Delete Relaxation with Axioms

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Planning Tasks

- Classical Planning
- Axioms
- Arbirary formulae as conditions
- Effect conditions

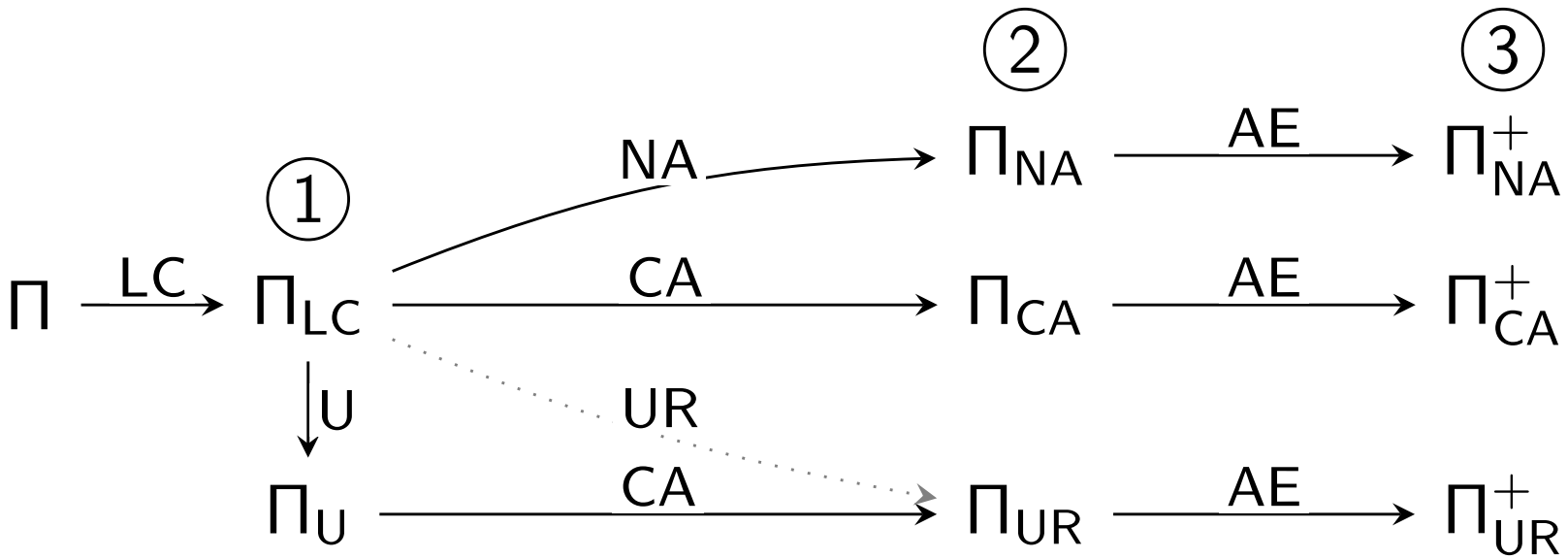
Contribution

- Introduce 3 new delete-relaxation heuristics

