Generalized Label Reduction for Merge-and-Shrink Heuristics

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Outline

1. Background
2. Generalized Label Reduction
3. Experiments
Merge-and-Shrink Heuristics

- Distance heuristics for state space search
  (Dräger et al. (2006), Helmert et al. (2007),
  Nissim et al. (2011), Helmert et al. (2014))

- Idea:
  - Represent state space as set of small finite automata
  - State space corresponds to product of automata
  - Transform automata to obtain distance heuristic for state space

- Applicable for classical planning and many other state space search problems
Example
Merge-and-Shrink Transformations (1)

- **Merge:** replace two automata by their product automaton
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Merge-and-Shrink Transformations (1)

- **Merge**: replace two automata by their product automaton

![Diagram showing the merge operation]

- **Exact transformation**: preserves distances in represented state space

![Diagram showing the exact transformation]
Merge-and-Shrink Transformations (2)

- **Shrink**: abstract one automaton
Merge-and-Shrink Transformations (2)

- **Shrink**: abstract one automaton
**Merge-and-Shrink Transformations (2)**

- **Shrink**: abstract one automaton

![Diagram showing an example of Shrink transformation](image)
Merge-and-Shrink Transformations (2)

- **Shrink**: abstract one automaton
**Merge-and-Shrink Transformations (2)**

- **Shrink**: abstract one automaton

- **Safe** transformation: does not increase distances in represented state space
  (Exact with bisimulation, Nissim et al. (2011))
Proof Sketch for Theorem 5.11 of Helmert et al. (2014)

We prove by induction over the construction of $T^\alpha$ that, for any intermediate merge-and-shrink abstraction $\beta$ over $V'$: $\Theta^\tau_\beta = \Theta^\beta$ if $v^* \notin V'$, and $\Theta^\tau_\beta = \Theta^\beta|_{\tau \overline{V'}}$ if $v^* \in V'$. The single tricky case in the induction is the case where $\beta = \alpha_1 \otimes \alpha_2$ and (WLOG) $v^* \in V_1$. Using the induction hypothesis, we then need to prove that

$$(\Theta^{\alpha_1}|_{\tau \overline{V_1}} \otimes \Theta^{\alpha_2}|_{\tau \overline{V_1}})|_{\tau \overline{V_1 \cup V_2}} = \Theta^{\alpha_1 \otimes \alpha_2}|_{\tau \overline{V_1 \cup V_2}}.$$ 

Since $\tau \overline{V_1}$ is conservative for $\Theta^\pi\overline{V_1}$, with $V_2 \subseteq \overline{V_1}$ and Proposition 5.4, it is conservative also for $\Theta^{\alpha_2}$. Hence, Lemma 5.6 reduces the left-hand side of our proof obligation to $((\Theta^{\alpha_1} \otimes \Theta^{\alpha_2})|_{\tau \overline{V_1}})|_{\tau \overline{V_1 \cup V_2}}$, which with $\tau \overline{V_1 \cup V_2} \circ \tau \overline{V_1} = \tau \overline{V_1 \cup V_2}$ is equal to $(\Theta^{\alpha_1} \otimes \Theta^{\alpha_2})|_{\tau \overline{V_1 \cup V_2}}$. The claim then follows with Theorem 4.5.
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Since $\tau V_1$ is conservative for $\Theta^{\pi V_1}$, with $V_2 \subseteq V_1$ and Proposition 5.4, it is conservative also for $\Theta^{\alpha_2}$. Hence, Lemma 5.6 reduces the left-hand side of our proof obligation to $((\Theta^{\alpha_1} \otimes \Theta^{\alpha_2})|_{\tau V_1})|_{\tau V_1 \cup V_2}$, which with $\tau V_1 \cup V_2 \circ \tau V_1 = \tau V_1 \cup V_2$ is equal to $(\Theta^{\alpha_1} \otimes \Theta^{\alpha_2})|_{\tau V_1 \cup V_2}$. The claim then follows with Theorem 4.5.
Previous Label Reduction and its Flaws

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- Full potential restricted to linear merge strategies
- Based on syntax of underlying planning operators
Outline

1. Background
2. Generalized Label Reduction
3. Experiments
Contribution

- Clear, easy and complete definition of label reduction
- Theoretic investigation: properties of label reduction (safeness and exactness)
- Empirical investigation for classical planning
Generalized Label Reduction

- Replace all labels of a chosen set by one chosen new label in all automata
Generalized Label Reduction

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Replace all labels of a chosen set by one chosen new label in all automata
Theorem: Safeness

_label reduction is safe, i.e. leaves the heuristic admissible._
Definitions

- Labels are **locally equivalent** in automaton $\Theta$ if they label the same set of transitions in $\Theta$.
- Labels are $\Theta$-**combinable** if they are locally equivalent in all automata but $\Theta$.
- Label $\ell_1$ **globally subsumes** label $\ell_2$ if the set of transitions labeled by $\ell_2$ is a subset of the transitions labeled by $\ell_1$ in all automata.
Theorem: Exactness

A label reduction which maps labels $\ell_1$ and $\ell_2$ onto a new label $\ell$ is exact, i.e. leaves the heuristic perfect, if and only if

1. $\ell_1$ globally subsumes $\ell_2$, or
2. $\ell_2$ globally subsumes $\ell_1$, or
3. $\ell_1$ and $\ell_2$ are $\Theta$-combinable for some automaton $\Theta$ of the set of automata.
Outline

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Results: Usefulness of Label Reduction

Expansions:

DFP-B-N50k, no label reduction
DFP-B-N50k, new label reduction
Results: Old vs. New Label Reduction Method

Construction time:

RL-B-N100k, original label reduction
RL-B-N100k, new label reduction
Conclusion

- **Generalized** label reduction for merge-and-shrink heuristics:
  - Cleaner and easier definition
  - Safe and unrestricted transformation
  - Exact transformation if based on $\Theta$-combinability

- Empirical **performance gain** for merge-and-shrink heuristics in classical planning

- Opened possibilities to develop even better merge-and-shrink heuristics
Thank you!
## Results: Coverage

### Coverage:

<table>
<thead>
<tr>
<th>merge/shrink strategy</th>
<th>Label Reduction</th>
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