

# Bounded Intention Planning Revisited

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## 1 INTRODUCTION

Bounded Intention Planning (BIP) [7] is a pruning approach for optimal planning with unary operators. BIP has the flavor of partial order reduction, which has recently found increasing interest for optimal planning [4, 6]. However, although BIP is claimed to be a variant of *stubborn sets* [3] in the original paper, no proof is given for this claim. In this paper, we shed light on the relationship of BIP and stubborn sets. In particular, we show that BIP’s operator partitions *sometimes* correspond to strong stubborn sets defined for planning [5].

## 2 PRELIMINARIES

We consider SAS<sup>+</sup> planning with action costs. A *planning task* is a 4-tuple  $\Pi = \langle \mathcal{V}, \mathcal{O}, s_0, s_* \rangle$ , where  $\mathcal{V}$  is a finite set of *state variables*,  $\mathcal{O}$  is a finite set of *operators*,  $s_0$  is the *initial state* and  $s_*$  is the *goal*. Every  $v \in \mathcal{V}$  has a finite domain  $\mathcal{D}(v)$ . A variable assignment on a subset of  $\mathcal{V}$  is called a *partial state*  $s$ ; we denote the set of variables mentioned in  $s$  by  $\text{vars}(s)$ . A partial state is a *state* if  $\text{vars}(s) = \mathcal{V}$ . By  $s[v]$  we refer to the value of  $v$  in  $s$ . A (partial) state  $s$  *complies* with a (partial) state  $s'$  iff  $s[v] = s'[v]$  for all  $v \in \text{vars}(s) \cap \text{vars}(s')$ . The initial state  $s_0$  is a state and  $s_*$  is a partial state. Every  $o \in \mathcal{O}$  has a *precondition*  $\text{pre}_o$ , an *effect*  $\text{eff}_o$  and a *prevail-condition*  $\text{prv}_o$ , which are partial states, and associated *cost*  $c(o) \in \mathbb{R}_0^+$ . If  $v \in \text{vars}(\text{eff}_o)$ , then  $v \notin \text{vars}(\text{prv}_o)$  and possibly  $v \in \text{vars}(\text{pre}_o)$ ; otherwise  $v \notin \text{vars}(\text{pre}_o)$  and possibly  $v \in \text{vars}(\text{prv}_o)$ . An operator  $o$  is *applicable* in  $s$  if both  $\text{pre}_o$  and  $\text{prv}_o$  comply with  $s$ . The result of applying  $o$  in  $s$  is the *successor state*  $s'$  that complies with  $\text{eff}_o$  and satisfies  $s'[v] = s[v]$  for all  $v \notin \text{vars}(\text{eff}_o)$ . A sequence of operators  $o_1, \dots, o_n \in \mathcal{O}$  is called an *s-plan* if applying all operators in sequence, starting at state  $s$ , results in a state complying with  $s_*$ . A *plan* for a task  $\Pi$  is defined as an  $s_0$ -plan for  $\Pi$ . A plan is *optimal* if its cost  $\sum_{i=1}^n c(o_i)$  is minimal among all plans. The objective of optimal planning is to find an optimal plan or to prove that no plan exists.

### 2.1 BOUNDED INTENTION PLANNING

We introduce the essential parts of bounded intention planning (BIP), which we will relate to stubborn sets afterwards.

BIP is defined for *unary* planning tasks ( $|\text{vars}(\text{eff}_o)| = 1$  for all  $o \in \mathcal{O}$ ). Roughly speaking, BIP augments the original planning task  $\Pi$  with *intention variables* and defines several “intermediate” operators for each original operator. The resulting augmented planning task  $\bar{\Pi} = (\bar{\mathcal{V}}, \bar{\mathcal{O}}, \bar{s}_0, \bar{s}_*)$  can then be exploited for pruning. Let  $CG$  be the *causal graph* [1] of  $\Pi$ , which is a directed graph with nodes  $\mathcal{V}$  and edges from  $v$  to  $w$  iff there exists an operator  $o \in \mathcal{O}$  with  $v \in \text{prv}_o$  and  $w \in \text{eff}_o$  (recall that  $\mathcal{O}$  only contains unary operators,

so there are no edges between effect variables). We denote the successors of  $v$  in  $CG$  by  $CG(v)$ . Furthermore, let  $\mathcal{O}[v] \subseteq \mathcal{O}$  denote the operators  $o$  with  $v \in \text{vars}(\text{eff}_o)$ .

