University of Basel

Operator-counting Constraints for Implicit Abstractions

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Introduction

Implicit Abstractions

Constraints for Forward Forks

Constraints for Inverted Forks

Results

Classical Planning

Definition (Planning Task)

- Variables $v \in V$ that can each take a value in $\text{dom}(v)$
- States $s \in S$ assign variables to a value
- Operators $o \in O$ transition between states
  - preconditions
  - effects
  - a cost

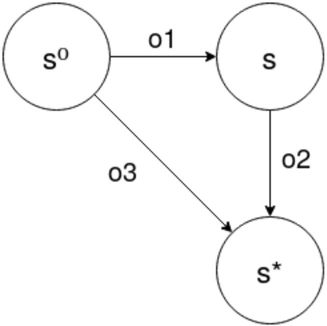
Goal: Find plans from the initial state s^0 to a goal state s^*

Optimal Planning

$$cost(o_1) = cost(o_2) = cost(o_3) = 1$$

$$\pi_1 = s^0 \xrightarrow{o_1} s \xrightarrow{o_2} s^* \qquad cost(\pi_1) = 2$$

$$\pi_2 = s^0 \xrightarrow{o_3} s^* \qquad cost(\pi_2) = 1$$



To find an **optimal** plan we can use A* search with any admissible heuristic h.

An **admissible heuristic** is an underestimation of the true goal distance.

Abstractions

Definition (Abstraction)

Function that abstracts the state space.

- $\alpha : S \rightarrow S_\alpha$
- s^0 remains the initial state in the abstraction
- s^* remains a goal state in the abstraction

The **abstraction heuristic** h^α is the true goal distance in the abstract state space S_α.

h^α is **admissible** if α does not increase the goal distance for any state.

Explicit Abstractions

How do we ensure admissibility?

Most well-known abstractions

- preserve the transitions of the original planning task,
- search **explicitly** for optimal plans in the abstract space.

Problem: Abstract state space must be bounded!

Implicit abstractions decompose the planning task until it is tractable to compute.

Implicit Abstractions

Instead of reflecting a few state variables perfectly,

- create an abstraction around one variable,
- combine those abstractions to reflect many variables.

We ensure admissibility by preserving the cost between two states instead of preserving the transitions:

$$cost(\alpha(s), \alpha(s')) \leq cost(s, s')$$

We want to be able to search the abstract space **implicitly** in polynomial time.

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Example

- $V = \{a, b, c, d\}$ with $\text{dom}(v) = \{0, 1\}$
- $O = \{o_1, o_2\}$
  - $o_1 = \langle \{b = 0\}, \{a = 1, b = 1\} \rangle$ with $cost(o_1) = 1$
  - $o_2 = \langle \{b = 1, d = 0\}, \{c = 1\} \rangle$ with $cost(o_2) = 1$
- $s^0 = \{a = 0, b = 0, c = 0, d = 0\}$
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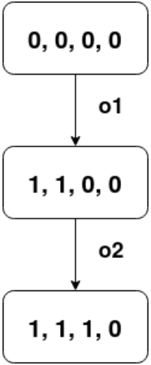

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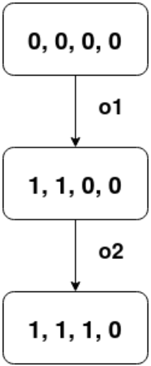

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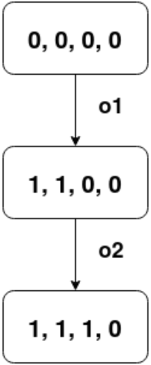

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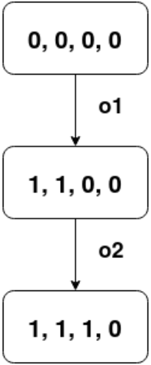

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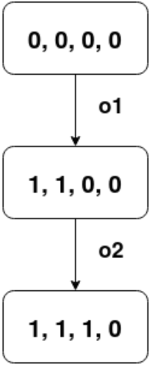

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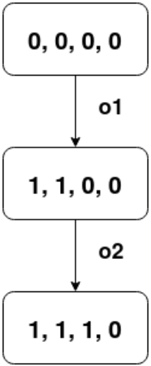


