

Master's Thesis Presentation

Generalization of
Cycle-Covering Heuristics

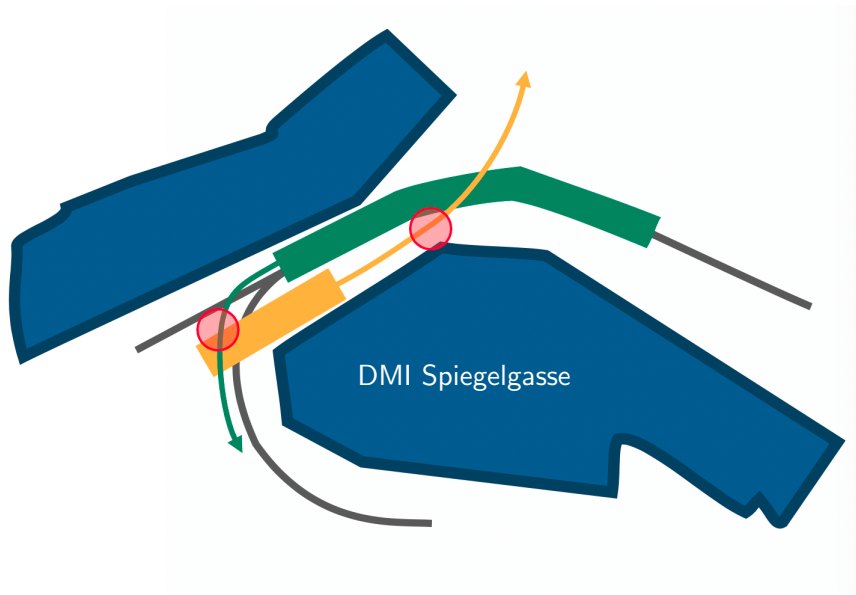
Clemens Büchner

Department of Mathematics and Computer Science
University of Basel

May 14, 2020



University
of Basel



Outline

1. Background
2. Cycle-covering heuristic
3. Experimental results

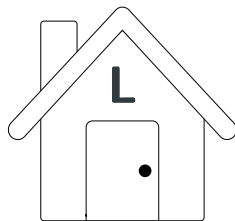
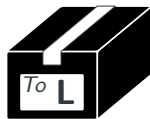
Background

Planning

Simplistic world model for specific problem purposes:

Definition (planning task)

A **planning task** is a 4-tuple $\mathcal{T} = \langle \mathcal{V}, s_0, \gamma, \mathcal{A} \rangle$ with