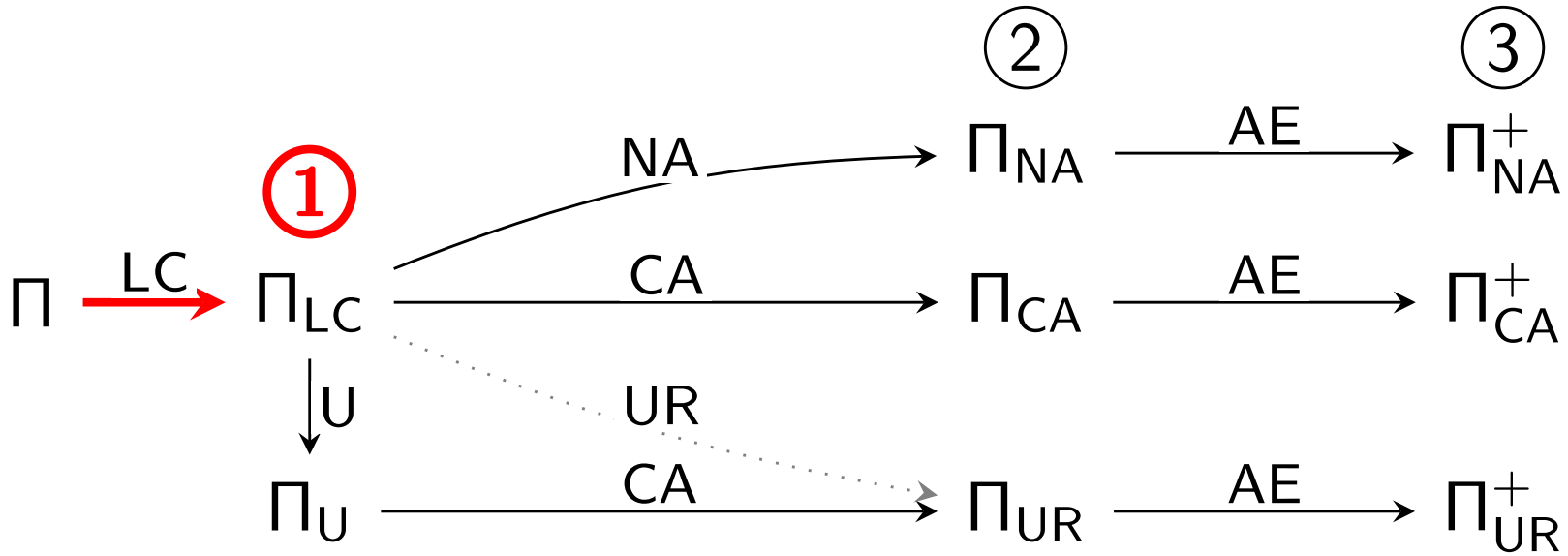
$$h^+_{\mathrm{NA}} \leq h^+_{\mathrm{CA}} \leq h^+_{\mathrm{UR}} \stackrel{?}{=} \underbrace{h^+_{\mathrm{3VL}} \leq h^+_{\mathrm{ASP}}}_{\text{Ivankovic \& Haslum, 2015}}$$

- Address issues with existing delete-relaxation heuristics
  - Thiébaux & Hoffmann, 2005
  - Helmert, 2006

