Merge-and-Shrink Heuristics for Classical Planning: Efficient Implementation and Partial Abstractions

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Abstract

Merge-and-shrink heuristics are a successful class of abstraction heuristics used for optimal classical planning. With the recent addition of generalized label reduction, merge-and-shrink can be understood as an algorithm framework that repeatedly applies transformations to a factored representation of a given planning task to compute an abstraction. In this paper, we describe an efficient implementation of the framework and its transformations, comparing it to its previous implementation in Fast Downward. We further discuss partial merge-and-shrink abstractions that do not consider all aspects of the concrete state space. To compute such partial abstractions, we stop the merge-and-shrink computation early by imposing simple limits on the resource consumption of the algorithm. Our evaluation shows that the efficient implementation indeed improves over the previous one, and that partial merge-and-shrink abstractions further push the efficiency of merge-and-shrink planners.

Introduction

A∗ search (Hart, Nilsson, and Raphael 1968) with an admissible heuristic (Pearl 1984) is a state-of-the-art approach to optimally solving classical planning problems (Ghallab, Nau, and Traverso 2004). A popular method for obtaining admissible heuristics is based on abstractions. Merge-and-shrink (Dräger, Finkbeiner, and Podelski 2009; Helmert et al. 2014) is a state-of-the-art algorithm framework for computing abstractions of planning tasks. With the recent addition of generalized label reduction (Sievers, Wehrle, and Helmert 2014), the algorithm can be understood as a framework that repeatedly applies transformations to a given state space to compute an abstraction.

Broadly speaking, the key idea of merge-and-shrink is to operate on compact representations of large transition systems, called factored transition systems. Planning tasks induce factored transition systems that consist of atomic factors which each represent a single variable of the task. Starting from this factored transition system, the merge-and-shrink framework repeatedly either merges two factors by computing their product, shrinks a factor by applying an abstraction to it, prunes a factor by discarding dead states, or applies label reductions to the common label set of the factored transition system, which initially corresponds to the set of operators of the task. The computation ends if only one factor is left, which induces the merge-and-shrink heuristic.

Besides factored representations of transition systems, the merge-and-shrink framework also computes factored mappings (e.g., Sievers 2017) to represent the state mapping of the abstraction. Factored mappings are tree-like data structures that map states of the planning task to states of the factors of the transformed factored transition system.

In the literature on merge-and-shrink, the algorithmic aspect as well as the question of how to efficiently implement a merge-and-shrink planner has not been addressed a lot. Helmert et al. (2014) describe some techniques used in their implementation which we base ours on. Recently, Fan, Holte, and Müller (2018) suggested to compute very small merge-and-shrink abstractions quickly and to complement them with the usual, larger abstractions by computing the maximum heuristic over both.

In this paper, we address the algorithmic aspect as well as the question of how to efficiently compute merge-and-shrink abstractions by making the following contributions:

1. We provide an algorithmic description of how to compute exact generalized label reductions, which has only briefly been mentioned in the original work introducing generalized label reduction (Sievers, Wehrle, and Helmert 2014).
2. We describe an efficient implementation of the merge-and-shrink framework that exploits local label equivalence relations to compactly represent transition systems. In particular, we describe the implementation of merge-and-shrink in Fast Downward (Helmert 2006) and compare it to the previous implementation.
3. We discuss partial merge-and-shrink abstractions that do not cover all variables of a planning task, which we compute by imposing simple limits on the resource consumption of the merge-and-shrink computation, further pushing the efficiency of the overall approach.

Background

In this section, we briefly review the concept of classical planning and provide a rather extensive exposition of the merge-and-shrink framework to build a solid foundation based on which we describe algorithms and implementation.
**Classical Planning**

A planning task in the SAS+ formalism (Bäckström and Nebel 1995), augmented with action costs, is a tuple \( \Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\ast \rangle \) with the following components. \( \mathcal{V} \) is a finite set of variables \( v \), each associated with a finite domain \( \text{dom}(v) \). A partial state \( s \) is a partial assignment over a subset \( \text{vars}(s) \subseteq \mathcal{V} \) of the variables. We write \( s[v] \) to denote the value of \( v \in \text{vars}(s) \). If \( \text{vars}(s) = \mathcal{V} \), \( s \) is called a state. We write \( S(\mathcal{V}) \) for the set of states over \( \mathcal{V} \). Two partial states \( s \) and \( s' \) are consistent if \( s[v] = s'[v] \) for all \( v \in \text{vars}(s) \cap \text{vars}(s') \). \( \mathcal{O} \) is a finite set of operators \( o \), each with associated partial states \( \text{pre}(o) \) and \( \text{eff}(o) \), called precondition and effect, and an associated non-negative value \( \text{cost}(o) \in \mathbb{R}_+^\ast \), called the cost. Finally, \( s_{0} \) is a state called the initial state, and \( S_{\ast} \) is a partial state called the goal.

