Strengthening Canonical Pattern Databases with Structural Symmetries

Silvan Sievers[1] Martin Wehrle[1] Malte Helmert[1]
Michael Katz[2]

[1]University of Basel, Switzerland
[2]IBM Watson Health, Haifa, Israel

June 16, 2017

Motivation

- Structural symmetries in recent work:
  - Symmetry-based pruning in forward search
  - Symmetric lookups
  - Enhancing merge-and-shrink heuristics

Motivation

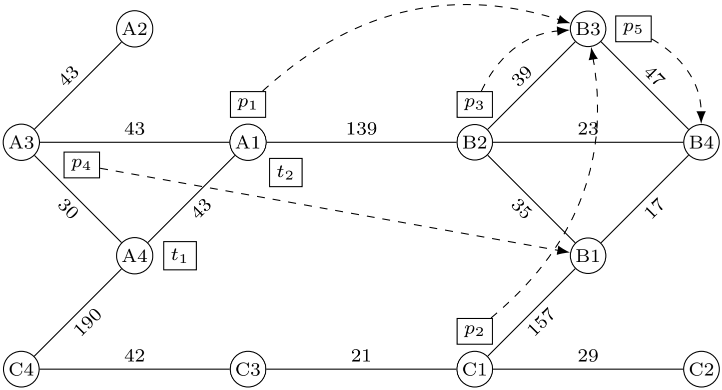
- Structural symmetries in recent work:
  - Symmetry-based pruning in forward search
  - Symmetric lookups
  - Enhancing merge-and-shrink heuristics
- In this work:
  - Symmetric pattern databases
  - Canonical PDB heuristic invariant under symmetry

Outline

Classical Planning

- Deterministic, fully observable, single-agent problems
- Initial state, many goal states
- Operators to transform states
- Find optimal plans
- Formalization: finite-domain state variables

Example



TRANSPORT-OPT11, #5

- v^{p_i}: variable for package p_i, v^{t_i}: variable for truck t_i

Pattern Databases

- Pattern:
  - Subset P of the state variables $\mathcal{V}$ of planning task Π
  - Induces abstract planning task Π^P
  - Pattern Database h^P: perfect heuristic values for Π^P

Pattern Databases

- Pattern:
  - Subset P of the state variables $\mathcal{V}$ of planning task Π
  - Induces abstract planning task Π^P
  - Pattern Database h^P: perfect heuristic values for Π^P
- Admissible combination of PDBs:
  - Maximum: always possible
  - Sum: disjoint-additive PDBs

Canonical PDB Heuristic

- Maximal-disjoint-additive subsets A of pattern collection C
- Sum PDB values whenever possible, maximize otherwise

$$h^{\mathcal{C}_C}(s) = \max_{B \in A} \sum_{P \in B} h^{P}(s)$$

Canonical PDB Heuristic

- Maximal-disjoint-additive subsets A of pattern collection C
- Sum PDB values whenever possible, maximize otherwise

$$h^{\mathcal{C}_C}(s) = \max_{B \in A} \sum_{P \in B} h^P(s)$$

- Example:

$C \quad \{v^{p_2}\}\ \{v^{p_3}\}\ \{v^{p_4}\}\ \{v^{p_5}\}\ \ \{v^{t_1}, v^{t_2}, v^{p_1}\}$

Canonical PDB Heuristic

- Maximal-disjoint-additive subsets A of pattern collection C
- Sum PDB values whenever possible, maximize otherwise

$$h^{\mathcal{C}_C}(s) = \max_{B \in A} \sum_{P \in B} h^P(s)$$

- Example:

C $\{v^{p_2}\}\ \{v^{p_3}\}\ \{v^{p_4}\}\ \{v^{p_5}\}\ \{v^{t_1}, v^{t_2}, v^{p_1}\}$

Canonical PDB Heuristic

- Maximal-disjoint-additive subsets A of pattern collection C
- Sum PDB values whenever possible, maximize otherwise

$$h^{\mathcal{C}_C}(s) = \max_{B \in A} \sum_{P \in B} h^P(s)$$

- Example:

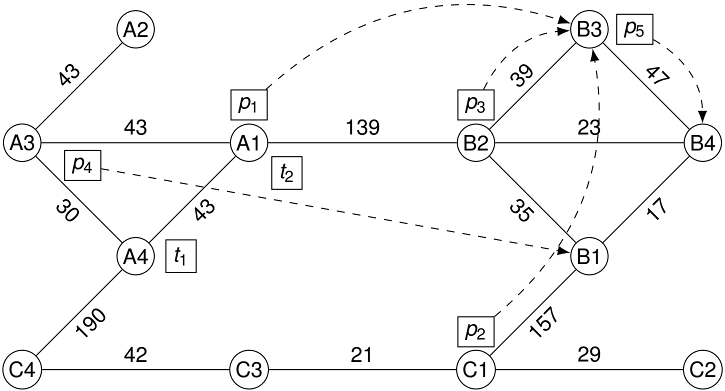
$$C \quad \{v^{p_2}\}\ \{v^{p_3}\}\ \{v^{p_4}\}\ \{v^{p_5}\}\ \{v^{t_1}, v^{t_2}, v^{p_1}\}$$

$$h^{\mathcal{C}_C}(s) = \max\{h^{\{v^{p_2}\}}(s) + h^{\{v^{p_3}\}}(s) + h^{\{v^{p_4}\}}(s) + h^{\{v^{p_5}\}}(s) + h^{\{v^{t_1}, v^{t_2}, v^{p_1}\}}(s)\}$$

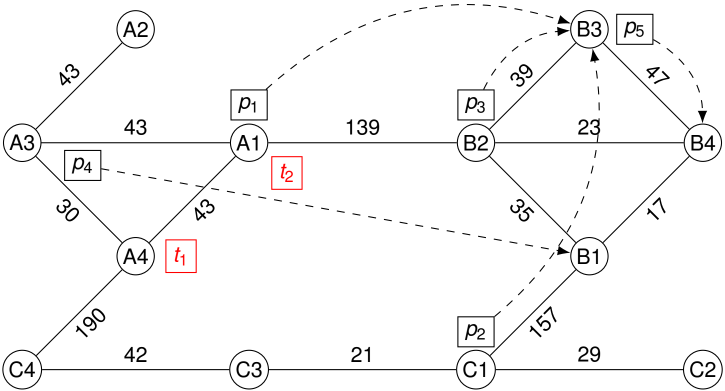
Structural Symmetries

- Permutation of variables, operators, and facts
- Goal-stable automorphisms: preserve structure

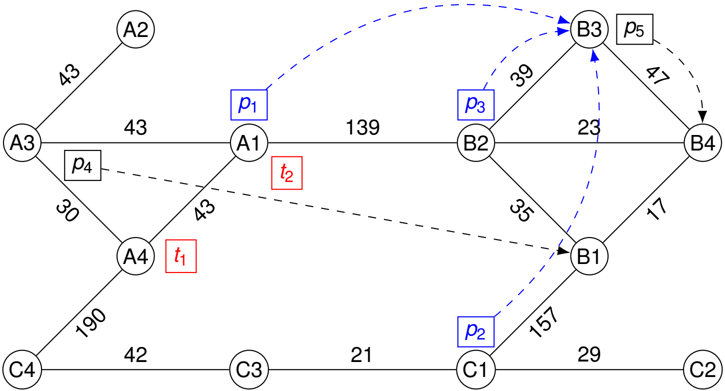
Example


A2
43
A3
43
A1
139
B2
23
B4
B3
39
47
p1
p3
p5
p4
t2
30
43
35
17
A4
t1
B1
190
157
p2
C4
42
C3
21
C1
29
C2

Example


A2
43
A3
43
p_1
A1
139
p_3
39
B3
p_5
47
B2
23
B4
p_4
t_2
30
43
35
17
A4
t_1
B1
190
p_2
157
C4
42
C3
21
C1
29
C2

Example


A2
43
A3
43
A1
139
B2
23
B4
B3
p_5
39
47
p_1
p_3
p_4
t_2
30
43
35
17
A4
t_1
B1
190
157
p_2
C4
42
C3
21
C1
29
C2

Outline

Symmetric Patterns

Definition

For pattern $P = \{v_1, \ldots, v_n\}$ and symmetry σ of planning task Π, the symmetric pattern is $\sigma(P) = \{\sigma(v_1), \ldots, \sigma(v_n)\}$.