For every  $v \in \mathcal{V}$ , the augmented variable set  $\bar{\mathcal{V}}$  contains  $v$  and two additional intention variables  $O_v$  and  $C_v$ .  $O_v$  has domain  $\mathcal{D}(O_v) = \mathcal{O}[v] \cup \{\text{free}, \text{frozen}\}$ , and  $C_v$  domain  $\mathcal{D}(C_v) = CG(v) \cup \{\text{free}\}$ .

For all  $v \in \mathcal{V}$ , the augmented operator set  $\bar{\mathcal{O}}$  contains the following operators: first, for every  $o \in \mathcal{O}[v]$ , there is a “set operator intention” operator  $\text{SetO}(o)$  with  $\text{pre}[O_v] = \text{free}$ ,  $\text{eff}[O_v] = o$  and  $\text{prv}[v] = \text{pre}_o[v]$ , with cost  $c(o)$ ; second, for every  $x \in \mathcal{D}(v)$ , there is a zero-cost “freeze operator intention” operator  $\text{Freeze}(v, x)$  with  $\text{pre}[O_v] = \text{free}$  and  $\text{eff}[O_v] = \text{frozen}$ ; third, for every  $v \in \mathcal{V}$  and  $c \in CG(v)$ , there is a zero-cost “set child intention” operator  $\text{SetC}(v, c)$  with  $\text{pre}[C_v] = \text{free}$  and  $\text{eff}[C_v] = c$ ; fourth, for every  $o \in \mathcal{O}[v]$ , there is a zero-cost “fire” operator  $\text{Fire}(o)$  which has the same conditions and effects as  $o$  and in addition  $\text{pre}[O_v] = o$ ,  $\text{eff}[O_v] = \text{free}$ , and for all  $w \in \text{vars}(\text{prv}_o)$ ,  $\text{pre}[O_w] = \text{frozen}$ ,  $\text{eff}[O_w] = \text{free}$  and  $\text{pre}[C_w] = v$ ,  $\text{eff}[C_w] = \text{free}$ .

The augmented initial state  $\bar{s}_0$  extends  $s_0$  by setting all new  $O_v$  and  $C_v$  variables to *free*. The augmented goal  $\bar{s}_*$  is the same as  $s_*$ .

BIP *partitions* the operators into partitions of three types: for each  $v \in \mathcal{V}$  and  $x \in \mathcal{D}(v)$ , there is a partition  $\text{SetO}_{v=x} = \{\text{SetO}(o) \mid o \in \mathcal{O}[v] \wedge \text{pre}_o[v] = x\} \cup \{\text{Freeze}(v, x)\}$ ; for each  $v \in \mathcal{V}$ , there is a partition  $\text{SetC}_v = \{\text{SetC}(v, c) \mid c \in CG(v)\}$ ; for each  $o \in \mathcal{O}$ , there is a partition  $\text{Fire}_o = \{\text{Fire}(o)\}$ .

Let  $P$  denote the set of all such partitions. By definition, either *all* operators in a partition are applicable or *none*. We denote the set of applicable partitions for a given state  $s$  with  $P_s$ . The central theorem of Wolfe and Russell [7] states that we can choose a *single, arbitrary* partition from  $P_s$  in every state  $s$  and still preserve optimality. Branching is restricted to operators within this partition.

### 2.2 STUBBORN SETS

Stubborn sets were introduced by Valmari [3]. To state the definition (adapted to planning tasks), we need the concept of *necessary enabling sets (NES)*. For a state  $s$  and operator  $o$  not applicable in  $s$ , a NES for  $o$  and  $s$  is a set of operators such that all operator sequences that lead from  $s$  to some goal state and include  $o$  contain some operator from the NES before the first occurrence of  $o$ .