**Example 1.** Consider a simple logistics planning task with truck \( T \) and package \( P \). Variable \( v_T \) with \( \text{dom}(v_T) = \{A, B\} \) represents the position of the truck, and \( v_P \) with \( \text{dom}(v_P) = \{A, B, T\} \) that of the package. There is an operator \( \text{Dr}-A-B \) that drives the truck from \( A \) to \( B \) and has precondition \( \text{pre}(\text{Dr}-A-B) = \{v_T \rightarrow A\} \) and effect \( \text{eff}(\text{Dr}-A-B) = \{v_T \rightarrow B\} \). Analogously, \( U-X \) load the package at location \( X \) in \( \{A, B\} \). We assume unit-cost. The goal is to have the package at \( B \), which is initially at \( A \), and the truck starts at \( B \).

The semantics of a planning task can be naturally defined in terms of a labeled transition system. A transition system is a tuple \( \Theta = (S, L, c, T, s_0, S_\ast) \), where \( S \) is a finite state of states, \( L \) is a finite set of transition labels, \( c : L \rightarrow \mathbb{R}_+^\ast \) is a label cost function, \( T \subseteq S \times L \times S \) is a set of labeled transitions, \( s_0 \in S \) is the initial state, and \( S_\ast \subseteq S \) is the set of goal states. We write \( s \leadsto s' \) to denote a transition \( (s, l, s') \) from \( s \) to \( s' \) with label \( l \). A path from \( s \) to \( s' \) is a sequence \( \pi = \langle t_1, \ldots, t_n \rangle \) of transitions such that there exist states \( s = s_0, \ldots, s_n = s' \) with \( t_i = \langle s_{i-1}, l_i, s_i \rangle \in T \) for all \( i \in \{1, \ldots, n\} \). The cost of such a path is the accumulated cost of the labels of the transitions, i.e., \( \sum_{i=1}^{n} c(t_i) \).

Let \( \Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_\ast \rangle \) be a planning task. Its induced transition system \( \Theta(\Pi) \) is defined as \( (S, L, c, T, s_0, S_\ast) \), where \( S = \mathcal{S}(\mathcal{V}) \), \( L = \mathcal{C}(\mathcal{\ell}) = \text{cost}(\ell) \) for all \( \ell \in L \), \( t \in T \) iff \( s \) is consistent with \( \text{pre}(\ell) \) and \( t \) is the state consistent with \( \text{eff}(\ell) \) and \( t[v] = s[v] \) for all \( v \notin \text{vars}(\text{eff}(\ell)) \), and \( S_\ast = \{s \in S | s \text{ consistent with } s_{\ast}\} \). A plan is a path from \( s_0 \) to a state in \( S_\ast \). Optimal planning deals with finding plans of minimal cost or proving that no plan exists.

Figure 1 shows the induced transition system \( \Theta(\Pi) \) for the example task. Goal states are marked by double circles and the initial state has an ingoing arrow. In this example, we label states with two letters indicating the position of the package and the truck.

A heuristic for a transition system \( \Theta \) with states \( S \) is a function \( h_{\Theta} : S \rightarrow \mathbb{R}_0^\ast \cup \{\infty\} \) that maps each state to an estimate of the cost of reaching a goal state from \( s \). \( h_{\Theta} \) is perfect, denoted \( h_{\Theta}^p \), if it computes the true cost for all states. It is admissible if \( h_{\Theta}(s) \leq h_{\Theta}^p(s) \) for all states \( s \).

**Merge-and-Shrink**

The goal of the merge-and-shrink framework is to compute an abstraction heuristic for a large transition system such as the one induced by a planning task. To do so, it not only computes the abstract transition system, but also the state and label mappings of the abstraction. However, the given transition system is usually too large to be represented explicitly. A key aspect of the merge-and-shrink framework to address this issue is to work with factored representations of both transition systems and state mappings.

In particular, a factored transition system \( F \) is a tuple of transition systems (also called factors or variables of \( F \)), \( F = \langle \Theta_1, \ldots, \Theta_n \rangle \), where each element has the same label set and label cost function. It serves as a representation of the synchronized product (product for short) of all of its elements, which is the transition system defined as \( \bigotimes F = \langle S^\otimes, L, T^\otimes, S^\otimes_0, S^\otimes_\ast \rangle \), i.e., the Cartesian product of the sets of states, \( T^\otimes = \{t^1, \ldots, t^n \} \rightarrow \{t^1, \ldots, t^n \} \mid s^1 \rightarrow t^1 \in T^1 \) for all \( 1 \leq i \leq n \), \( s^0 = \langle s^1_0, \ldots, s^n_0 \rangle \), and \( S^\otimes_\ast = \prod_{i=1}^{n} S_i^\ast \).

A planning task \( \Pi \) induces a factored transition system \( F(\Pi) \) which consists of atomic factors \( \Theta^i \) that reflect the behavior of all variables \( v \) of \( \Pi \): the states of \( \Theta^i \) correspond to values of \( v \) and all operators induce transitions depending on their preconditions and effects on \( v \). The top part of Figure 2 shows \( F(\Pi) \) for the example task. It consists of two atomic factors for the two variables of the task. The product of \( F(\Pi) \) corresponds to the transition system shown in Figure 1, apart from names of states, which formally would read, e.g., \( (0, 0) \) instead of \( AA \) and \( (2, 1) \) instead of \( TB \).
Algorithm 1 Merge-and-shrink Framework

Input: Planning task $\Pi$, function $\text{SELECTTF}$ that selects the next basic transformation to apply.
Output: Merge-and-shrink heuristic $h^\text{M&S}$ for $\Pi$.

