Operator-Counting Heuristics for Domain-Independent Dynamic Programming: Supplementary Material

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1 DyPDL Expression and their SMT encoding

In order to leverage satisfiability modulo theories (SMT) solvers for reasoning over DyPDL models, it is necessary to establish a systematic translation from DyPDL expressions into their corresponding SMT encodings. This mapping ensures that the semantics of DyPDL, covering variables, constants, and composite expressions, are preserved within the logical framework of SMT. Table 1 summarizes the core translation rules, where each DyPDL expression e is associated with its encoded form.

2 Experiment Results

We present extensive visualizations of each algorithm's behavior that could not be included in the main manuscript because of space constraints. Figure 1 shows the performance profile of each algorithm. The first row of plots shows the performance profile of each solver across all instances, highlighting the specific computational resources most stressed by each algorithm in plot 1a, the proportion of total time used by different solver components in plot 1b, and the proportion of instances the algorithm ran out of memory at a particular component. The remaining plots in Figure 1 present the same information for each problem class.

Figures 2 to 5 present coverage over multiple features of the algorithm, including total time, search time, memory and expansion count. Figures 6 and 7 present the progress of dual bound over search time and expansion count. Plots in Figure 8 compare expansion count between OII and USR and OTI and USR.

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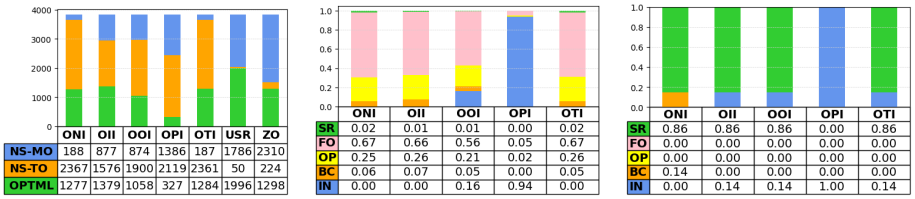
DyPDL expression e	SMT expression $\eta(e, \mathcal{V})$
General expressions	
v for $v \in \mathcal{V}$	v
k for $k \in \mathcal{K}$	k
$ITE(b, x_1, x_2)$	ITE$(\eta(b, \mathcal{V}), \eta(x_1, \mathcal{V}), \eta(x_2, \mathcal{V}))$
$T[e_1] \cdots [e_k]$	$f_T(\eta(e_1), \ldots, \eta(e_k))$
Element expressions	
$e_1 \circ e_2$ for $\circ \in \{+, -, *, /, \%\}$	$\eta(e_1, \mathcal{V}) \circ \eta(e_2, \mathcal{V})$
$\min(e_1, e_2)$	$\eta(\text{ITE}(e_1 \leq e_2, e_1, e_2), \mathcal{V})$
$\max(e_1, e_2)$	$\eta(\text{ITE}(e_1 \geq e_2, e_1, e_2), \mathcal{V})$
Numeric expressions	
analogous expressions as for element expressions	
$-n$, $\lvert n \rvert$, $\lvert s \rvert$	$-\eta(n, \mathcal{V})$, $\lvert \eta(n, \mathcal{V}) \rvert$, $\lvert \eta(s, \mathcal{V}) \rvert$
$\sqrt{n}$, $n_1^{n_2}$	$\sqrt{\eta(n, \mathcal{V})}$, $\eta(n_1, \mathcal{V})^k$ if $n_2 = k$
$\lfloor n \rfloor$	$\lfloor \eta(n, \mathcal{V}) \rfloor$
$\lceil n \rceil$	$\eta(\text{ITE}(n = \lfloor n \rfloor, n, \lfloor n \rfloor + 1, \mathcal{V})$
round(n)	$\eta(\lfloor n + 0.5 \rfloor, \mathcal{V})$
trunc(n)	$\eta(\text{ITE}(n \geq 0, \lfloor n \rfloor, -\lfloor -n \rfloor), \mathcal{V})$
reduce$(T, \circ, s_1, \ldots, s_k)$ for $\circ \in \{+, *, \max, \min\}$	$\eta(\bigcirc_{x_1, \ldots, x_k}$ ITE$(\bigwedge_i (x_i \in s_i), T[x_1] \cdots [x_k], 0_\circ))$
Set expressions	
$\bar{s}$	$E_{\tau(s)} \setminus \eta(s, \mathcal{V})$
$s_1 \circ s_2$ for $\circ \in \{\cap, \cup, \setminus\}$	$\eta(s_1, \mathcal{V}) \circ \eta(s_2, \mathcal{V})$
reduce$(T, \circ, s_1, \ldots, s_k)$ for $\circ \in \{\cup, \cap\}$	$\eta(\bigcirc_{x_1, \ldots, x_k}$ ITE$(\bigwedge_i (x_i \in s_i), T[x_1] \cdots [x_k], 0_\circ))$
Boolean expressions	
$x_1 \circ x_2$ for $\circ \in \{=, \neq\}$	$\eta(x_1, \mathcal{V}) \circ \eta(x_2, \mathcal{V})$
$b_1 \circ b_2$ for $\circ \in \{\wedge, \vee\}$, $\neg b$	$\eta(b_1, \mathcal{V}) \circ \eta(b_2, \mathcal{V})$, $\neg \eta(b, \mathcal{V})$
$n_1 \circ n_2$ for $\circ \in \{\geq, >, \leq, <\}$	$\eta(n_1, \mathcal{V}) \circ \eta(n_2, \mathcal{V})$
$s_1 \subseteq s_2$	$\eta(s_1, \mathcal{V}) \subseteq \eta(s_2, \mathcal{V})$
$e \in s$	$\eta(e, \mathcal{V}) \in \eta(s, \mathcal{V})$