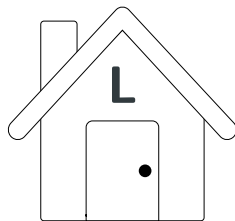
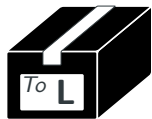
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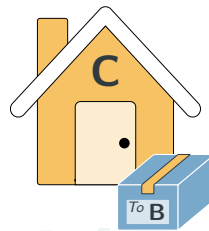
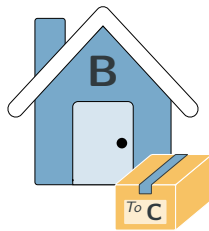
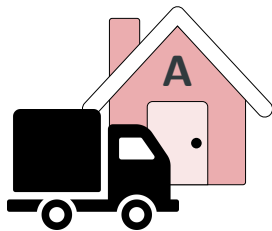
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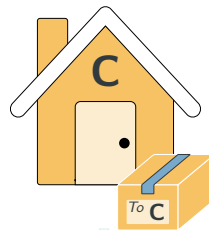
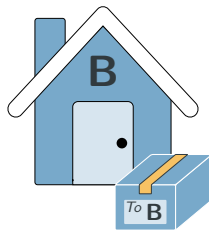
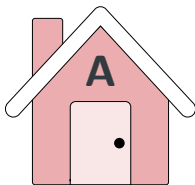
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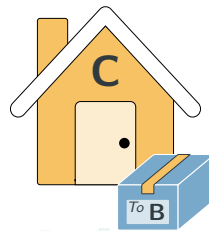
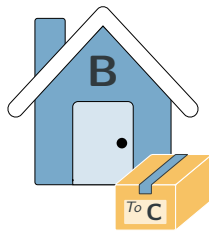
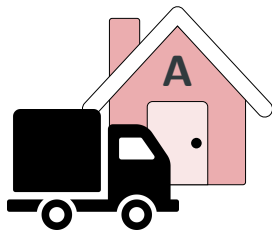
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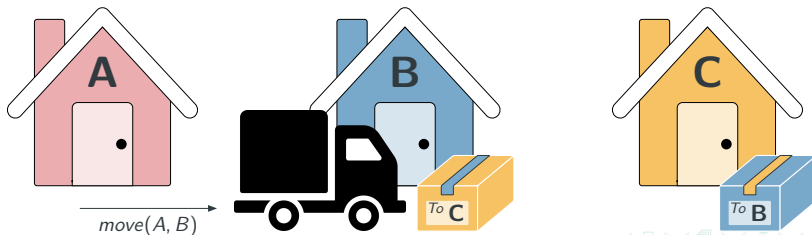
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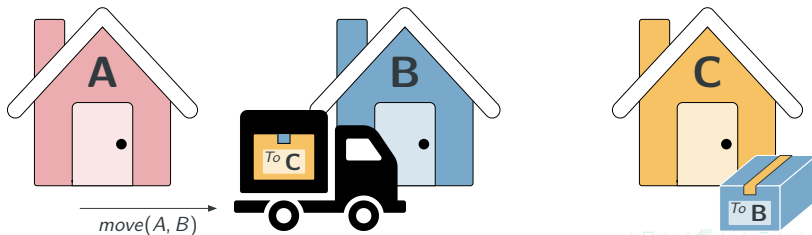
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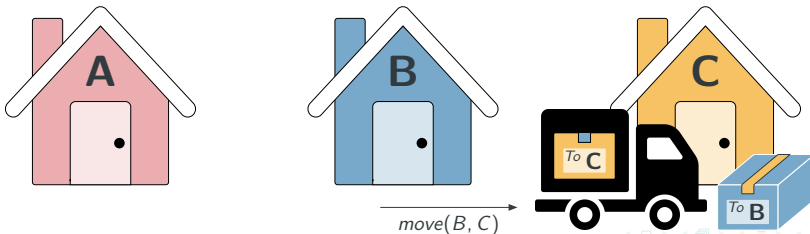
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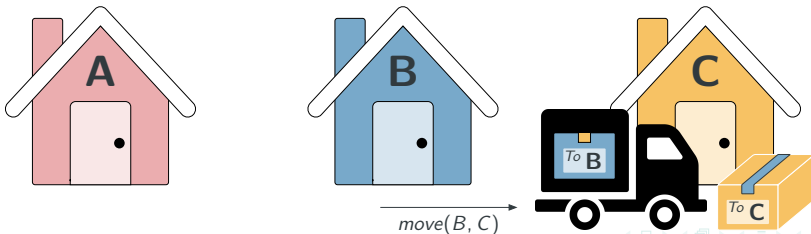
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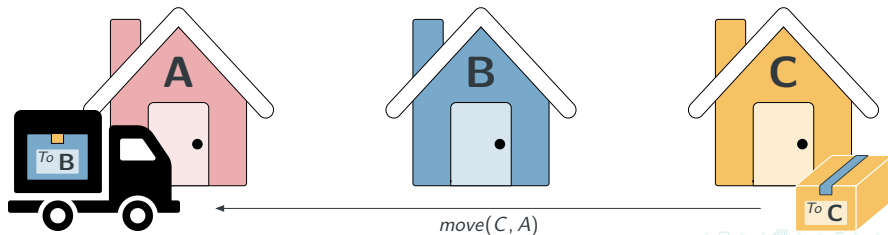
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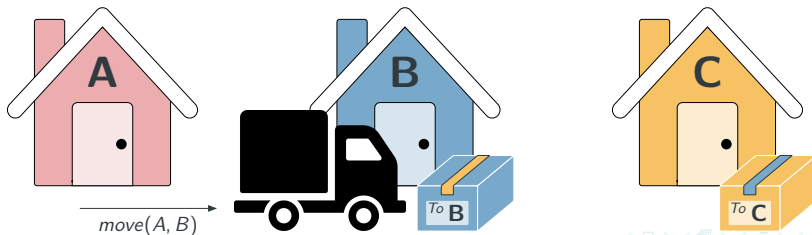
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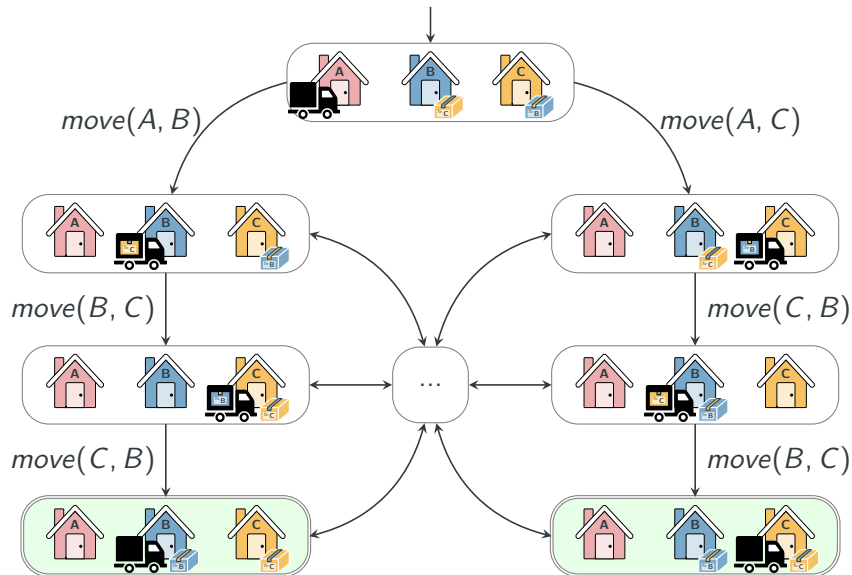
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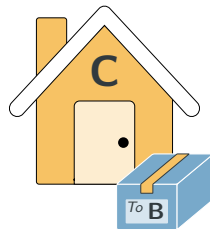
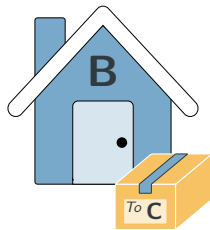
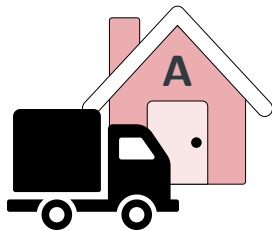
State Space and Heuristic Search