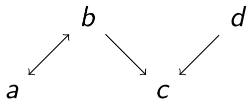
Causal Graph

Definition (Causal Graph)

- Nodes over the variables V
- Edges $\langle v, v' \rangle$ if an operator o
  - has a precondition or effect on v
  - has an effect on v'

$$o_1 = \langle \{b = 0\}, \{a = 1, b = 1\} \rangle$$

$$o_2 = \langle \{b = 1, d = 0\}, \{c = 1\} \rangle$$



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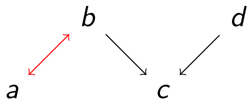

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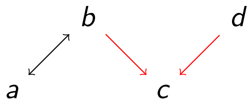

Forward Forks

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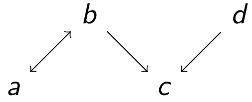
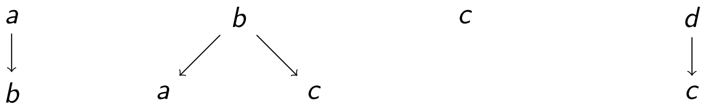


$CG(\Pi)$



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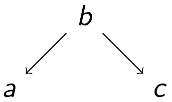

$CG(\Pi_b^{\mathrm{f}})$

Inverted Forks

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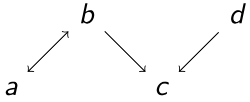

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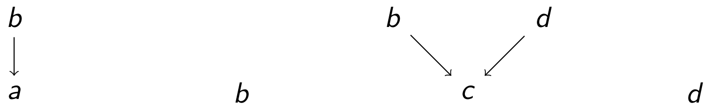

Inverted Forks

$$o_1 = \langle \{b = 0\}, \{a = 1, b = 1\} \rangle$$

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$O^{\mathrm{i}}_c = \{o^b_1, o^c_2\}$ with

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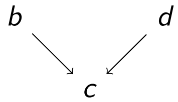

$CG(\Pi^{\mathrm{i}}_c)$

Tractability

Fork abstractions can be implicitly searched in polynomial time if

- for forward forks: $\mathrm{dom}(r) = \{0, 1\}$,
- for inverted forks: $|\mathrm{dom}(r)| = \mathcal{O}(1)$.

Compositions of Fork Abstraction Heuristics

We can **admissibly** combine the fork abstractions obtained for each variable by

- using an optimal cost partitioning,
- using operator-counting constraints.

Optimal Cost Partitioning Constraints

The optimal cost partitioning heuristic can be obtained by a linear program (LP):

$$\text{Maximize } \sum_{i=1}^{m} h_i(\alpha_i(s)) \text{ subject to } C(s)$$

Where the cost is distributed among all unary-effect operators of all fork abstractions.

We can use those constraints to derive operator-counting constraints.

Operator-counting Constraints

The operator-counting heuristic can be obtained by a linear program (LP):

$$\text{Minimize} \sum_{o \in O} cost(o) \cdot \mathrm{Y}_o \text{ subject to } C(s)$$

Where Y_o denotes how often the operator o is used in a plan.

Combines different LP based heuristics by combining their constraints.

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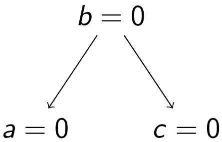
$\sigma(r) = \langle 0, 1, 0, \ldots \rangle$



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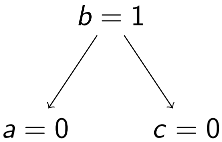

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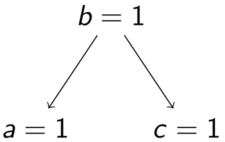

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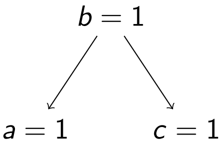
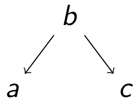


No unary root-effect operator to change *b* back to 0.