Table 1: DyPDL expressions and their encoding SMT formulas. $n, n_i, e, e_i, s, s_i, b, b_i$ are numeric, element, set, and boolean expressions. x, x_i are expressions of any non-boolean type. T is a k-dimensional table of constants with indices from $E_{\tau(e_i)}$ in dimension i. Expression types match, e.g. in $e \in s$ we have $\tau(e) = \tau(s)$. $0_\circ$ is the neutral for $\circ$.



	ONI	OII	OOI	OPI	OTI	USR	ZO
NS-MO	188	877	874	1386	187	1786	2310
NS-TO	2367	1576	1900	2119	2361	50	224
OPTML	1277	1379	1058	327	1284	1996	1298

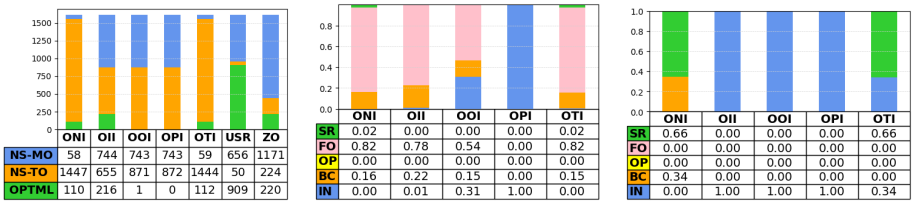
(a) Performance profile

	ONI	OII	OOI	OPI	OTI
SR	0.02	0.01	0.01	0.00	0.02
FO	0.67	0.66	0.56	0.05	0.67
OP	0.25	0.26	0.21	0.02	0.26
BC	0.06	0.07	0.05	0.00	0.05
IN	0.00	0.00	0.16	0.94	0.00

(b) Time usage

	ONI	OII	OOI	OPI	OTI
SR	0.86	0.86	0.86	0.00	0.86
FO	0.00	0.00	0.00	0.00	0.00
OP	0.00	0.00	0.00	0.00	0.00
BC	0.14	0.00	0.00	0.00	0.00
IN	0.00	0.14	0.14	1.00	0.14