Landmarks

Properties that **must hold along all plans**:

- ▶ move to B to **pick up** yellow package
- ▶ move to C to **deliver** yellow package
- ▶ the blue package induces the same landmarks



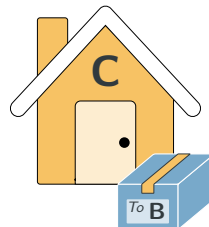
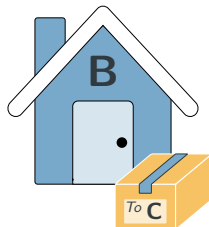
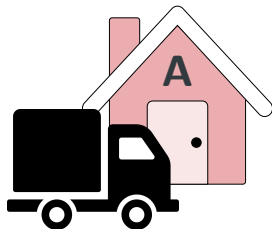
Landmarks

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Definition (disjunctive action landmark)

Let $\mathcal{T} = \langle \mathcal{V}, s_0, G, \mathcal{A} \rangle$ be a planning task and let s be a state of \mathcal{T} .

A **disjunctive action landmark** of s is a non-empty set of actions $\ell \subseteq \mathcal{A}$ such that every s -plan contains an action $a \in \ell$.



Landmarks

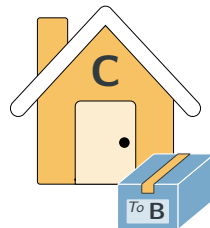
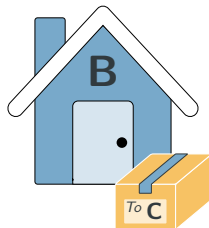
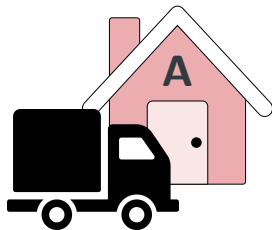
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Landmarks

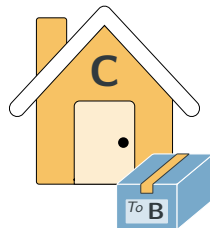
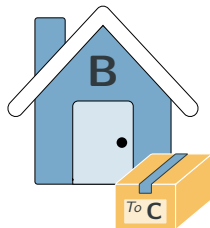
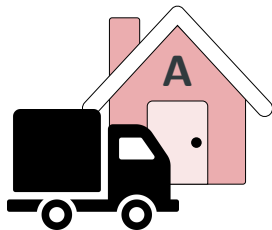
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- ▶ in example: $\{move(A, B), move(C, B)\}$ and $\{move(A, C), move(B, C)\}$
- ▶ **landmark generation** is not the topic of this thesis



Landmark Heuristic h^{LM}

- ▶ one action from each landmark must be **part of every plan**
- ▶ **minimum hitting set** approach
 - ▶ cheapest set of actions that **hits** each landmark
- ▶ solve with **linear programming**

$$\min \sum_{a \in \mathcal{A}} Y_a \cdot \text{cost}(a) \quad \text{s.t.}$$

$$Y_a \geq 0 \quad \text{for all } a \in \mathcal{A} \text{ and}$$

$$\sum_{a \in \ell} Y_a \geq 1 \quad \text{for all } \ell \in \mathcal{L}$$

- ▶ this corresponds to the **operator-counting framework**
- ▶ use **objective value** as heuristic estimate

Landmark Orderings

Landmark orderings denote **dependencies** between landmarks.