Notation

1. Y_{o^v} denotes how often the unary-effect operator o^v is used to change the value of a leaf variable v.
2. $Y_l(v, \theta, \theta')$ denotes how often v is changed from θ to θ' at root sequence step l.
3. Y_{σ_l} denotes how often a partial root sequence σ of length l is taken.

$$\sigma(r) = \langle 0, 1 \rangle$$



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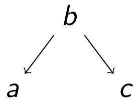

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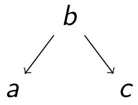
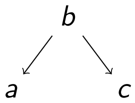


Operator Count Inequalities

$$Y_{o_1} \geq Y_{\sigma_2}$$
$$Y_{o_1} \geq Y_{o_1^a}$$
$$Y_{o_2} \geq Y_{o_2^c}$$



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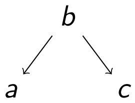

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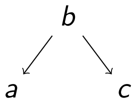

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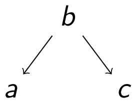
Path Inequalities

$$\mathrm{Y}_{o_1^a} \geq \mathrm{Y}_{l=2}(a, 0, 1)$$

$$\mathrm{Y}_{o_2^c} \geq \mathrm{Y}_{l=2}(c, 0, 1)$$

$$\sigma(r) = \langle 0, 1 \rangle$$



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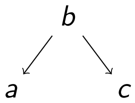

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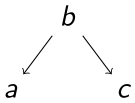
Goal Inequality

$$\mathrm{Y}_{\sigma_1} + \mathrm{Y}_{\sigma_2} \geq 1$$

- $\sigma_1 = \langle 0 \rangle$
- $\sigma_2 = \langle 0, 1 \rangle$



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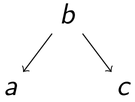
Root-sequence Flow Inequalities

$$
\begin{aligned}
Y_{l=1}(a,0,0) - Y_{l=2}(a,0,0) - Y_{l=2}(a,0,1) &\geq 0 \\
Y_{l=1}(a,0,1) - Y_{l=2}(a,1,0) - Y_{l=2}(a,1,1) &\geq Y_{\sigma_1} \\
Y_{l=2}(a,0,0) + Y_{l=2}(a,1,0) &\geq 0 \\
Y_{l=2}(a,0,1) + Y_{l=2}(a,1,1) &\geq Y_{\sigma_2}
\end{aligned}
$$

Similar for $v = c$.

$\sigma(r) = \langle 0, 1 \rangle$



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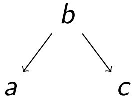
Y_{l=1}(a,0,0) - Y_{l=2}(a,0,0) - Y_{l=2}(a,0,1) &\geq 0 \\
Y_{l=1}(a,0,1) - Y_{l=2}(a,1,0) - Y_{l=2}(a,1,1) &\geq Y_{\sigma_1} \\
Y_{l=2}(a,0,0) + Y_{l=2}(a,1,0) &\geq 0 \\
Y_{l=2}(a,0,1) + Y_{l=2}(a,1,1) &\geq Y_{\sigma_2}
\end{aligned}
$$

$\sigma(r) = \langle 0, 1 \rangle$



- $\sigma_1 = \langle 0 \rangle$
- $\sigma_2 = \langle 0, 1 \rangle$

Similar for $v = c$.

Root-sequence Flow Inequalities

$$\sigma(r) = \langle 0, 1 \rangle$$

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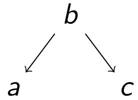

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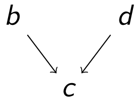
Constraints for Forward Forks

Constraints for Inverted Forks

Results

Notation

1. Y_{o^v} denotes how often a unary-effect operator o^v is used to change the value of a parent variable v.
2. $Y_{\pi^c_l}$ denotes how often a particular plan for the sink π^c of length l is taken.



$$\pi^c = \alpha_c(s)[c] \rightarrow \cdots \rightarrow G^{\mathrm{i}}_c[c]$$

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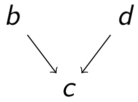

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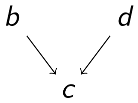
Operator Count Inequalities

$$\mathrm{Y}_{o_1} \geq \mathrm{Y}_{o_1^b}$$

$$\mathrm{Y}_{o_2} \geq \mathrm{Y}_{\pi_1^c}$$

$$\pi_1^c = \{c = 0\} \xrightarrow{o_2^c} \{c = 1\}$$



$O_c^{\mathrm{i}} = \{o_1^b, o_2^c\}$ with

- $o_1^b = \langle \{b = 0\}, \{b = 1\} \rangle$,
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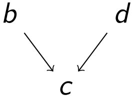