Symmetric Patterns

Definition

For pattern $P = \{v_1, \ldots, v_n\}$ and symmetry σ of planning task Π, the symmetric pattern is $\sigma(P) = \{\sigma(v_1), \ldots, \sigma(v_n)\}$.

Theorem

For all states s of Π: $h^P(s) = h^{\sigma(P)}(\sigma(s))$.

Implicit PDBs

- Patterns P, Q with $\sigma(Q) = P$
- Alternative to computing both PDBs:
  - Compute h^P
  - Keep $\langle h^P, \sigma \rangle$ as implicit representation
  - Computation of implicit PDB: $h^Q(s) = h^P(\sigma(s))$

Symmetric and Disjoint-additive Pattern Collections

Definition

Pattern collection C is closed under symmetry group Γ if for all $\sigma \in \Gamma$ and for all $P \in C$, $\sigma(P) \in C$.

- $\overline{C}$ symmetric closure of C if $P, \sigma(P) \in \overline{C}$ for all $P \in C$

Symmetric and Disjoint-additive Pattern Collections

Definition

Pattern collection C is closed under symmetry group Γ if for all $\sigma \in \Gamma$ and for all $P \in C$, $\sigma(P) \in C$.

- $\overline{C}$ symmetric closure of C if $P, \sigma(P) \in \overline{C}$ for all $P \in C$

Theorem

If pattern collection C is disjoint-additive, then also $\overline{C}$ is disjoint-additive.

Invariance and Dominance of the CPDB Heuristic

Theorem

If pattern collection C is closed under symmetry group Γ, then for all states s of Π: $h^{\mathcal{C}_C}(s) = h^{\mathcal{C}_C}(\sigma(s))$.

Invariance and Dominance of the CPDB Heuristic

Theorem

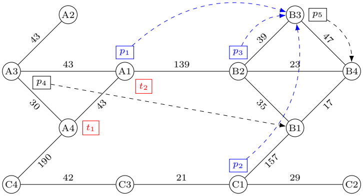
If pattern collection C is closed under symmetry group Γ, then for all states s of Π: $h^{\mathcal{C}_C}(s) = h^{\mathcal{C}_C}(\sigma(s))$.

Theorem

For pattern collection C and symmetry group Γ, for all states s of Π: $h^{\mathcal{C}_C}_{SL}(s) \leq h^{\mathcal{C}_{\overline{C}}}(s)$.

Algorithm – Example

C $\{v^{p_2}\}\ \{v^{p_3}\}\ \{v^{p_4}\}\ \{v^{p_5}\}\ \ \{v^{t_1}, v^{t_2}, v^{p_1}\}$



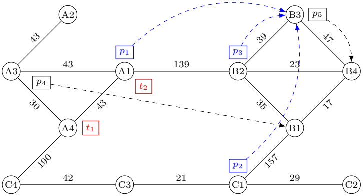
Algorithm – Example

C $\{v^{p_2}\}\ \{v^{p_3}\}\ \{v^{p_4}\}\ \{v^{p_5}\}\ \{v^{t_1}, v^{t_2}, v^{p_1}\}$

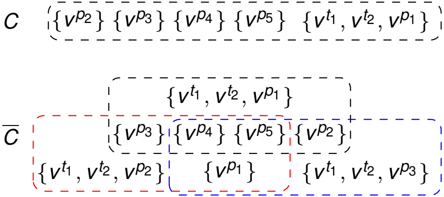
$\{v^{t_1}, v^{t_2}, v^{p_1}\}$

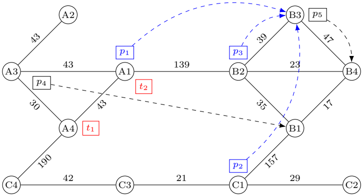
$\overline{C}$ $\{v^{p_3}\}\ \{v^{p_4}\}\ \{v^{p_5}\}\ \{v^{p_2}\}$