1: function $\text{MERGEANDSHRINK}(\Pi, \text{SELECTTF})$
2: \quad $F \leftarrow F(\Pi)$, $\Sigma \leftarrow \text{id}^P$, $\lambda \leftarrow \text{id}^L$
3: \quad while $|F| > 1$ do
4: \quad \quad $(F', \Sigma', \lambda') \leftarrow \text{SELECTTF}(F, \Sigma, \lambda)$
5: \quad \quad $F \leftarrow F'$, $\Sigma \leftarrow \Sigma' \circ \Sigma$, $\lambda \leftarrow \lambda' \circ \lambda$
6: \quad end while
7: \quad $h^\text{M&S}_\Pi \leftarrow h^\text{M&S}$
8: \quad return $h^\text{M&S}$
9: end function

A factored mapping $\sigma$ (Sievers 2017), also called cascading table (Helmert et al. 2014) or merge-and-shrink representation (Helmert, Röger, and Sievers 2015), is a tree-like data structure inductively defined over a set of variables $V$. If $\sigma$ is atomic (a leaf), then it has an associated variable $v \in V$ and a table function $\sigma^\text{tab}$ mapping from $\text{dom}(v)$ to an arbitrary value set $\text{vals}(\sigma)$. If $\sigma$ is a merge (an inner node), then it has two component factored mappings $\sigma_L$ and $\sigma_R$ and a table function $\sigma^\text{tab}$ mapping from $\text{vals}(\sigma_L) \times \text{vals}(\sigma_R)$ to an arbitrary value set $\text{vals}(\sigma)$. A factored mapping $\sigma$ represents a function $[\sigma] : S(\mathcal{V}) \rightarrow \text{vals}(\sigma)$ inductively defined as follows: if $\sigma$ is atomic with $v$, then $[\sigma](\alpha) = \sigma^\text{tab}(\alpha[v])$. If it is a merge with components $\sigma_L$ and $\sigma_R$, then $[\sigma](\alpha) = \sigma^\text{tab}(\langle \sigma_L(\alpha), \sigma_R(\alpha) \rangle)$.

We call such factored mappings $\sigma$ factored-to-non-factored (F2N) mappings because $[\sigma]$ maps states to single values. To represent state mappings between factored transition systems, we define factored-to-factored (F2F) mappings. An F2F mapping from a factored transition system $F$ to a factored transition system $F' = (\Theta_1', \ldots, \Theta_n')$ is a tuple $\Sigma = \langle \sigma_1, \ldots, \sigma_n \rangle$ where each $\sigma_i$ is an F2N mapping defined over $F$ (viewing factors as variables with their states as values) which corresponds to $\Theta_i'$, i.e., it maps to the states of $\Theta_i'$. Thus $\Sigma$ thus represents the function $[\Sigma] : S(F) \rightarrow S(F')$.

The bottom part of Figure 2 shows $\Sigma = \langle \sigma_P, \sigma_T \rangle$, an F2N mapping from $F(\Pi)$ to $F'(\Pi)$. Its component F2N mappings $\sigma_P$ and $\sigma_T$ each project states of $F(\Pi)$ to their corresponding state in $\Theta^P$ and $\Theta^T$. Thus $\Sigma$ represents the identity function on states of $F(\Pi)$. In the illustration, we only show the node of the F2N mapping and its table, omitting its value set which can be inferred from the the entries of the table.

We now discuss the pseudo-code of the merge-and-shrink framework shown in Algorithm 1. The algorithm first computes the induced factored transition system $F = F(\Pi)$ of the given task $\Pi$ and further initializes the F2F mapping $\Sigma$ to the identity mapping from $F$ to $F$ and the labeling $\lambda$ to the identity mapping on the label set $L$ of $F$ (line 2). In the main loop (lines 3–6), the algorithm then repeatedly selects a transformation (line 4) of the current factored transition system $F$ using the user-specified $\text{SELECTTF}$ function. Such transformations of $F$ specify the transformed factored transition system $F'$, the F2F mapping $\Sigma$ from $F$ to $F'$, and the labeling $\lambda$ defined on $L$. To “apply” the selected transformation, the algorithm replaces $F$ by $F'$ and composes the state and label mappings with the previous one (line 5). When there is only one factor $\Theta$ of $F$ left, the algorithm computes the heuristic which is defined as $h^\text{M&S}_{\Pi}\sigma(s) = h^\text{M&S}_{\Pi}\sigma([\Sigma](s))$ for all states $s \in S(F(\Pi))$.

We call the algorithm an algorithm framework rather than a concrete algorithm because it leaves one important choice point unspecified, namely how to select the transformations using function $\text{SELECTTF}$. Before discussing a concrete instantiation of the framework, we briefly review the four types of merge-and-shrink transformations.