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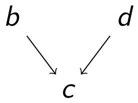

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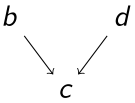

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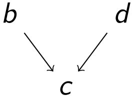
Goal Inequality

$$Y_{\pi_1^c} \geq 1$$

$$\pi_1^c = \{c = 0\} \xrightarrow{o_2^c} \{c = 1\}$$



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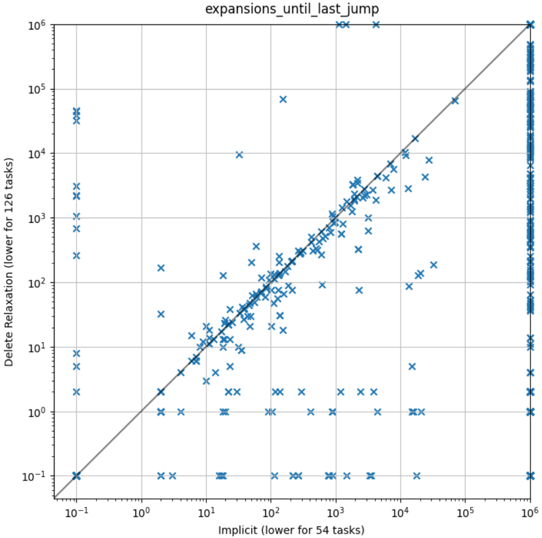
Coverage

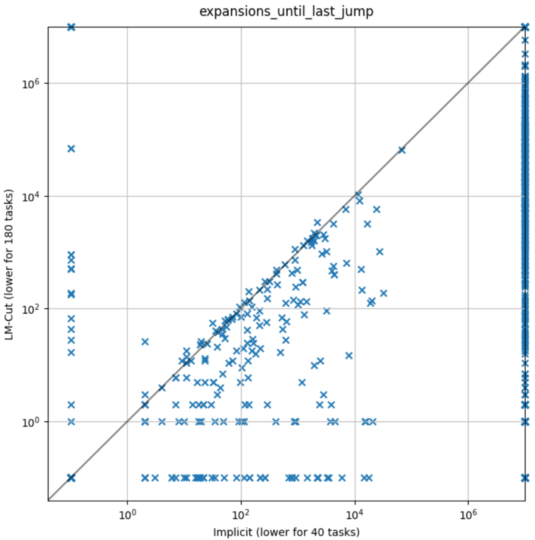
	Success	Out-of-Memory	Out-of-Time
Implicit	281	637	**894**
Delete Relaxation	577	207	1027
LM-Cut	**909**	**0**	901
Post-Hoc	748	2	1058
State Equation	770	**0**	1041

Coverage comparison of 1827 planning tasks. **Implicit** denotes the operator-counting heuristic for forward fork abstractions.