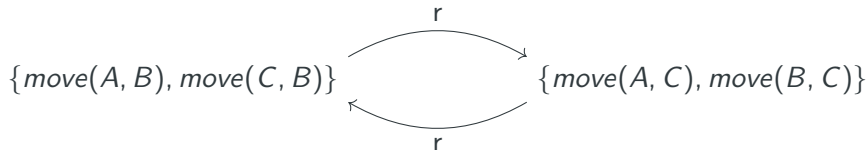
- ▶ **natural orderings** must hold along all plans
 - ▶ e.g., impossible to *unload* package before *loaded*
- ▶ **reasonable orderings** are rather “suggestions”
 - ▶ e.g., *move* to the package’s origin before its destination

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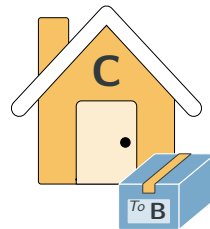
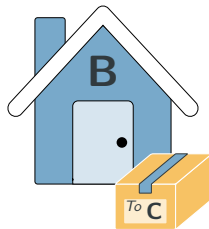
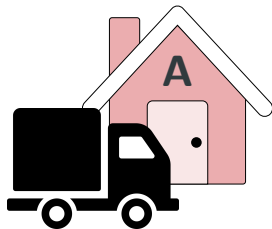
Represent landmarks and orderings in **landmark graphs**:



Cycle-Covering

Valuable Information in Landmark Graphs

- ▶ **cyclical dependencies** between landmarks
- ▶ sub-goal must be achieved multiple times to **resolve** cycle
- ▶ one landmark per cycle necessary **twice in every plan**
- ▶ again **minimum hitting set** problem for cycles



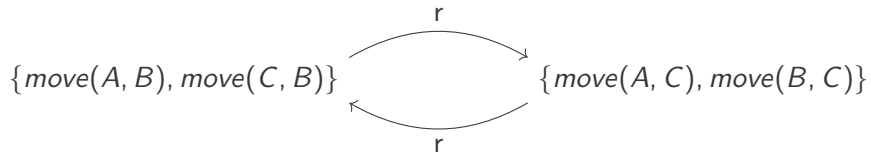
Cycle-Covering Heuristic h^{cycle}

Extending the landmark heuristic with cycle constraints:

$$\begin{aligned} \min \sum_{a \in \mathcal{A}} Y_a \cdot \text{cost}(a) \quad \text{s.t.} \\ Y_a \geq 0 \quad \text{for all } a \in \mathcal{A} \text{ and} \\ \sum_{a \in \ell} Y_a \geq 1 \quad \text{for all } \ell \in \mathcal{L} \text{ and} \\ \sum_{\ell \in \mathcal{C}} \sum_{a \in \ell} Y_a \geq |\mathcal{C}| + 1 \quad \text{for all } \mathcal{C} \in \mathcal{C} \end{aligned}$$

with \mathcal{C} the set of cycles in the landmark graph

Heuristics Applied to Running Example

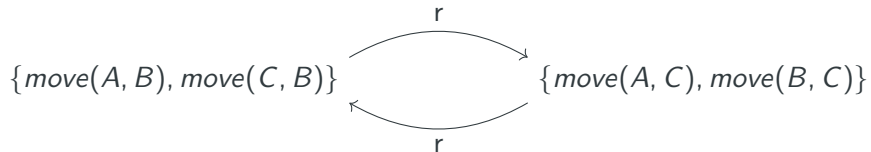


$$\min Y_{A \rightarrow B} + Y_{C \rightarrow B} + Y_{A \rightarrow C} + Y_{B \rightarrow C} \quad \text{s.t.}$$

$$Y_{A \rightarrow B} + Y_{C \rightarrow B} \geq 1$$

$$Y_{A \rightarrow C} + Y_{B \rightarrow C} \geq 1$$

Heuristics Applied to Running Example



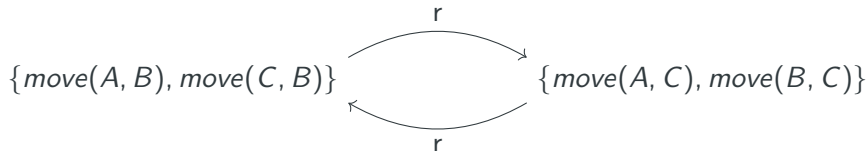
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► $h^{LM}(s_0) = 2 \quad (Y_{A \rightarrow B} = 1, Y_{B \rightarrow C} = 1)$

