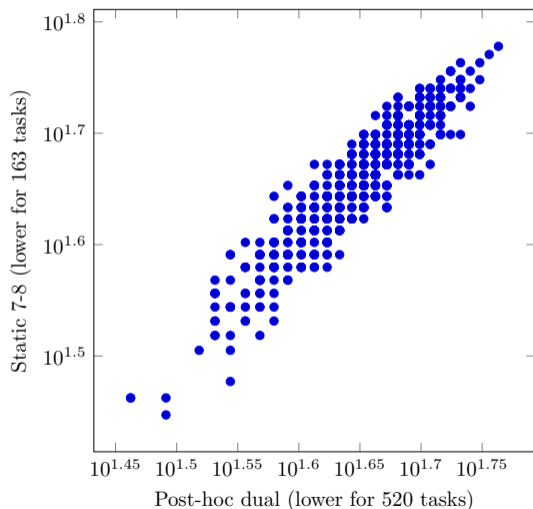
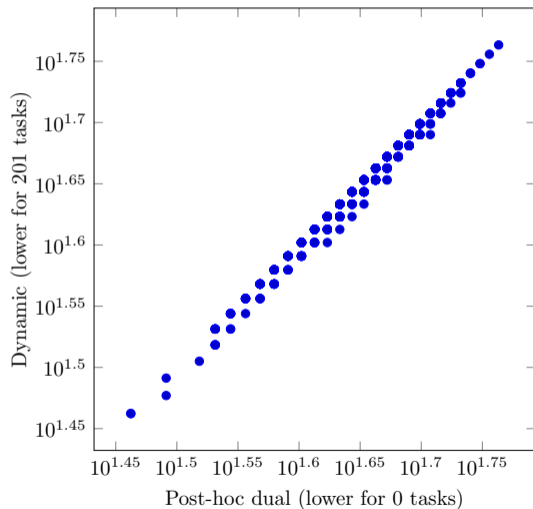
## Post-hoc Optimization Heuristic

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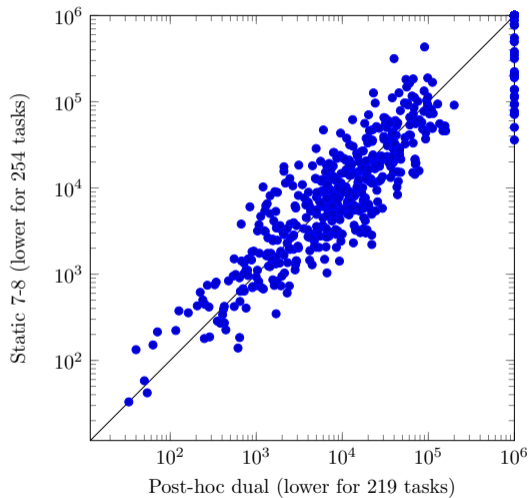
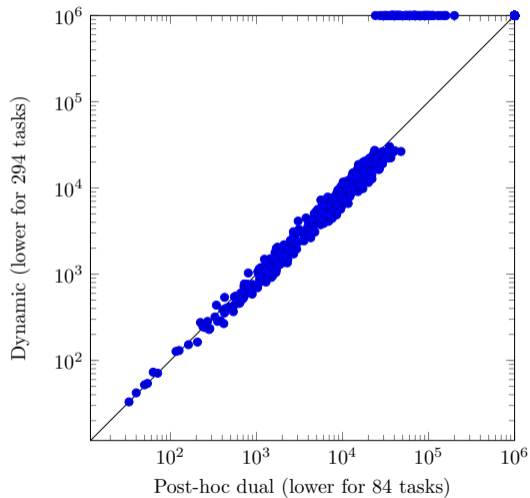
1. Manhattan Distance: 5
2. Linear Conflicts: 9
3. Statically: 11
4. Dynamically: 13
5. PhO: 15

1	0	2
3	5	4
7	6	8

# Experimental Evaluation: Initial Heuristic



# Experimental Evaluation: Expansion





Questions?

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