A Comparison of Cost Partitioning Algorithms for Optimal Classical Planning

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Cost Partitioning

Distributing operator costs among multiple cost functions $C = \{c_1, \ldots, c_n\}$, where

$$\sum_{i=1}^{n} c_i(o) \leq c_i(o) \text{ for all operators } o,$$

makes sum of heuristic values under $C$ admissible:

$$h^\tau(s) := \sum_{i=1}^{n} h_i(s, c_i).$$

Example Abstractions

$\begin{array}{c}
\begin{array}{c}
O_1 \quad O_2 \quad O_3
\end{array}
\begin{array}{c}
S_1, S_2 \quad S_3 \quad S_4, S_5
\end{array}
\begin{array}{c}
O_1 \quad O_2 \quad O_3
\end{array}
\begin{array}{c}
S_1 \quad S_2, S_3, S_4 \quad S_5
\end{array}
\end{array}$

$\begin{array}{c}
\text{cost}(o_1) = \text{cost}(o_2) = 4, \quad \text{cost}(o_3) = 1
\end{array}$

$\Rightarrow h^{OCP} = h^{SCP} = 8, \quad h^{OUCP} = 7, \quad h^{PHO} = h^{CAN} = h^{GZOCP} = 5$

Cost Partitioning Algorithms

- Optimal Cost Partitioning
  - Cost partitioning with highest heuristic value for a given state among all cost partitionings

- Post-hoc Optimization
  - Let $(w_1, \ldots, w_n)$ be a solution to the linear program that maximizes
    $\sum_{i=1}^{n} w_i \cdot h_i(s)$ subject to
    $\sum_{i=1}^{n} w_i \leq 1$ for all operators $o$ relevant for $h_i$
    $w_i \geq 0$.
  - Use costs $w_i \cdot \text{cost}(o)$ if $o$ is relevant for $h_i$, otherwise 0

- Greedy Zero-one Cost Partitioning
  - Order heuristics
  - Use full costs for the first relevant heuristic

- Saturated Cost Partitioning
  - Order heuristics
  - For each heuristic $h_i$:
    - Use minimum costs preserving all heuristic estimates for $h_i$
    - Use remaining costs for subsequent heuristics

- Uniform Cost Partitioning
  - Distribute costs uniformly among relevant heuristics

- Opportunistic Uniform Cost Partitioning (New)
  - Order heuristics
  - For each heuristic $h_i$:
    - Distribute costs uniformly among relevant, unconsidered heuristics
    - Use remaining costs for subsequent heuristics

- Canonical Heuristic
  - Maximum over additive (independent) heuristic subsets

Theoretical Comparison

$\begin{array}{c}
\begin{array}{c}
\mu^{SCP} \quad \mu^{OCP} \quad \mu^{GZOCP} \quad \mu^{CAN}
\end{array}
\begin{array}{c}
\gamma \geq 1 \text{ order}
\end{array}
\end{array}$

Experimental Comparison: Systematic PDBs

| $\mu^{OCP}$ | $\mu^{SCP}$ | $\mu^{OUCP}$ | $\mu^{GZOCP}$ | $\mu^{CAN}$ | $\mu^{OUCP}_\text{one}$ | $\mu^{GZOCP}_\text{one}$ | $\mu^{CAN}_\text{one}$ | $\mu^{PHO}$ | $\mu^{OCP}$ | Coverage | Std. Dev. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 3 | 15 | 4 | 11 | 10 | 30 | 709.0 | – |
| 14 | 9 | 22 | 8 | 6 | 0 | 14 | 13 | 31 | 744.9 | 3.07 |
| 13 | 8 | 22 | 7 | 6 | 0 | 14 | 13 | 31 | 734.6 | 2.01 |
| 3 | 1 | 4 | 3 | 0 | 9 | 11 | 29 | 694.0 | 2.58 |
| 15 | 12 | 14 | 20 | 9 | 0 | 13 | 13 | 30 | 749.9 | 1.66 |
| 20 | 19 | 17 | 23 | 16 | 0 | 18 | 21 | 32 | 775.7 | 4.47 |
| 27 | 26 | 24 | 28 | 22 | 23 | 26 | 33 | 854.9 | 2.33 |
| 8 | 7 | 7 | 17 | 5 | 8 | 2 | 13 | 28 | 656.0 | – |
| 9 | 7 | 15 | 7 | 6 | 3 | 10 | 31 | 737.0 | – |
| 4 | 4 | 4 | 4 | 4 | 3 | 5 | 3 | 471.0 | – |

Discussion of Experimental Comparison

- Results for hill climbing PDBs, Cartesian abstractions and landmark heuristics in paper
- Beneficial to reuse unused costs, to assign them greedily and to use multiple orders
- Saturated cost partitioning method of choice in all tested settings

Comparison of Different Heuristics (Using $h^2$ Mutexes)

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<th>Cart + SCP</th>
<th>LM + SCP</th>
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