Heuristics Applied to Running Example



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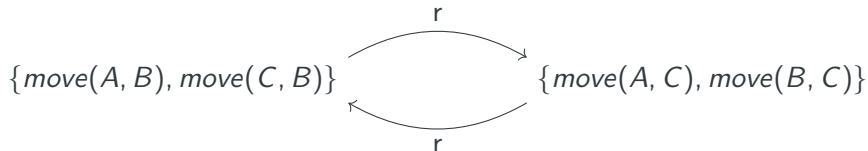
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Heuristics Applied to Running Example



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$$Y_{A \rightarrow B} + Y_{C \rightarrow B} + Y_{A \rightarrow C} + Y_{B \rightarrow C} \geq 3$$

▶ $h^{\text{LM}}(s_0) = 2$ ($Y_{A \rightarrow B} = 1, Y_{B \rightarrow C} = 1$)

▶ $h^{\text{cycle}}(s_0) = 3$ ($Y_{A \rightarrow B} = 1, Y_{B \rightarrow C} = 1, Y_{C \rightarrow B} = 1$)

Ordering-Aware Cycle-Covering Heuristic h^{ord}

- ▶ natural orderings are **acyclic** by definition
- ▶ candidates for resolving cycles must have an **incoming reasonable ordering**

$$\min Y_a + Y_b + Y_c \quad \text{s.t.}$$

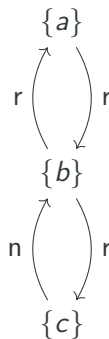
$$Y_a \geq 1$$

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$$Y_a + Y_b \geq 3$$

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- ▶ $h^{cycle}(s) = 4$ ($Y_a = 1, Y_b = 2, Y_c = 1$)

Ordering-Aware Cycle-Covering Heuristic h^{ord}

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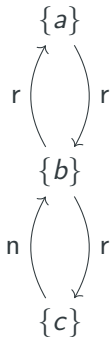
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$$Y_a + Y_b \geq 3$$

~~$$Y_b + Y_c \geq 2$$~~



▶ $h^{cycle}(s) = 4$ ($Y_a = 1, Y_b = 2, Y_c = 1$)

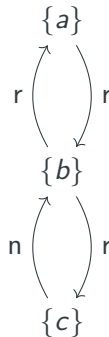
▶ $h^{ord}(s) = 5$ ($Y_a = 1, Y_b = 2, Y_c = 2$)

Ordering-Aware Cycle-Covering Heuristic h^{ord}

- ▶ natural orderings are **acyclic** by definition
- ▶ candidates for resolving cycles must have an **incoming reasonable ordering**

Ordering-aware cycle-covering heuristic:

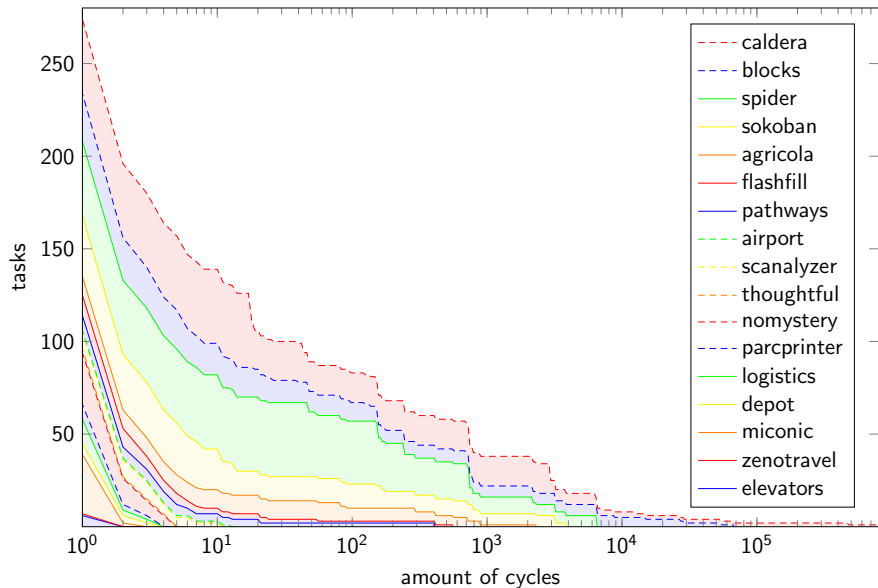
$$\begin{aligned} \min \sum_{a \in \mathcal{A}} Y_a \cdot \text{cost}(a) \quad \text{s.t.} \\ Y_a \geq 0 \quad \text{for all } a \in \mathcal{A} \text{ and} \\ \sum_{a \in \ell} Y_a \geq 1 \quad \text{for all } \ell \in \mathcal{L} \text{ and} \\ \sum_{\ell \in c_r} \sum_{a \in \ell} Y_a \geq |c_r| + 1 \quad \text{for all } c \in \mathcal{C} \end{aligned}$$