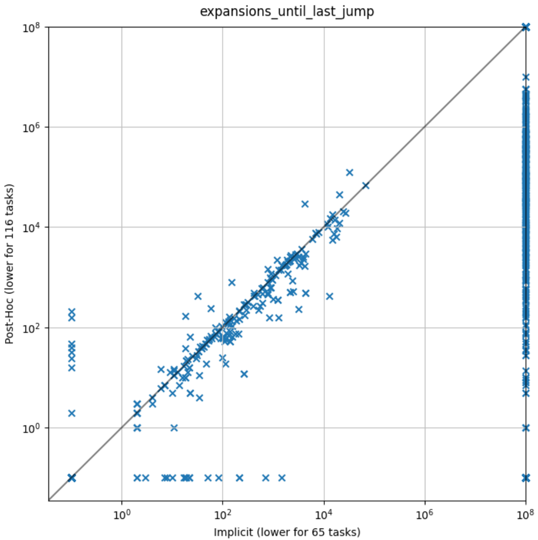
Implicit vs Delete Relaxation



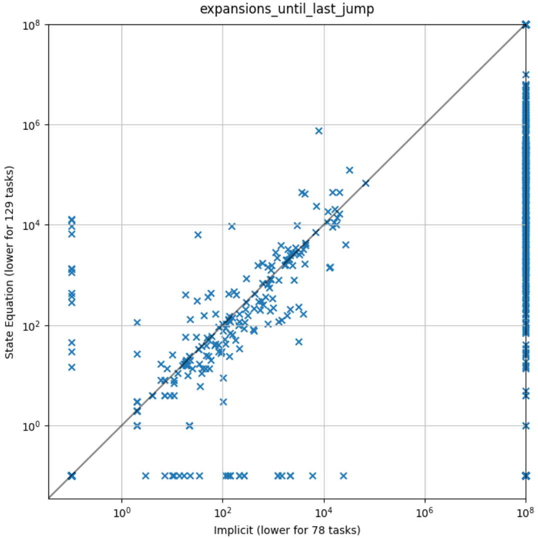
Implicit vs LM-Cut



Implicit vs Post-Hoc



Implicit vs State Equation



Conclusion

- Implicit abstractions tractably decompose large state spaces
- We derived operator-counting constraints from cost-partitioning for forks
- Forward fork constraints are too expensive in practice

- Future work: practicality of inverted fork constraints

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Questions?

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Extension Results

	Success	Out-of-Memory	Out-of-Time
Implicit	281	637	894
Implicit-General	274	650	**888**
Delete Relaxation	577	207	1027
LM-Cut	**909**	**0**	901
Post-Hoc	748	2	1058
State Equation	770	**0**	1041

Coverage comparison of all 1827 planning tasks. The winner of each category is highlighted in bold. We note that the reason for **Implicit-General** having the lowest out-of-time error is due to it running out of memory for those tasks before running out of time. We omitted 56619 out of 173555 fork abstractions as they did not contain any goal variables.

Extension Results

	Implicit	Delete Relaxation	Combined
Implicit	–	270	0
Delete Relaxation	**493**	–	0
Combined	**172**	**411**	–

	Implicit	State Equation	Combined
Implicit	–	396	0
State Equation	**570**	–	0
Combined	**714**	**619**	–

Comparison of the initial h-value. We compare the row heuristic to the column heuristic and denote in each cell for how many tasks it yields a higher value in the initial state. The winner of each pairwise comparison is highlighted in bold.

Extension Results

	Implicit	LM-Cut	Combined
Implicit	–	135	0
LM-Cut	**903**	–	0
Combined	**958**	**208**	–

	Implicit	Post-Hoc	Combined
Implicit	–	313	0
Post-Hoc	**486**	–	0
Combined	**525**	**484**	–

Comparison of the initial *h*-value. We compare the row heuristic to the column heuristic and denote in each cell for how many tasks it yields a higher value in the initial state. The winner of each pairwise comparison is highlighted in bold.

Forward Fork Constraint: Operator Count Inequalities

For the unary-effect operator $o^r \in O^{\mathrm{f}}_r[r](o)$ and $o^v \in O^{\mathrm{f}}_r[v](o)$ for all $v \in V^{\mathrm{f}}_r$, where $\mathrm{Pre}(o^v)[r]$ is the set of root values for which o^v can be applied. For each $o \in O$:

$$\mathrm{Y}_o \geq \begin{cases} \sum\limits_{\sigma^*_l \in \sigma(r)} \sum\limits_{\substack{o' \in O^{\mathrm{f}}_r[r] \\ \mathrm{eff}(o')[r] = 1 - \mathrm{eff}(o^r)[r]}} \left\lceil \frac{l-1}{2} \right\rceil \cdot \mathrm{Y}^i_{\sigma^*_l}(o^r, o') & \text{if } \mathrm{eff}(o^r)[r] \neq s_i[r] \\ \sum\limits_{\sigma^*_l \in \sigma(r)} \sum\limits_{\substack{o' \in O^{\mathrm{f}}_r[r] \\ \mathrm{eff}(o')[r] = 1 - \mathrm{eff}(o^r)[r]}} \left\lfloor \frac{l-1}{2} \right\rfloor \cdot \mathrm{Y}^i_{\sigma^*_l}(o', o^r) & \text{if } \mathrm{eff}(o^r)[r] = s_i[r] \end{cases},$$

$$\mathrm{Y}_o \geq \sum_{\substack{\theta \in \mathrm{dom}(v) \\ \theta \neq \mathrm{eff}(o^v)[r]}} \sum_{\theta_r \in \mathrm{Pre}(o^v)[r]} \mathrm{Y}^i_{\theta_r}(v, \theta, \mathrm{eff}(o^v)[r], o^v).$$