(c) Memout stage



	ONI	OII	OOI	OPI	OTI	USR	ZO
NS-MO	58	744	743	743	59	656	1171
NS-TO	1447	655	871	872	1444	50	224
OPTML	110	216	1	0	112	909	220

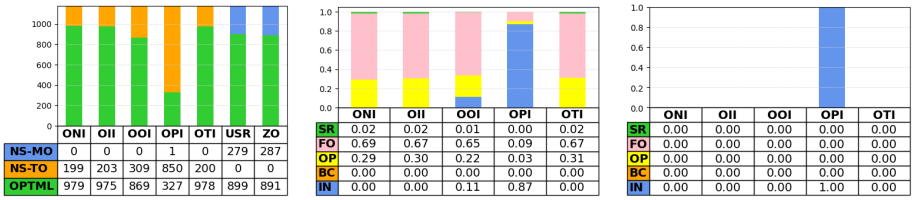
(d) BPP Performance profile

	ONI	OII	OOI	OPI	OTI
SR	0.02	0.00	0.00	0.00	0.02
FO	0.82	0.78	0.54	0.00	0.82
OP	0.00	0.00	0.00	0.00	0.00
BC	0.16	0.22	0.15	0.00	0.15
IN	0.00	0.01	0.31	1.00	0.00

(e) BPP Time usage

	ONI	OII	OOI	OPI	OTI
SR	0.66	0.00	0.00	0.00	0.66
FO	0.00	0.00	0.00	0.00	0.00
OP	0.00	0.00	0.00	0.00	0.00
BC	0.34	0.00	0.00	0.00	0.00
IN	0.00	1.00	1.00	1.00	0.34

(f) BPP Memout stage



	ONI	OII	OOI	OPI	OTI	USR	ZO
NS-MO	0	0	0	1	0	279	287
NS-TO	199	203	309	850	200	0	0
OPTML	979	975	869	327	978	899	891

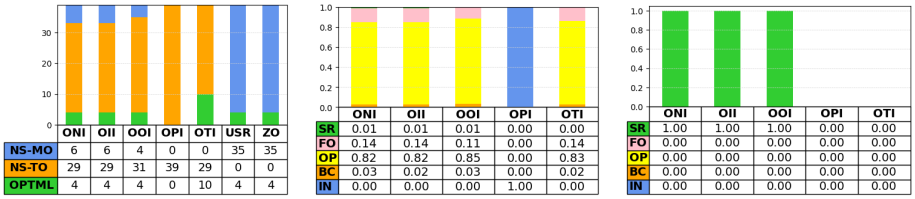
(g) m-PD Performance profile

	ONI	OII	OOI	OPI	OTI
SR	0.02	0.02	0.01	0.00	0.02
FO	0.69	0.67	0.65	0.09	0.67
OP	0.29	0.30	0.22	0.03	0.31
BC	0.00	0.00	0.00	0.00	0.00
IN	0.00	0.00	0.11	0.87	0.00

(h) m-PD Time usage

	ONI	OII	OOI	OPI	OTI
SR	0.00	0.00	0.00	0.00	0.00
FO	0.00	0.00	0.00	0.00	0.00
OP	0.00	0.00	0.00	0.00	0.00
BC	0.00	0.00	0.00	0.00	0.00
IN	0.00	0.00	0.00	1.00	0.00

(i) m-PD Memout stage



	ONI	OII	OOI	OPI	OTI	USR	ZO
NS-MO	6	6	4	0	0	35	35
NS-TO	29	29	31	39	29	0	0
OPTML	4	4	4	0	10	4	4

(j) TSP Performance profile

	ONI	OII	OOI	OPI	OTI
SR	0.01	0.01	0.01	0.00	0.00
FO	0.14	0.14	0.11	0.00	0.14
OP	0.82	0.82	0.85	0.00	0.83
BC	0.03	0.02	0.03	0.00	0.02
IN	0.00	0.00	0.00	1.00	0.00