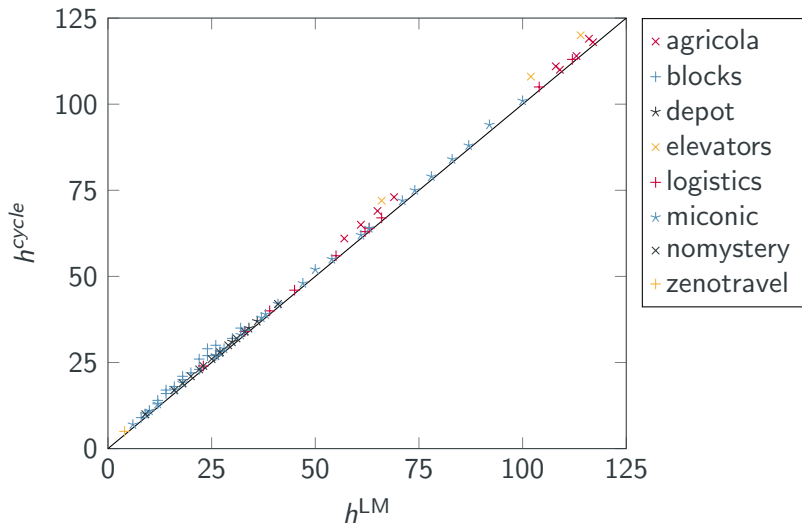


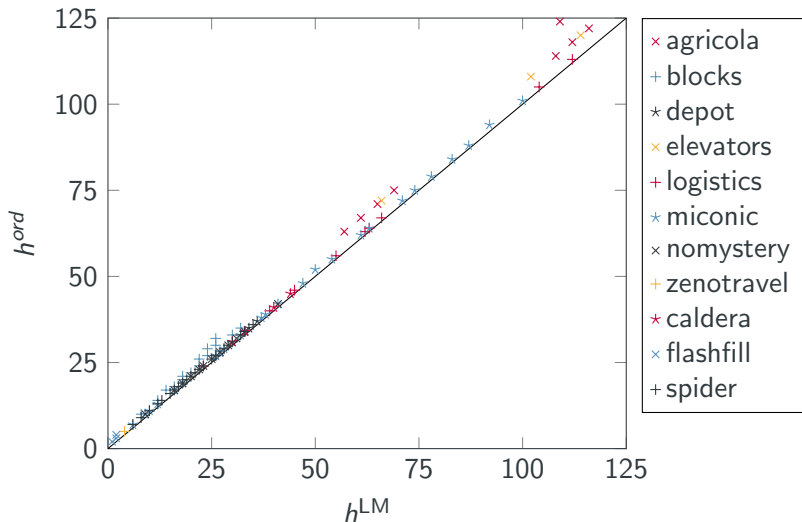
with $c_r \subseteq c$ the set of landmarks with incoming reasonable orderings

Experimental Evaluation

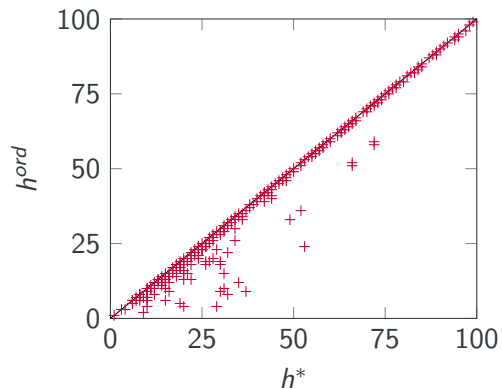
Cycles in Landmark Graphs



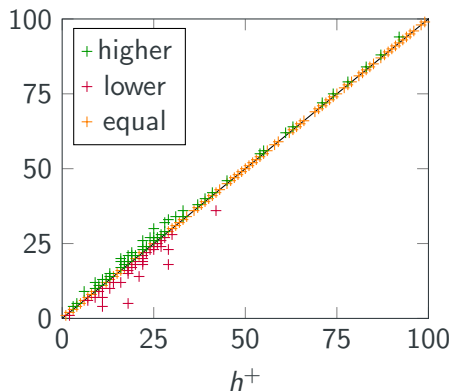
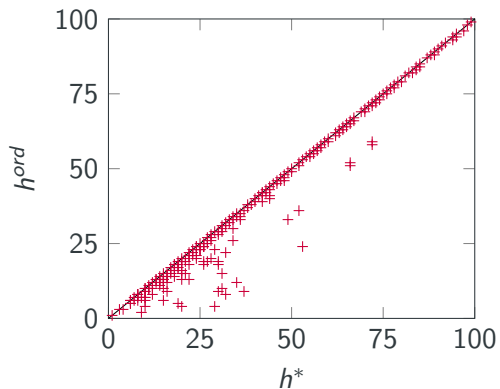
Initial h -Value – h^{LM} vs. h^{cycle} 