A shrink transformation computes an abstraction $\alpha$ of the states of a given factor $\Theta$ of $F$ and replaces $\Theta$ by the induced abstract factor $\alpha(\Theta)$. The F2N mapping corresponding to $\alpha(\Theta)$ represents the abstraction mapping $\alpha$. An example shrink transformation of $F(\Pi)$ of the example task selects states 1 and 2 of $\Theta^P$ to the same state $x$, which thus inherits the transitions of both 1 and 2, turning, e.g., 1 $\xrightarrow{L-B}$ 2 into a self-looping transition. The transformed F2N mapping $\alpha(\sigma_P)$ maps both B and T to x (and still A to 0).

A merge transformation replaces two factors $\Theta, \Theta'$ of $F$ by their product $\Theta \otimes \Theta'$. The F2N mapping for the new factor is a merge factored mapping that maps pairs of states of $\Theta$ and $\Theta'$ to the corresponding product state in $\Theta \otimes \Theta'$. An example merge transformation for $F(\Pi)$ of the example task replaces the two atomic factors $\Theta_P$ and $\Theta_T$ by their product $\Theta_\otimes$ shown in Figure 1. The two corresponding factored mappings $\sigma_P$ and $\sigma_T$ are replaced by the merge factored mapping $\sigma_\otimes$ with components $\sigma_P$ and $\sigma_T$, shown in Figure 3. It maps values as computed by $\sigma_P$ and $\sigma_T$ (which correspond to states of the corresponding factors) to values corresponding to the product states in the product system (we annotated states of $\Theta_\otimes$ in Figure 1 with corresponding numbers from the value set of $\sigma_\otimes$). For example, for the assignment $v_P \rightarrow 2, v_T \rightarrow 1$, $\sigma_\otimes$ first computes values 2 and 1 from its components and then retrieves the result 5 from its table, which indeed corresponds to the state TB.

A prune transformation combines unreachable states or states from which no goal can be reached of some factor $\Theta$ of $F$ into an isolated dummy state, removing all corresponding transitions. In the corresponding F2N mapping, this change is reflected like for shrink transformations.

Finally, a label reduction transformation applies a label mapping $\lambda$ to the label set $L$ with label costs $c_F$ of $F$, mapping it to some (usually smaller) label set $L'$ with label costs $c'_F$. In all factors of the transformed factored transition system, all transitions $s \xrightarrow{t} t$ must be replaced by transitions $s \xrightarrow{\lambda(t)} t$. 

![Diagram](image-url)
Algorithm 2 The main loop of the merge-and-shrink framework as implemented in Fast Downward.

1: while $|F| > 1$ do
2: \quad $\Theta_1, \Theta_2 \leftarrow \text{SELECT}(F)$
3: \quad $(F; \Sigma, \lambda) \leftarrow \text{COMPOSETF}(\text{LABELRED}(F))$
4: \quad $(F; \Sigma, \lambda) \leftarrow \text{COMPOSETF}(\text{SHRINK}(F, \Theta_1, \Theta_2))$
5: \quad $(F; \Sigma, \lambda) \leftarrow \text{COMPOSETF}(\text{MERGE}(F, \Theta_1, \Theta_2))$
6: \quad $(F; \Sigma, \lambda) \leftarrow \text{COMPOSETF}(\text{PRUNE}(F, \Theta_\infty))$
7: end while

Label costs must not increase to guarantee admissibility, i.e., $c'(\ell') := \min_{\ell \in X \cap \{\ell'\}} c(\ell)$ for all $\ell' \in L'$. An example label reduction for $F(\Pi)$ of the example task reduces U-A, U-B, L-A, L-B to a new label X. In the transformed factors, transitions are relabeled accordingly, collapsing identical transitions. For example, $0 \stackrel{1\cdot A}{\rightarrow} 0$ and $0 \stackrel{U-A}{\rightarrow} 0$ of $\Theta_T$ are combined into $0 \stackrel{X}{\rightarrow} 0$ in the transformed factor.

Sievers, Wehrle, and Helmert (2014) provide a full characterization of label reductions, describing sufficient and necessary conditions under which they are exact. Exact transformations preserve perfect heuristic values and are thus desirable. The exclusive condition used in previous work to compute exact generalized label reductions is based on $\Theta$-combinability. Consider a factored transition system $F$ with labels $L$ and some factor $\Theta$ of $F$. Labels $\ell, \ell' \in L$ are locally equivalent in $\Theta$ if they label the same transitions in $\Theta$, and they are $\Theta$-combinable in $F$ if they are locally equivalent in all factors $\Theta' \neq \Theta$ of $F$. A label reduction that only combines $\Theta$-combinable labels of the same cost for some $\Theta$ of $F$ is exact. In $F(\Pi)$ of the example task, labels DR-A-B and DR-B-A are $\Theta_T$-combinable because they both induce the same self-looping transitions in the only other factor $\Theta_P$.

We conclude this section with a description of the concrete instantiation of the merge-and-shrink main loop in Fast Downward (Helmert 2006). Algorithm 2 shows pseudocode. The layout is based on the decision to perform one merge transformation in each iteration. Selecting the factors $\Theta_1, \Theta_2$ to merge is thus the first step (line 2), followed by possibly applying a label reduction (line 3), shrinking one or both factors that will be merged in order to satisfy a given size limit imposed on transition systems (line 4), performing the merge transformation by replacing $\Theta_1$ and $\Theta_2$ by $\Theta_\infty$ (line 5), and finally pruning $\Theta_\infty$ (line 6). Recall that performing a transformation means to compose it with the previous transformation as described in line 5 of Algorithm 1, for which COMPOSETF is a short-hand notation. This algorithm still has parameters, namely the transformation strategies that need to specify how to compute a specific transformation. For example, a shrink strategy needs to specify how to compute an abstraction for a given factor.