Operator  $o$  *disables* operator  $o'$  if there is a variable  $v$  with values  $\{x, x'\} \subseteq \mathcal{D}(v)$  such that  $x \neq x'$ ,  $\text{eff}_o[v] = x$  and either  $\text{prv}_{o'}[v] = x'$  or  $\text{pre}_{o'}[v] = x'$ . Operators  $o$  and  $o'$  *interfere* if  $o$  disables  $o'$ , or vice versa, or  $\text{eff}_o[v] = x$  and  $\text{eff}_{o'}[v] = x'$  for  $x' \neq x$ .

**Definition 1.** A set of operators  $T_s \subseteq \mathcal{O}$  of a task  $\Pi$  is a strong semistubborn set in state  $s$  iff for all  $o \in T_s$  not applicable in  $s$ ,  $T_s$  contains a necessary enabling set for  $s$ , and for all  $o \in T_s$  applicable in  $s$ ,  $T_s$  contains all operators that interfere with  $o$ .

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A set of operators  $T_s \subseteq \mathcal{O}$  is a strong stubborn set in the Valmari sense iff  $T_s$  is a strong semistubborn set, and  $T_s$  contains at least one applicable operator in  $s$  if such an operator exists.

Strong stubborn sets in the Valmari sense guarantee to preserve deadlocks, but not goal reachability. However, goal reachability can be reduced to deadlock detection, which yields corresponding definitions for planning tasks [4, 5]. We provide the definition of generalized strong stubborn sets given by Wehrle and Helmert [5], simplified to the setting needed for this paper. A state  $s$  is called *solvable* iff there exists an  $s$ -plan. Furthermore, following Wehrle and Helmert, we call a plan  $\pi$  *strongly optimal* iff  $\pi$  is an optimal plan with minimum number of zero-cost operators.

**Definition 2.** A set of operators  $T_s \subseteq \mathcal{O}$  of a task  $\Pi$  is a generalized strong stubborn set (GSSS) in the planning sense in the solvable state  $s$  iff  $T_s$  is a strong semistubborn set in  $s$ , and  $T_s$  contains at least one operator from at least one strongly optimal plan starting in  $s$ .

### 3 RELATION TO STUBBORN SETS

BIP’s applicable operator partitions induce strong semistubborn sets that contain exactly the same applicable operators.

**Theorem 1.** Let  $s$  be a state,  $X \in P_s$  be an applicable partition. Then  $T_s := X \cup \{o \mid o \text{ interferes with } o' \in X\}$  is a strong semistubborn set with the same applicable operators as  $X$ .

*Proof sketch.* We exemplarily prove the claim for the case that  $X$  is a partition  $\text{Fire}_o = \{\text{Fire}(o)\}$ . All remaining cases follow a similar argumentation. The full proof can be found in a technical report [2].

Without loss of generality, assume  $v \in \text{vars}(\text{eff}_o)$ . We have to show that all operators that interfere with  $\text{Fire}(o)$  are not applicable in  $s$  and that  $T_s$  already contains a NES for those operators. We show the claim for the most involved case where an operator  $\text{Fire}(o')$  interferes with  $\text{Fire}(o)$ . Let us assume that  $\text{Fire}(o')$  disables  $\text{Fire}(o)$  via variable  $w \in \mathcal{V}$ ,  $w \neq v$ , with  $\text{eff}_{o'}[w] \neq \text{prv}_o[w]$ .

We claim that  $\text{Fire}(o')$  is not applicable in  $s$ . By the definition of  $\text{Fire}$  operators, we have  $\text{pre}_{\text{Fire}(o)}[O_w] = \text{frozen}$  and thus  $s[O_w] = \text{frozen}$  because  $\text{Fire}(o)$  is applicable in  $s$ . Again by definition, we have  $\text{pre}_{\text{Fire}(o')}[O_w] = o' \neq s[O_w]$ , proving the claim.