Initial h -Value – h^{LM} vs. h^{ord} 

Aiming for Optimality



Aiming for Optimality



Planning with the Cycle-Covering Heuristic

Coverage **barely affected**

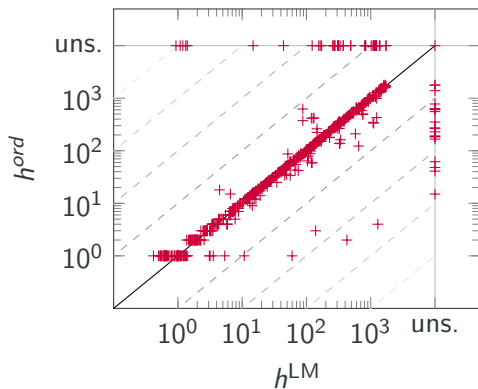
- ▶ tasks with many cycles prone to exceed **memory limit**
- ▶ **increased complexity** of optimization problems

Overall results are **inconclusive** and vary depending on several factors:

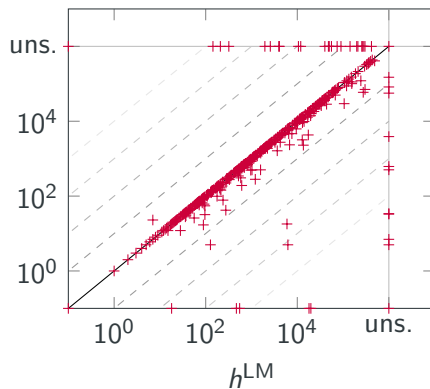
- ▶ Which **landmark generator** is used?
 - ▶ various options in Fast Downward
- ▶ How to **update landmarks** in encountered states?
 - ▶ **recomputing** vs. **tracking** based on previous state

Planning with the Cycle-Covering Heuristic

Search Time



Expanded States



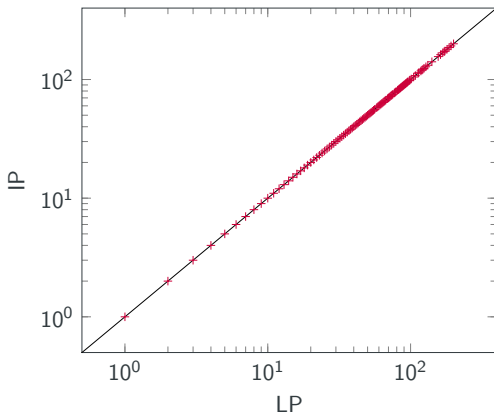
Summary

- ▶ **cyclical dependencies** between landmarks contain **valuable information**
- ▶ **cycle-covering heuristic** dominates **minimum hitting set** landmark heuristic for the same landmark graph
- ▶ considering **ordering types** improves cycle-covering heuristic
- ▶ increased **heuristic accuracy** in practice
- ▶ does not (yet) pay off in **coverage**

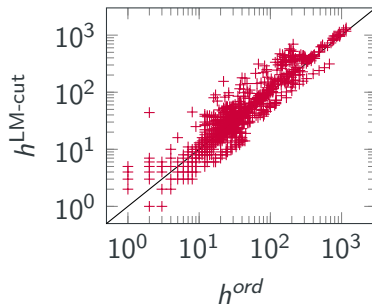
Appendix

Initial h -Values – LP vs. IP

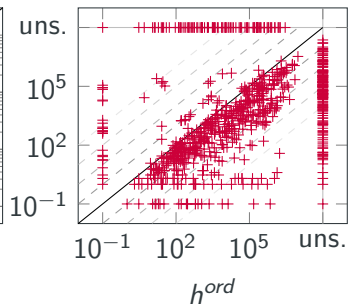
- ▶ IP solutions **identical** to LP-relaxation
- ▶ possible explanation: **totally unimodular** matrices
 - ▶ all squared sub-matrices have $\det \in \{-1, 0, 1\}$
 - ▶ LP solutions are integral
- ▶ but **not generally** the case
 - ▶ Counterexample with **LP \neq IP**