Cheapest Fixed-root Path Inequalities

For all goal variables $v \in V^{\mathrm{f}}_r \setminus \{r\}$, each $\theta, \theta' \in \mathrm{dom}(v)$, and $\theta_r \in \{0, 1\}$. Let $l \geq 1$ if $s_i[v] = \theta$, and $l \geq 2$ otherwise:

For $\theta = \theta'$, we have:

$$Y^i_{\theta_r}(v, \theta, \theta, \square) \geq \sum_{\substack{l \leq |\sigma(r)| \\ \sigma(r)[l] = \theta_r}} Y^i_l(v, \theta, \theta) + \sum_{\substack{o' \in O^{\mathrm{f}}_r[v] \\ \mathrm{pre}(o')[v] = \theta \\ \theta \neq \mathrm{eff}(o')[v] \\ \theta_r \in \mathrm{Pre}(o')[r]}} Y^i_{\theta_r}(v, \theta, \mathrm{eff}(o')[v], o')$$

For $\theta \neq \theta'$, we have:

$$\sum_{\substack{o \in O^{\mathrm{f}}_r[v] \\ \mathrm{eff}(o)[v] = \theta' \\ \theta_r \in \mathrm{Pre}(o)[r]}} Y^i_{\theta_r}(v, \theta, \theta', o) \geq \sum_{\substack{l \leq |\sigma(r)| \\ \sigma(r)[l] = \theta_r}} Y^i_l(v, \theta, \theta') + \sum_{\substack{o' \in O^{\mathrm{f}}_r[v] \\ \mathrm{pre}(o')[v] = \theta' \\ \theta \neq \mathrm{eff}(o')[v] \\ \theta_r \in \mathrm{Pre}(o')[r]}} Y^i_{\theta_r}(v, \theta, \mathrm{eff}(o')[v], o')$$

Root-sequence-induced-distance Flow Inequalities

For all goal variables $v \in V_r^{\mathrm{f}} \setminus \{r\}$, each $\theta' \in \mathrm{dom}(v)$, and $1 \leq l \leq |\sigma(r)|$:
For $l = 1$, we have:

$$\mathrm{Y}_1^i(v, s_i[v], \theta') - \sum_{\theta \in \mathrm{dom}(v)} \mathrm{Y}_2^i(v, \theta', \theta) \geq \begin{cases} \sum_{\sigma_1^*(o,o')} \mathrm{Y}_{\sigma_1^*}^i(o, o') & \text{if } \theta' = G_r^{\mathrm{f}}[v] \\ 0 & \text{otherwise} \end{cases}$$

For $l \geq 2$, we have:

$$\sum_{\theta \in \mathrm{dom}(v)} \mathrm{Y}_l^i(v, \theta, \theta') - \sum_{\theta'' \in \mathrm{dom}(v)} \mathrm{Y}_{l+1}^i(v, \theta', \theta'') \geq \begin{cases} \sum_{\sigma_l^*(o,o')} \mathrm{Y}_{\sigma_l^*}^i(o, o') & \text{if } \theta' = G_r^{\mathrm{f}}[v] \\ 0 & \text{otherwise} \end{cases}$$

Goal Inequality

A goal inequality:

$$\sum_{\sigma_l^* \in \sigma(r)} \sum_{\substack{o \in O_r^{\mathrm{f}}[r] \\ \mathrm{eff}(o)[r] = 1 - s_i[r]}} \sum_{\substack{o' \in O_r^{\mathrm{f}}[r] \\ \mathrm{eff}(o')[r] = s_i[r]}} \mathrm{Y}^i_{\sigma_l^*}(o, o') \geq 1$$

