Theoretical Foundations for Structural Symmetries of Lifted PDDL Tasks

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Motivation

Symmetries arise in many areas:
- Model checking
- SAT
- Petri nets
- Planning

Planning symmetries (mostly) of ground representations

Potential application of symmetries before grounding:
- Invariant synthesis/Speed up grounding
- Task transformations
Contributions

- Transfer **structural symmetries** to **lifted** planning tasks
- Investigate **relationship** between lifted and ground symmetries
- Provide **graph representation** of planning tasks for computing symmetries
- Quantitative analysis of IPC benchmarks
Outline

1. Structural Symmetries
2. Relationship to Ground Symmetries
3. Graph Representation
4. Quantitative Analysis
Abstract Structures

- $S$: set of symbols $s$, each with type $t(s)$
- Abstract structures over $S$:
  - $s \in S$ abstract structure
  - If $A_1, \ldots, A_n$ abstract structures, then also $\langle A_1, \ldots, A_n \rangle$ and $\{A_1, \ldots, A_n\}$ abstract structures
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- **Example**:
  - $S = \{a, b, c, d\}$, $t(a) = t(b) = t_1$, $t(c) = t(d) = t_2$
  - $A = \{\langle a, c \rangle, \langle b, d \rangle, \{c, d\}\}$
Symbol mapping $\sigma$: permutation of $S$ with $t(\sigma(s)) = t(s)$

Induced abstract structure mapping $\tilde{\sigma}$:

$$\tilde{\sigma}(A) := \begin{cases} 
\sigma(A) & \text{if } A \in S \\
\{\tilde{\sigma}(A_1), \ldots, \tilde{\sigma}(A_n)\} & \text{if } A = \{A_1, \ldots, A_n\} \\
\langle \tilde{\sigma}(A_1), \ldots, \tilde{\sigma}(A_n) \rangle & \text{if } A = \langle A_1, \ldots, A_n \rangle
\end{cases}$$

$\sigma$ structural symmetry for abstract structure $A$ if $\tilde{\sigma}(A) = A$
Example

- Abstract structure:
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- Symmetry:
  - Symbol mapping \( \sigma \): swap \( a \) with \( b \), swap \( c \) with \( d \)
  - \( \sigma \) is structural symmetry: \( \tilde{\sigma}(A) = \{\langle b, d \rangle, \langle a, c \rangle, \{d, c\}\} = A \)
Lifted Planning Tasks as Abstract Structures

- Lifted representation: normalized PDDL with action costs
- Lifted planning task $\Pi$ as abstract structure:
  - Symbols with the following types: object, variable, predicate, function, negation, $n \in \mathbb{N}$
  - Abstract structures for modeling atoms, literals, function terms, operators, axioms
Example Operator (Spanner)

(:action pick-up
  :parameters (?s ?l)
  :precondition
    (and (LOCATION ?l)
      (SPANNER ?s)
      (bob-at ?l)
      (spanner-at ?s ?l))
  :effect
    (and (not (spanner-at ?s ?l))
      (carrying ?s)
      (increase (total-cost) 1)))

⟨{s, l},
  {⟨location, l⟩, ⟨spanner, s⟩, ⟨bob-at, l⟩, ⟨spanner-at, s, l⟩},
  {⟨∅, ∅, ⟨¬, ⟨spanner-at, s, l⟩⟩}, ⟨∅, ∅, ⟨carrying, s⟩⟩\}
  1⟩
Full Grounding

- \( \text{ground}(\Pi) \): fully grounded planning task \( \Pi \)
Full Grounding

- $ground(\Pi)$: fully grounded planning task $\Pi$

Theorem

If $\sigma$ is a structural symmetry for planning task $\Pi$, then $\sigma$ is a structural symmetry for $ground(\Pi)$. 
Rational Grounding

- Full grounding infeasible in practice
- Optimized grounding: remove some irrelevant part of the task representation (reachability analysis)
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- Optimized grounding: remove some irrelevant part of the task representation (reachability analysis)
- Rational grounding (\(\text{ground}_{\text{rat}}(\Pi)\)): remove all or no symmetric irrelevant parts

**Theorem**

*If \(\sigma\) is a structural symmetry for planning task \(\Pi\), then \(\sigma\) is a structural symmetry for \(\text{ground}_{\text{rat}}(\Pi)\).*
Propositional STRIPS tasks: set of symbols contains atoms
Relationship to Propositional STRIPS Symmetries

- Propositional STRIPS tasks: set of symbols contains atoms
- Representational differences:
  - Example symmetry of STRIPS task $\Pi$: 
    $\sigma(P(a)) = P(a)$ and $\sigma(P(b)) = Q(b)$
  - No analogous symmetry with abstract structures: cannot map predicate $P$ to both $Q$ and $P$
Propositional STRIPS tasks: set of symbols contains atoms

Representational differences:

- Example symmetry of STRIPS task $\Pi$:
  \[ \sigma(P(a)) = P(a) \text{ and } \sigma(P(b)) = Q(b) \]
- No analogous symmetry with abstract structures: cannot map predicate $P$ to both $Q$ and $P$

Other direction:

- If $\sigma$ symmetry of ground task $\Pi$ (in our definition), then $\sigma$ also symmetry of $\Pi$ (in STRIPS)
- If $\sigma$ symmetry of lifted task $\Pi$, then $\sigma$ also transition graph symmetry
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Properties

Let $A$ be an abstract structure.

**Theorem**

*Every colored graph automorphism of $\text{ASG}_A$ induces a structural symmetry of $A$.*
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**Theorem**

*Every structural symmetry of $A$ induces a colored graph automorphism of $\text{ASG}_A$.*
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Summarized Results

- Roughly 53% of IPC tasks with lifted symmetries
- Ground symmetry groups often larger than lifted ones
- Quick computation using abstract structure graphs
Size of PDG vs. Abstract Structure Graph

The graph shows the relationship between the size of Partially Defined Graphs (PDGs) and Abstract Structure Graphs (ASGs) across various domains. Each domain is represented by a different marker, with labels indicating the specific problem or domain. The x-axis represents the size of PDGs, while the y-axis represents the size of ASGs. The graph includes markers for other domains, AIRPORT, HIKING, STRIPS, LOGISTICS, MICONIC-FULLADL, MPRIME, MYSTERY, NOMYSTERY-OPT11, NOMYSTERY-SAT11, PARKING-OPT14, PARKING-SAT14, PIPESWORLD-TANKAGE, SATELLITE, SCANALYZER-OPT11, SCANALYZER-SAT11, TETRIS-OPT14, TETRIS-SAT14, TIDYBOT-SAT11, TPP, TRUCKS, and ZENOTRAVEL.
Conclusions

Summary:

- Structural symmetries of the lifted representation
- Lifted symmetries also ground symmetries
- Graph representation of planning tasks
- Many lifted symmetries in IPC benchmarks
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Future work:
- Accelerated computation of invariants/grounding: consider only subset of (symmetric) objects (ICAPS 2018)
- Task transformations