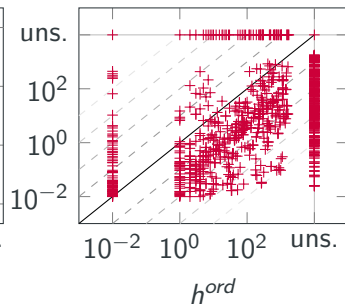
Comparison to LM-Cut

Initial h -Value

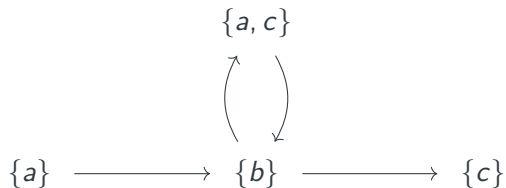
Expanded States



Search Time

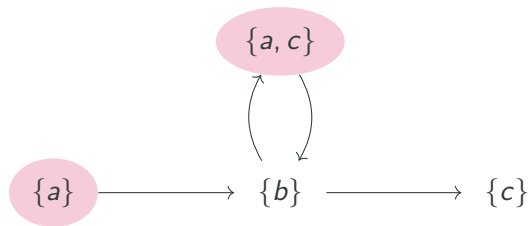


Decomposing Cycle-Covering from Landmark Hitting Set



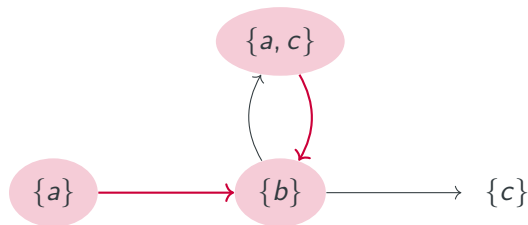
- ▶ Minimum landmark hitting set: 3, minimum cycle hitting set: 1 $\Rightarrow h = 4$

Decomposing Cycle-Covering from Landmark Hitting Set



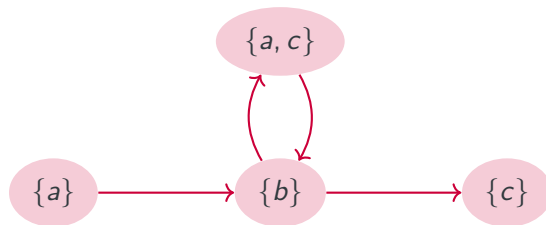
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- ▶ Optimal plan: $\langle a, b, c \rangle \Rightarrow h^* = 3$

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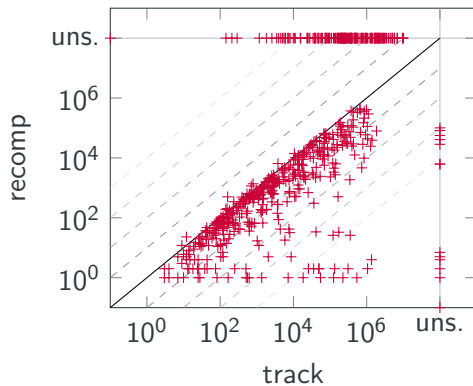
Recomputing vs. Tracking Landmarks

Update landmarks in every state:

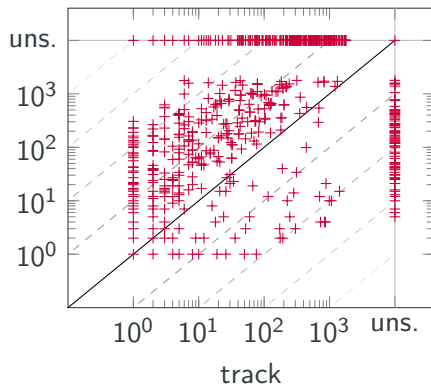
- ▶ always **recompute**
 - + potentially find **new** landmarks
 - might be **time-consuming**
- ▶ or **track** along paths
 - + compute landmarks **only once** in the beginning
 - applying actions can only **decrease heuristic** estimates
 - ± heuristic is **path-dependent**

Recomputing vs. Tracking Landmarks

Expanded States



Search Time



Coverage Results

- ▶ 870 planning tasks from domains with cyclical initial states

	LM ^{RHW}		LM ^{h^m}	
	recomp	track	recomp	track
h^{LM}	342	308	222	298
h^{cycle}	336	305	228	305
h^{ord}	340	306	231	308

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- ▶ success of recomputing vs. tracking depends on **landmark generator**
- ▶ considering cycles is not always beneficial
 - ▶ **memory** is a limitation
 - ▶ optimization problems have **increased complexity**