Inverted Fork Constraint: Operator Count Inequalities

For the unary-effect operator $o^v \in O^{\mathrm{i}}_r[v](o)$ for all $v \in V^{\mathrm{i}}_r$ and $o^r \in O^{\mathrm{i}}_r[r](o)$. For each operator $o \in O$:

$$\mathsf{Y}_o \geq \sum_{\substack{\theta \in \mathrm{dom}(v) \\ \theta \neq \mathrm{eff}(o^v)[v]}} \mathsf{Y}^i(v, \theta, \mathrm{eff}(o^v)[v], o^v),$$

$$\vdots$$

$$\mathsf{Y}_o \geq \sum_{\pi^*_m \in \mathcal{P}(r)} Y^{\pi^*_m}_{o^r} \cdot \mathsf{Y}^i_{\pi^*_m},$$

where $Y^{\pi^*_m}_{o^r}$ denotes the number of occurrences of o^r in π^*_m.

Inverted Fork Constraint: Cheapest Path Inequalities

For all parent variables $v \in V_r^{\mathrm{i}} \setminus \{r\}$ and each $\theta, \theta' \in \mathrm{dom}(v)$:
For $\theta = \theta'$, we have:

$$\mathrm{Y}^i(v, \theta, \theta, \Box) \geq \sum_{\pi_m^* \in \mathcal{P}(r)} \sum_{\substack{0 \leq j \leq m \\ p_j[v] = \theta \\ p_{j+1}[v] = \theta'}} \mathrm{Y}^i_{\pi_m^*} + \sum_{\substack{o' \in O_r^{\mathrm{i}}[v] \\ \mathrm{pre}(o')[v] = \theta \\ \theta \neq \mathrm{eff}(o')[v]}} \mathrm{Y}^i(v, \theta, \mathrm{eff}(o')[v], o')$$

For $\theta \neq \theta'$, we have:

$$\sum_{\substack{o \in O_r^{\mathrm{i}}[v] \\ \mathrm{eff}(o)[v] = \theta'}} \mathrm{Y}^i(v, \theta, \theta', o) \geq \sum_{\pi_m^* \in \mathcal{P}(r)} \sum_{\substack{0 \leq j \leq m \\ p_j[v] = \theta \\ p_{j+1}[v] = \theta'}} \mathrm{Y}^i_{\pi_m^*} + \sum_{\substack{o' \in O_r^{\mathrm{i}}[v] \\ \mathrm{pre}(o')[v] = \theta' \\ \theta \neq \mathrm{eff}(o')[v]}} \mathrm{Y}^i(v, \theta, \mathrm{eff}(o')[v], o')$$

Inverted Fork Constraint: Goal Inequality

A goal inequality:

$$\sum_{\pi^*_m \in \mathcal{P}(r)} \mathrm{Y}^i_{\pi^*_m} \geq 1$$

Forward Forks

$$o_1 = \langle \{b = 0\}, \{a = 1, b = 1\} \rangle$$

$$o_2 = \langle \{b = 1, d = 0\}, \{c = 1\} \rangle$$

$O^{\mathrm{f}}_a = \{o^a_1, o^b_1\}$ with

- $o^a_1 = \langle \{\ \}, \{a = 1\} \rangle,$
- $o^b_1 = \langle \{a = 1, b = 0\}, \{b = 1\} \rangle.$

$CG(\Pi^{\mathrm{f}}_a)$

Forward Forks

$$o_1 = \langle \{b = 0\}, \{a = 1, b = 1\} \rangle$$

$$o_2 = \langle \{b = 1, d = 0\}, \{c = 1\} \rangle$$

$O^{\mathrm{f}}_d = \{o^c_2\}$ with

- $o^c_2 = \langle \{d = 0\}, \{c = 1\} \rangle$.

$CG(\Pi^{\mathrm{f}}_d)$

Inverted Forks

$$o_1 = \langle \{b = 0\}, \{a = 1, b = 1\} \rangle$$

$$o_2 = \langle \{b = 1, d = 0\}, \{c = 1\} \rangle$$

$O^{\mathrm{i}}_a = \{o_1^b, o_1^a\}$ with

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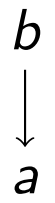
- $o_1^a = \langle \{b = 1\}, \{a = 1\} \rangle$.



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