Negated Occurrences of Predicates in PDDL Axiom Bodies

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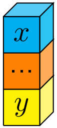
PDDL Axioms

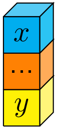
$$\textit{above}(x, y) \leftarrow \textit{on}(x, y) \lor \exists z\, (\textit{on}(x, z) \land \textit{above}(z, y))$$



PDDL Axioms

derived predicate   basic predicate

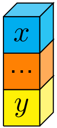
$$\textit{above}(x, y) \leftarrow \textit{on}(x, y) \lor \exists z \, (\textit{on}(x, z) \land \textit{above}(z, y))$$



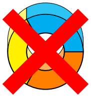
PDDL Axioms

derived predicate basic predicate

$$\textit{above}(x, y) \leftarrow \textit{on}(x, y) \lor \exists z\,(\textit{on}(x, z) \land \textit{above}(z, y))$$



$$\textit{illegal}() \leftarrow \exists x\ \textit{above}(x, x)$$



Axiom Formalisms

$A(x) \leftarrow \forall y\ \neg B(y) \wedge \neg C(x)$

Stratified Axioms[1]

Semipositive Axioms[2]

$A(x) \leftarrow \forall y\ B(y) \wedge \neg C(x)$

Stratified Datalog

$A(x) \leftarrow \exists y\ \neg B(y) \wedge \neg C(x)$

Semipositive Datalog

$A(x) \leftarrow \exists y\ B(y) \wedge \neg C(x)$

[1] Sylvie Thiébaux, Jörg Hoffmann, Bernhard Nebel (2005). In Defense of PDDL Axioms. AIJ.

[2] Stefan Edelkamp, Jörg Hoffmann (2004). PDDL2.2: The Language for the Classical Part of the 4th International Planning Competition. Technical Report.

Axiom Formalisms

$A(x) \leftarrow \forall y\ \neg B(y) \wedge \neg C(x)$

Stratified Axioms[1]

$\not\leq$ $\not\geq$

Semipositive Axioms[2]

$A(x) \leftarrow \forall y\ B(y) \wedge \neg C(x)$

Stratified Datalog $>$ Semipositive Datalog

$A(x) \leftarrow \exists y\ \neg B(y) \wedge \neg C(x)$

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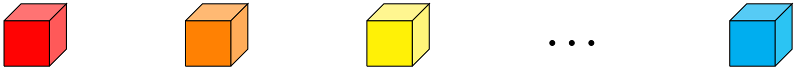
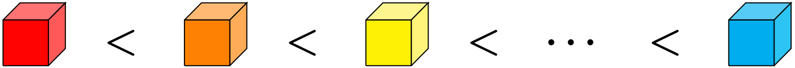
Linear Order

Even number of objects?



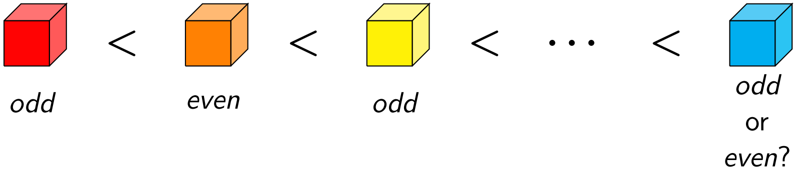
Linear Order

Even number of objects?



Linear Order

Even number of objects?



Capturing PTIME

Stratified Axioms

linear order

Semipositive Axioms = Stratified Datalog

With linear orders all three axiom formalisms can express any polynomial-time algorithm that can decide if a state is legal.

(Immerman-Vardi Theorem)

In General

$A(x) \leftarrow \forall y\ \neg B(y) \wedge \neg C(x)$

Stratified Axioms

? ?

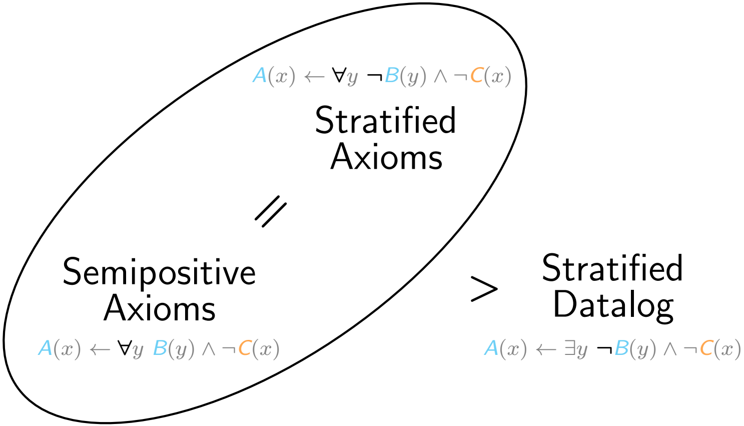
Semipositive Axioms

$A(x) \leftarrow \forall y\ B(y) \wedge \neg C(x)$

Stratified Datalog

$A(x) \leftarrow \exists y\ \neg B(y) \wedge \neg C(x)$

In General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Stratified Axioms
=
Semipositive Axioms
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>
Stratified Datalog
$A(x) \leftarrow \exists y\ \neg B(y) \wedge \neg C(x)$

In General

$A(x) \leftarrow \forall y\ \neg B(y) \wedge \neg C(x)$

Stratified Axioms

LFP = Stratified Axioms = Semipositive Axioms

LFP > BFP

Semipositive Axioms

$A(x) \leftarrow \forall y\ B(y) \wedge \neg C(x)$

> Stratified Datalog = BFP

$A(x) \leftarrow \exists y\ \neg B(y) \wedge \neg C(x)$

Summary

With linear order:

Stratified Axioms = Semipositive Axioms = Stratified Datalog

In General:

Stratified Axioms = Semipositive Axioms > Stratified Datalog

Link to our ICAPS 2024 paper
Formal Representations of Classical Planning Domains

Link to our PuK 2024 paper
Negated Occurrences of Predicates in PDDL Axiom Bodies

Relationships to Fixed Point Logics

Libkin, 2004 (Corollaries 10.8 & 10.13):
LFP-sim $=$ LFP $=$ LFP$_0$
Trivial: AP$_0 \leq$ AP
New: LFP$_0 \leq$ AP$_0$, AP $\leq$ LFP-sim
Thus: AP $=$ AP$_0$

Ebbinghaus and Flum, 1995 (Theorems 7.7.2 & 8.1.1):
Dat $=$ BFP, BFP $<$ LFP
Thus: Dat $<$ AP, Dat $<$ AP$_0$

AP $=$ stratified axioms
AP$_0$ $=$ semipositive axioms
Dat $=$ stratified Datalog

LFP $=$ least fixed-point logic
LFP-sim $=$ LFP with simultaneous fixed points
LFP$_0$ $=$ first-order logic extended with single fixed point
BFP $=$ bounded LFP

Libkin, L. (2004). Elements of Finite Model Theory. Springer Berlin, Heidelberg.
Ebbinghaus, H.-D. and Flum, J. (1995). Finite Model Theory. Springer-Verlag.