Furthermore, we claim that  $\{\text{Fire}(o)\}$  is a NES for  $\text{Fire}(o')$  in  $s$ . We observe that the value of  $O_w$  must change from *frozen* to *free* before it can be set to  $o'$  as required by  $\text{Fire}(o')$ . Only some operator  $\text{Fire}(\hat{o})$  with  $w \in \text{vars}(\text{prv}_{\hat{o}})$  can set  $O_w$  to *free*. Let  $v' \in \text{vars}(\text{eff}_{\hat{o}})$ . If  $v' = v$ ,  $\text{pre}_{\text{Fire}(\hat{o})}[O_v] = \hat{o} \neq o = s[O_v]$  and thus  $\text{Fire}(o)$  must be applied first. If  $v' \neq v$ ,  $\text{pre}_{\text{Fire}(\hat{o})}[C_w] = v' \neq v = s[C_w]$  and only some operator  $\text{Fire}(\hat{o}')$  with  $v \in \text{vars}(\text{eff}_{\hat{o}'})$  and  $w \in \text{vars}(\text{prv}_{\hat{o}'})$  can change  $C_w$  from  $v$  to *free* (required before setting it to  $v'$ ). Because  $s[O_v] = o$ , this must be  $\text{Fire}(o)$ , proving the claim.  $\square$

As the induced semistubborn set  $T_s$  contains an applicable operator partition by definition, it contains at least one applicable operator.

**Corollary 1.** Let  $s$  be a state,  $X \in P_s$  be an applicable operator partition. Then  $T_s$  induced by  $X$  defined in Theorem 1 is a strong stubborn set in the Valmari sense.

As shown by Wolfe and Russell, every applicable operator partition starts an optimal plan. However, not all such partitions contain an operator that starts a *strongly optimal* plan<sup>2</sup>, which is the missing

<sup>2</sup> Consider a task with variables  $v$  and  $w$ , initially 0, goal  $w = 1$  and two operators that can set  $v$  and  $w$  to 1. Applying the zero-cost operator  $\text{Freeze}(v, 0)$  from partition  $\text{SetO}_{v=0}$  is neither required nor corrupting an optimal plan (as it does not disable any operator from other partitions).

criterion for  $T_s$  to be a GSSS in the planning sense (cf. Definition 2). Nevertheless, there always exists at least one partition in  $P_s$  which induces a GSSS in the planning sense.

**Theorem 2.** Let  $s$  be a solvable state. Then, for at least one operator partition  $X \in P_s$ , the induced strong semistubborn set  $T_s$  defined in Theorem 1 contains an operator that starts a strongly optimal plan in  $s$ . Hence  $T_s$  is a GSSS in the planning sense.

*Proof.* Because  $s$  is solvable, there exists an operator  $o$  that starts a strongly optimal plan in  $s$ . As  $P_s$  contains exactly the applicable operators, one of these partitions contains  $o$ .  $\square$

We observe that only partitions inducing a GSSS in the planning sense are needed to find strongly optimal plans, whereas the others could ultimately be ignored. However, deciding if a partition induces a GSSS in the planning sense is computationally hard. Wolfe and Russell propose a heuristic criterion to select “good” partitions, which prefers partitions that resolve existing intentions. This in turn corresponds to selecting partitions inducing a GSSS in the planning sense. Hence, our analysis in particular sheds light on *what* the heuristic proposed in BIP computes and *why* it is reasonable.

### 4 CONCLUSION

BIP’s operator partition pruning can be viewed as a stubborn set method: every applicable operator partition  $X$  induces a strong stubborn set in the Valmari sense with the same pruning power as  $X$ , and for every state, there must be at least one such partition that induces a generalized strong stubborn set in the planning sense.

Apart from the theoretical results obtained so far, our analysis also points us to interesting future research directions. In particular, as the “good” operator partitions are related to strong stubborn sets which are defined for arbitrary (non-unary) operators, it will be interesting to investigate if BIP can be generalized to arbitrary operators based on this insight – this question is considered as “most important” in Wolfe and Russell’s future work description. We think that our analysis provides a promising starting point for this research goal.

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