Computation of Exact Label Reductions

In the following, we detail out the computation of exact label reductions based on $\Theta$-combinable labels (comparable labels for short) that Sievers, Wehrle, and Helmert (2014) only briefly mentioned. We begin with the following important observations. The relation on locally equivalent labels for a single factor is transitive; it is even an equivalence relation, called local label equivalence relation in the following. However, the combinable relation is the union over these local equivalence relations, and this union relation is not transitive. This means that if $\ell_1$ and $\ell_2$ are combinable and $\ell_2$ and $\ell_3$ are combinable for different factors of a factored transition system, this does not imply that $\ell_1$ and $\ell_3$ are combinable for any factor. Intuitively, after merging $\ell_1$ and $\ell_2$, the combined label is in general not locally equivalent with $\ell_3$ in all (but one) factors because we now have transitions with the new label that originated from $\ell_1$ and have no matching transition for $\ell_3$. An immediate consequence of this observation is that if we want to apply all possible label reductions by computing combinable labels for all factors until there are no more such combinable labels, the result depends on the order in which we consider the factors.

Due to the above observations, Fast Downward implements the fixed point algorithm illustrated in Algorithm 3 for computing exact label reductions based on $\Theta$-combinability. It keeps track of the number of iterations in which no more labels could be combined (line 2) and reaches a fixed point if the number of such unsuccessful iterations equals the number of factors in the given factored transition system $F$, i.e., if there are no more combinable labels in any of the factors of $F$ (line 3).

In each iteration, the algorithm chooses the next factor $\Theta \in F$ according to some user-specified order $O$ in which

Algorithm 3 Fixed point label reduction algorithm.

Input: Factored transition system $F$ with labels $L$ and label costs $c$; Order $O$ on factors of $F$.
Output: Transformed factored transition system $F'$.

1: function LABELREDUCTION($F$)
2: \quad $\#\text{UnsuccIt} \leftarrow 0$
3: \quad while $\#\text{UnsuccIt} < |F|$ do
4: \quad \quad $\Theta \leftarrow \text{NEXT}(F', O)$
5: \quad \quad $\text{eqRel} \leftarrow \Theta$-$\text{COMBINABLEREL}(F, \Theta)$
6: \quad \quad if $\text{eqRel}$ is trivial then
7: \quad \quad \quad $\#\text{UnsuccIt} \leftarrow \#\text{UnsuccIt} + 1$
8: \quad \quad else
9: \quad \quad \quad $\lambda, c' \leftarrow \text{LABELMAPPING}(L, \text{eqRel})$
10: \quad \quad \quad $F \leftarrow \text{APPLY}(F, \lambda, c')$
11: \quad \quad \quad $\#\text{UnsuccIt} \leftarrow 0$
12: \quad \quad end if
13: \quad end while
14: \quad return $F$
15: end function

16: function $\Theta$-$\text{COMBINABLEREL}(F, \Theta \in F)$
17: \quad $\text{eqRel} \leftarrow \text{UNIVEQREL}(L)$
18: \quad for $\Theta' \in F$ with $\Theta' \neq \Theta$ do
19: \quad \quad $\text{eqRel} \leftarrow \text{REFINE}(\text{eqRel}, \text{EQREL}(\Theta', L))$
20: \quad end for
21: \quad return eqRel
22: end function
The fact that the previous implementation could not store transitions of irrelevant labels explicitly due to a prohibitive memory consumption, which led to various error-prone special-casing. In $\Theta^P$ of the example, labels $\text{Dr-A-B}$ and $\text{Dr-B-A}$ are irrelevant and thus form a single label group (cf. Table 1).

A further, small conceptual difference of the two implementations is that the previous implementation computed pruning transformations as part of shrinking, whereas the optimized implementation only attempts pruning after merging, which is the only place where pruning opportunities can occur. Furthermore, we allow computing $g$- and $h$-values depending on which information is required by the transformation strategies, whereas the previous implementation always computed both. Both implementations compute distances of transition systems with Dijkstra’s algorithm (Dijkstra 1959). This is the only place where we need an explicit adjacency list representation of transition systems.