$\{v^{t_1}, v^{t_2}, v^{p_2}\}\quad \{v^{p_1}\}\quad \{v^{t_1}, v^{t_2}, v^{p_3}\}$

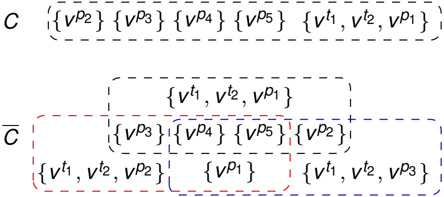


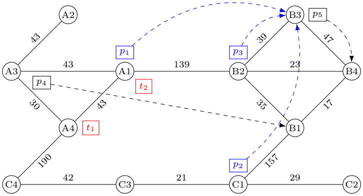
Algorithm – Example





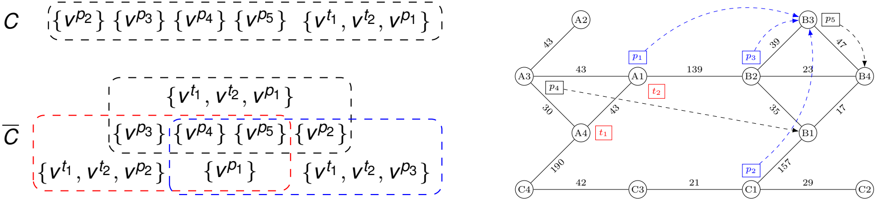
Algorithm – Example





- Example computations for the initial state:

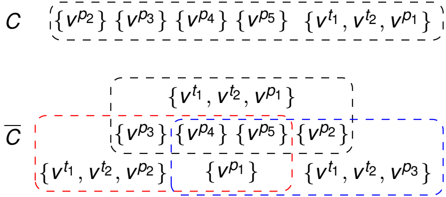
Algorithm – Example

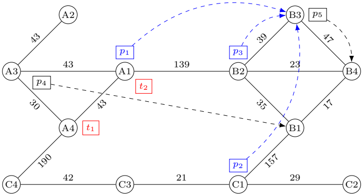


- Example computations for the initial state:

$$h^{\mathcal{C}_C}(s_0) = 2 + 2 + 2 + 2 + 180 = 188$$

Algorithm – Example





- Example computations for the initial state:

$$h^{\mathcal{C}_{\mathcal{C}}}(s_0) = 2 + 2 + 2 + 2 + 180 = 188$$

$$\begin{aligned} h^{\mathcal{C}_{\overline{\mathcal{C}}}}(s_0) = \max\{&180 + 2 + 2 + 2 + 2, \\ &476 + 2 + 2 + 2 + 2, \\ &180 + 2 + 2 + 2 + 2\} = 484 \end{aligned}$$

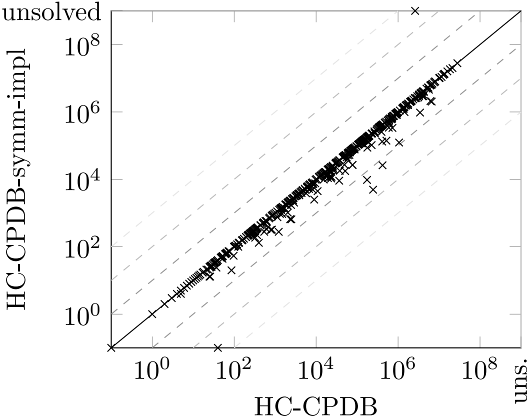
Outline

Results for A^*

	HC-CPDB		
	orig	**symm**	**symm-impl**
Coverage (# solved tasks)	814	813	813
Search out of memory	774	736	730
Search out of time	70	109	115

- Not shown: dominance over symmetric lookups

Expansions



(dominance in 194 task across 33 domains)

Results for Symmetry-based Pruning

	HC-CPDB with DKS		
	orig	**symm**	**symm-impl**
Coverage (# solved tasks)	887	893	891
Expansions 95th percentile	3510224	2584593	2584593

Conclusions

- Implicit PDBs: trade-off between memory and runtime
- CPDB heuristic invariant under symmetry if using symmetric closure of pattern collection
- Fruitful combination with symmetry-based pruning methods