(k) TSP Time usage

	ONI	OII	OOI	OPI	OTI
SR	1.00	1.00	1.00	0.00	0.00
FO	0.00	0.00	0.00	0.00	0.00
OP	0.00	0.00	0.00	0.00	0.00
BC	0.00	0.00	0.00	0.00	0.00
IN	0.00	0.00	0.00	0.00	0.00

(l) TSP Memout stage

	ONI	OII	OOI	OPI	OTI	USR	ZO
NS-MO	124	127	127	642	128	816	817
NS-TO	692	689	689	358	688	0	0
OPTML	184	184	184	0	184	184	183

(m) TS Performance profile

	ONI	OII	OOI	OPI	OTI
SR	0.01	0.01	0.01	0.00	0.01
FO	0.37	0.37	0.37	0.00	0.37
OP	0.61	0.61	0.61	0.00	0.61
BC	0.00	0.00	0.00	0.00	0.00
IN	0.00	0.00	0.00	1.00	0.00

(n) TS Time usage

	ONI	OII	OOI	OPI	OTI
SR	1.00	1.00	1.00	0.00	1.00
FO	0.00	0.00	0.00	0.00	0.00
OP	0.00	0.00	0.00	0.00	0.00
BC	0.00	0.00	0.00	0.00	0.00
IN	0.00	0.00	0.00	1.00	0.00

(o) TS Memout stage

Figure 1: Performance profiles of algorithms, over all instances in plots (a) to (c) and on each problem class in plots (d) to (o). It captures the number of instances with each run status: OPTML when optimality is *proved*, and NS-{TO, MO} when no solution is found within time or memory limit. Time usage plots shows the proportion of time used by Invariant Synthesis (IN), Computation of change in features by operators (FO), Operator synthesis (OP), Base case synthesis (BC) and Search (SR) in instances with OPTML or NS-TO status across *all* algorithms. The Memout stage highlights the proportion of NS-MO instances that run out of memory at a particular stage.

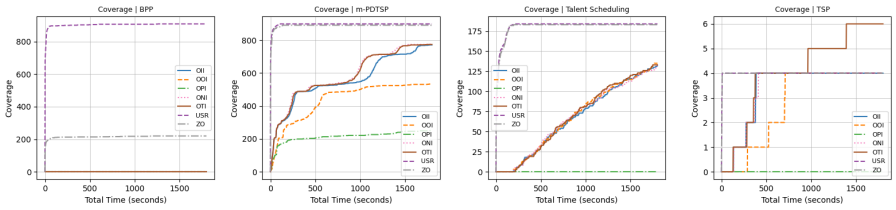


Figure 2: Optimally solved instances (Coverage) within a particular time budget. The total time includes the time consumed by *all* pre-preprocessing components.

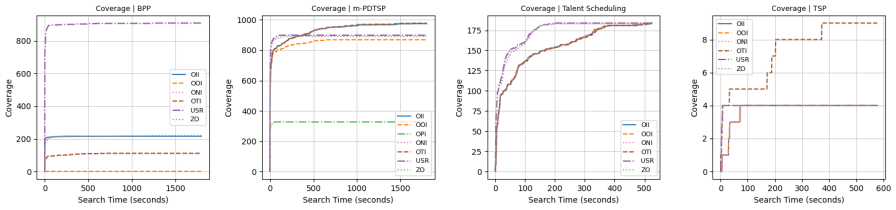


Figure 3: Optimally solved instances (Coverage) within a particular *search* time budget.

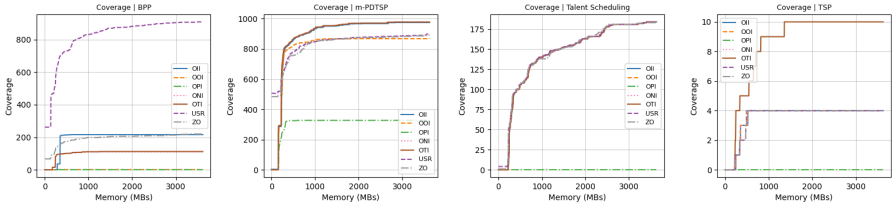


Figure 4: Optimally solved instances (Coverage) within a particular memory budget.