We now turn our attention to the different merge-and-shrink transformations, beginning with shrinking, which is

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\begin{table}
\centering
\begin{tabular}{ |c|c|c|c|c| }
\hline
& prev & & & opt \\
\hline
Dr-A-B & $\{ (0, 0), (1, 1), (2, 2) \}$ & & $\{ \text{Dr-A-B, Dr-B-A} \}$ & \\
Dr-B-A & $\{ (0, 0), (1, 1), (2, 2) \}$ & & $\{ (0, 0), (1, 1), (2, 2) \}$ & \\
L-A & $\{ (0, 2) \}$ & & $\{ \text{L-A} \}$ & \\
U-A & $\{ (2, 0) \}$ & & $\{ \text{U-A} \}$ & \\
L-B & $\{ (1, 2) \}$ & & $\{ \text{L-B} \}$ & \\
U-B & $\{ (2, 1) \}$ & & $\{ \text{U-B} \}$ & \\
\hline
\end{tabular}
\caption{Representing $\Theta^P$ of the example (cf. Figure 2) in the previous (prev) and the optimized (opt) implementation.}
\end{table}
done identically in both implementations. First, the shrink strategy computes a state equivalence relation for the given factor. An example shrink transformation combines states $0$ and $1$ of $\Theta^p$. We transform this equivalence relation into an explicit state mapping, assigning a consecutive number to each equivalence class. In the example, $0 \rightarrow 0$, $1 \rightarrow 0$, and $2 \rightarrow 1$. Then we use this state mapping for an in-place modification of the factor by going over all transitions and updating their source and target states. This can be done efficiently because we store transitions by labels or label groups, see Table 1. If we used an adjacency list representation of transition systems, we would possibly need to reorder all entries of the list, besides renumbering the entries within successor lists. Finally, we perform an in-place modification of the F2N mapping corresponding to the shrink factor by applying the state mapping to its table. In Figure 2, we would relabel the second row of $\sigma_P$ to read $0$, $0$, and $1$.

When merging two transition systems $\Theta_1$ and $\Theta_2$, the previous implementation considered each label separately and computed its transitions in the product by pair-wise combinations of its transitions in $\Theta_1$ and $\Theta_2$. In the optimized implementation, we do not compute this full product, because this would require to compute the local label equivalence relation from scratch afterwards. Instead, we use the following simple bucket-based approach to directly compute the refinement of the local label equivalence relations $\text{EqRel}_1$ and $\text{EqRel}_2$ of $\Theta_1$ and $\Theta_2$: for each label group $x$ of $\text{EqRel}_1$, first partition its labels according to $\text{EqRel}_2$, and then, for each resulting new label group $y$, compute the product of the transitions of $x$ and of the label group of $\Theta_2$ that corresponds to $y$. We remark that computing product transitions is facilitated by storing transitions grouped by labels rather than using an adjacency list, which allows to quickly access the transitions of a label or a label group.

Computing the F2N mapping $\sigma_\Theta$ corresponding to the product is straightforward and identical to the previous implementation: it has two components, which are the two F2N mappings of the previous transformation that map to the factors $\Theta_1$ and $\Theta_2$, and it has a table that maps pairs of component states to their product state. Starting from the factored transition system shown in Figure 2, Figures 1 and 3 illustrate the result of a merge transformation.

When pruning, the prune strategy computes a set of to-be-pruned states for the given factor. We then modify the factor in-place by removing these states together with their transitions. The table of the corresponding F2N mapping is updated in-place to map removed states to a special symbol which is evaluated to $\infty$ by the heuristic. This implementation of pruning is identical to the previous one.

Computing an exact label reduction with Algorithm 3 is favored by the representation that stores the local label equivalence relation of all factors: the loop of the method $\Theta$-COMBINABLE-REL can simply use the already computed local label equivalence relations (instead of calling $\text{EQUIVREL}(\Theta^p, L)$ in line 19). The previous implementation needed to compute this information on demand and cache it over the iterations of the algorithm.

For efficient refinement of label equivalence relations (REFINE), we store label groups in linked lists as discussed in the description of the algorithm. Computing the label mapping ($\text{LABELMAPPING}$) is straightforward and is done like computing a state mapping from a state equivalence relation when shrinking. Applying the label mapping ($\text{APPLY}$) can be efficiently done as follows. When updating a factor other than $\Theta$, all we have to do is to remove the old labels from their group (they are all in the same group) and to add the new label to it; the transitions remain. Otherwise, when updating $\Theta$, we need to remove the old labels from their groups (potentially different ones) and add a new singleton group for the new label. While doing so, we collect the transitions of all to-be-removed labels and combine them to form the transitions of the new label. If label groups become empty, we remove them together with their transitions. We also need to recompute the costs of all modified groups. This part of label reduction is identical to the previous implementation. In the optimized implementation, we additionally recompute the local label equivalence relation to restore the compact representation of transition systems.

We remark that our optimized implementation also greatly benefits two transformation strategies of the merge-and-shrink toolbox. The state-of-the-art shrink strategy based on bisimulation (Nissim, Hoffmann, and Helmert 2011) is accelerated significantly due to only considering transitions of label groups rather than of individual labels. For the same reason, computing factored symmetries (Sieviers et al. 2015) that can be used to enhance merge strategies is faster with the optimized implementation.

Partial Merge-and-Shrink Abstractions

The regular termination criterion of the merge-and-shrink framework is to stop when the transformed factored transition system only contains a single factor. Then we have exactly one abstraction for computing the heuristic. However, nothing prevents us from stopping the algorithm early, ending up with several factors and factored mappings, and hence with several abstractions that induce heuristics.

Paired with the observation that merge-and-shrink based planners fail to finish computing the heuristic in the given time and memory limits in a non-negligible number of cases (151–267 out of 1667 tasks for state-of-the-art-configurations), we will evaluate two simple ways of stopping the merge-and-shrink computation by limiting resource consumption. Both have been used before by Torralba and Hoffmann (2015) to limit the merge-and-shrink computation. However, they did not compute merge-and-shrink heuristics but used the framework as a basis for computing label dominance relations.

The first limit restricts the number of transitions that a factor may have at any time. This is reasonable because this number has a much higher impact on the runtime of merge-and-shrink than the number of states, which is controlled by the shrink strategy: both computing exact label reductions and product systems is dominated by the number of transitions. However, controlling the number of transitions is not possible in a straightforward way with an approach similar to how shrinking controls the number of states because

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See rows "# constr" of column "base" of Table 3 or 4.
removing or ignoring transitions can result in inadmissible heuristics. Therefore, we terminate the merge-and-shrink algorithm once the limit is reached by any factor. The second limit restricts the runtime of the merge-and-shrink algorithm, which we allow to be terminated even during the computation of atomic factors.

In both cases, when terminating the merge-and-shrink computation early, we are left with a factored transition system $F = \langle \Theta_1, \ldots, \Theta_n \rangle$ and an F2F mapping $\Sigma = \langle \sigma_1, \ldots, \sigma_n \rangle$ that contain several elements. Thus, instead of computing $h^{\text{M&S}} = h^*_{\Theta}$ for the single remaining factor $\Theta$ as usual, for each factor $\Theta_i$, we can compute the factor heuristic $h_i(s) = h^*_{\Theta_i}(\sigma_i(s))$ for all states $s$ of the task. The decision we then face is to compute a (merge-and-shrink) heuristic from this set of heuristics.

The first, possibly most obvious variant we consider is to compute the max-factor heuristic

$$h^{\text{M&S}} = \max_i h_i$$

that maximizes over all factor heuristics. The second, less expensive alternative is to choose a single factor heuristic $(h_i^F)$ and use it as the merge-and-shrink heuristic. We use the following simple rule of thumb for this choice: we prefer the heuristic with the largest estimate for the initial state (rational: better informed heuristic), breaking ties in favor of larger factors (rational: more fine-grained abstraction), and choose a random heuristic among all remaining candidates of equal preference. Other choices or tie-breakers are certainly possible, such as choosing the heuristic with the largest average estimate or the heuristic with the largest number of distinguishable estimates. We leave an evaluation of these options as future work.

**Experiments**

In the following, we first compare the previous to the optimized implementation of merge-and-shrink in Fast Downward and then evaluate partial merge-and-shrink abstractions. The technical setup is the same for all experiments: we use all (optimal) benchmarks of all IPCs up to 2014, a set comprised of 1667 planning tasks distributed across 57 domains. We limit time to 30 minutes and memory to 3.5 GB per task, using Downward-Lab (Seipp et al. 2017) for conducting the experiments on a cluster of machines with Intel Xeon Silver 4114 CPUs running at 2.2 GHz.

All results are obtained with the state-of-the-art shrink strategy based on bisimulation with a size limit of 50000. We allow shrinking even if the size limit is not violated to allow exploiting potential perfect shrinking opportunities. We use full pruning of dead states. Label reductions are computed with the fixed point algorithm described earlier. In the first experiment, we can only use the merge strategies that were available already in the previous implementation. This includes two representatives of simple linear strategies, causal graph goal level (CGGL) (Helmer, Haslum, and Hoffmann 2007) and reverse level (RL) (Nissim, Hoffmann, and Helmert 2011), as well as the non-linear merge strategies DFP (Dräger, Finkbeiner, and Podelski 2009; Sievers, Wehrle, and Helmert 2014) and maximum intermediate abstraction size minimizing (MIASM) (Fan, Müller, and Holte 2014), the latter using DFP as fallback mechanism (MIASMdfp or Mdfp for short). In the second experiment, we also use the most recent, state-of-the-art non-linear merge strategies score-based MIASM (shMIASM), also called DYN-MIASM, and the strategy based on strongly connected components of the causal graph (Sievers, Wehrle, and Helmert 2016), which uses DFP for internal merging (SCCdFp).

**Previous and Optimized Implementation**

Recall that in this comparison we want to reproduce results of an older implementation of merge-and-shrink in Fast Downward. Since Fast Downward versions that are several years apart usually have huge differences in performance, we integrated the previous implementation of merge-and-shrink into the most recent version of Fast Downward, attempting to keep the required modifications at a minimum. We think that this is the best way to allow for a fair comparison to state-of-the-art techniques.

We further remark that the previous implementation uses the first implementation of MIASM that its authors used for their original paper, whereas the optimized implementation uses a re-implementation of MIASM which its authors developed based on a newer version of Fast Downward. Unfortunately, integrating the newer implementation of MIASM into the previous implementation of merge-and-shrink was not possible without major changes. Hence the following comparison of MIASM has to be taken with a grain of salt.