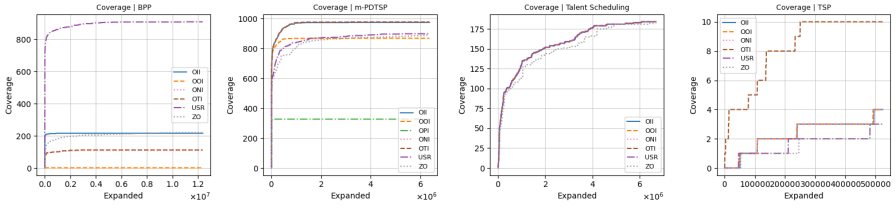


Figure 5: Optimally solved instances (Coverage) within a particular expansion count.

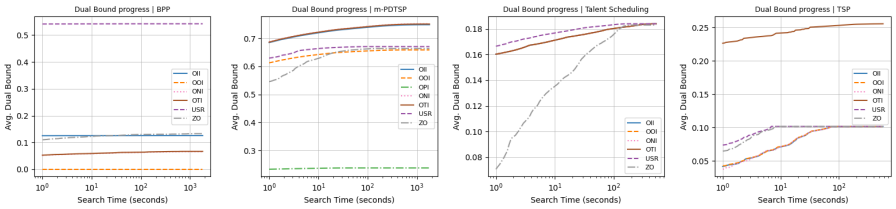


Figure 6: Dual bound progress over search time as mean fraction of the optimal cost.

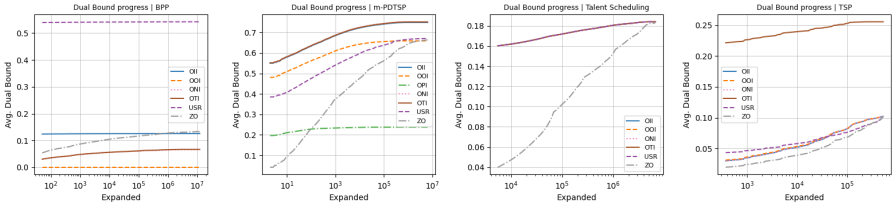


Figure 7: Dual bound progress over expanded as mean fraction of the optimal cost.

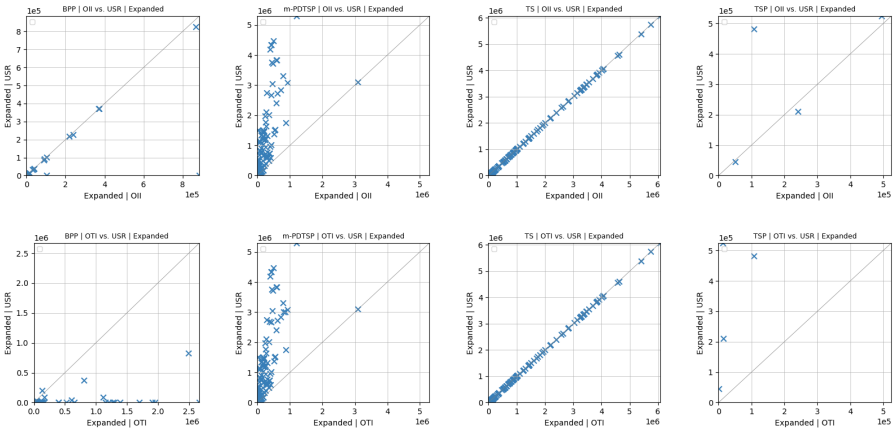


Figure 8: Pairwise comparison of expansion count for commonly solved instances.