Table 2 shows the comparison of the previous implementation (column prev) against the optimized one (column opt). For each merge strategy (vertical blocks), the table reports, in different rows, the number of solved tasks (coverage), the number of tasks for which the construction of the heuristic finished (No constr), the runtime of the construction in seconds (Constr time), aggregated using the geometric mean first over all tasks and then over all domains, the number of tasks for which the construction ran out of memory (oom) or time (oot), and the number of expansions (rounded to thousands) until the last f-layer, aggregated using the 75th percentile (E 75th perc). The table also includes columns displaying the difference (diff) between opt and prev, which is highlighted in bold if it is favorable for the optimized implementation, and the number of tasks for which opt is better (+) and worse (-) than prev.

We observe that the optimized implementation indeed leads to an improved efficiency of the merge-and-shrink computation: the smaller representation size of transition systems results in fewer tasks for which the computation runs out of memory, which also transfers to a great decrease of construction time on average. This increase of the number of successful heuristic constructions in turn yields better

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4Code: https://doi.org/10.5281/zenodo.1290524

5Suite “optimal strips” from https://bitbucket.org/aibasel/downward-benchmarks

6Dataset: https://doi.org/10.5281/zenodo.1290644

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However, for some tasks with a large number of variables or operators, the (expensive) fixed point computation of label reductions causes the heuristic construction to run out of time or memory in the optimized implementation because it explicitly represents irrelevant labels of each factor. We still prefer the cleaner implementation over an even more efficient computation for those few cases.
the computed heuristic and a larger limit may not stop the computation before running out of time.\textsuperscript{8} A second conclusion we draw from Table 3 is that there is no significant difference between \( h^{\text{mf}} \) and \( h^{\text{sg}} \). While \( h^{\text{mf}} \) theoretically dominates any factor heuristic by definition, evaluating the former can be slightly more expensive. Furthermore, in scenarios where there is one large factor and many small (e.g., atomic) factors in the end, the large one likely dominates the others, and thus \( h^{\text{sg}} \) is equally informed as \( h^{\text{mf}} \). This scenario always occurs with linear merge strategies because they maintain only one non-atomic factor.

We now evaluate adding a time limit to the computation. Table 4 shows the results for limits of 450s, 900s, and 1350s. As expected, this is a very effective measure for greatly increasing the number of successful heuristic constructions, which also directly transfers to a significant increase in coverage of all configurations, with 450s and 900s achieving slightly better results than 1350s. In contrast to limiting the number of transitions, stopping the computation early does not affect the heuristic quality at all. The likely reason is that with limiting the time, we catch precisely those tasks for which the construction otherwise does not terminate or terminates too late for a successful search. Tasks which we can already solve without imposing a time limit (base) usually require a rather short construction time, and therefore limiting the time to 900s or more does not stop the heuristic construction before running out of time.

Table 2: Comparison of the baseline against the optimized implementation of merge-and-shrink.

<table>
<thead>
<tr>
<th>prev</th>
<th>opt</th>
<th>diff</th>
<th>+</th>
<th>-</th>
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<td>754</td>
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<td>23</td>
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<tr>
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<td>57</td>
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<td>98</td>
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<td>3</td>
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Table 3: Comparison of the baseline against two versions of partial merge-and-shrink, using different limits on the number of transitions.

<table>
<thead>
<tr>
<th>base ( h^{\text{mf}} )</th>
<th>( t2m )</th>
<th>( t5m )</th>
<th>( t10m )</th>
<th>( h^{\text{sg}} )</th>
<th>( t2m )</th>
<th>( t5m )</th>
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<td>1084k</td>
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</tbody>
</table>

Table 3: Comparison of the baseline against two versions of partial merge-and-shrink, using different limits on the number of transitions.

\textsuperscript{8}We also tested to only exclude a factor from further consideration if it violates the transition limit rather than aborting the computation. This is only possible with merge strategies such as DFP and sbMIASTM that are able to base their selection on a subset of factors. In this variant, coverage decreased less compared to aborting the computation, but still did not increase for any tested limit.
tic computation early and hence does not reduce heuristic quality in these cases. Regarding the difference between $h^u$ and $h^{mf}$, it is again very small, showing no clear dominance trend.

**Conclusions**

In this paper, we provided an extensive theoretical and algorithmic description of the merge-and-shrink framework based on generalized label reduction. Furthermore, we described an efficient implementation in Fast Downward, highlighting improvements compared to a previous implementation. We also discussed partial merge-and-shrink abstractions and proposed imposing two simple limits on the algorithm for computing such abstractions. Our experimental evaluation showed that the optimized implementation clearly improves over the previous one. While limiting the number of transitions did not result in partial abstractions that are informed enough, imposing a time limit significantly pushed the performance of merge-and-shrink planners.

We think that partial merge-and-shrink abstractions have further potential. In particular, we would like to come up with a way of deterministically computing them rather than imposing a time limit which does not allow full reproducibility due to computational fluctuations. Furthermore, we want to investigate other alternatives for computing a merge-and-shrink heuristic given a set of partial factor heuristics, such as using cost partitioning techniques.

**Acknowledgments**

We thank the anonymous reviewers who helped improving the clarity of the paper. This work was supported by the European Research Council as part of the project “State Space Exploration: Principles, Algorithms and Applications”.

**References**


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**Table 4**: Compassion of the baseline against two versions of partial merge-and-shrink, using different time limits.

<table>
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<tr>
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