Pipeline

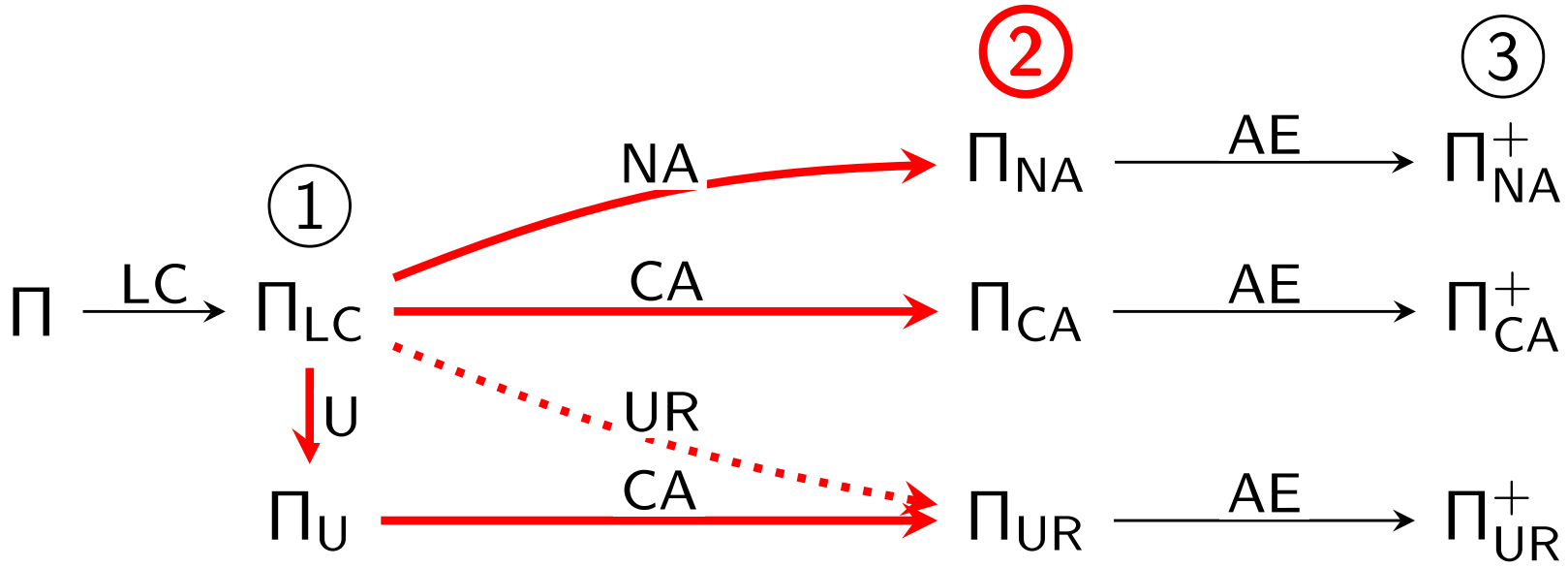


Pipeline



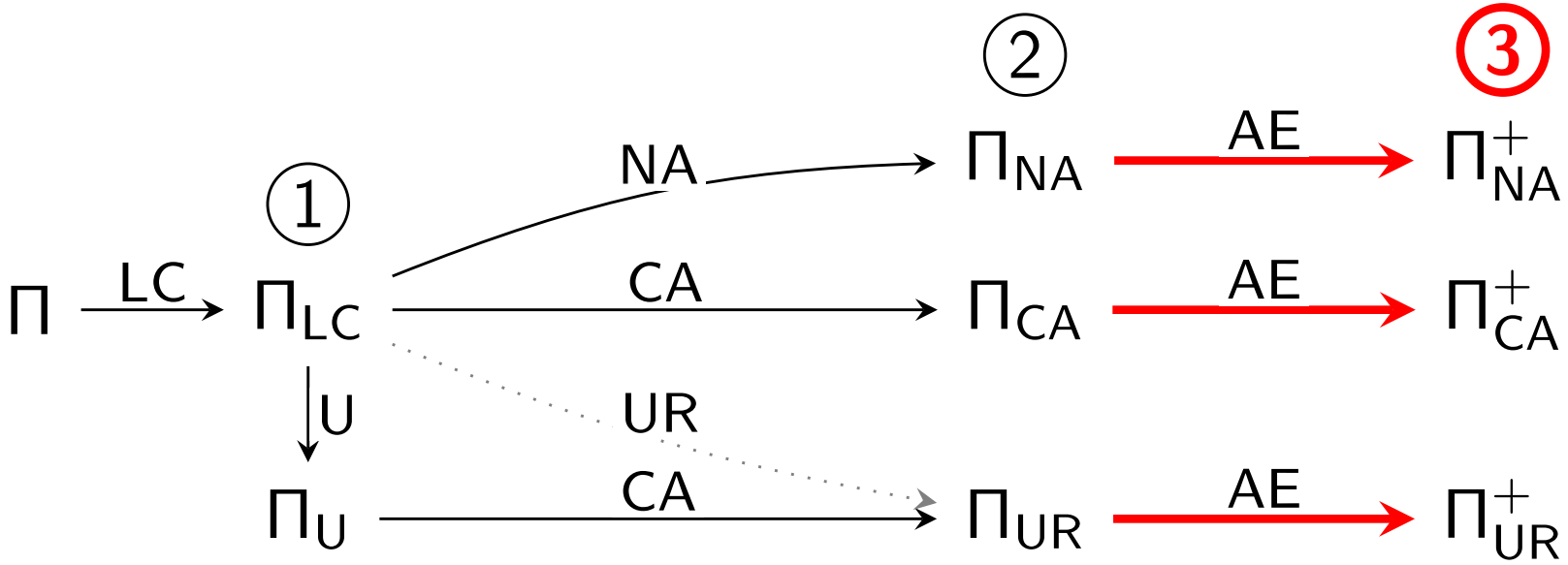
remove $\vee$

Pipeline



remove ¬

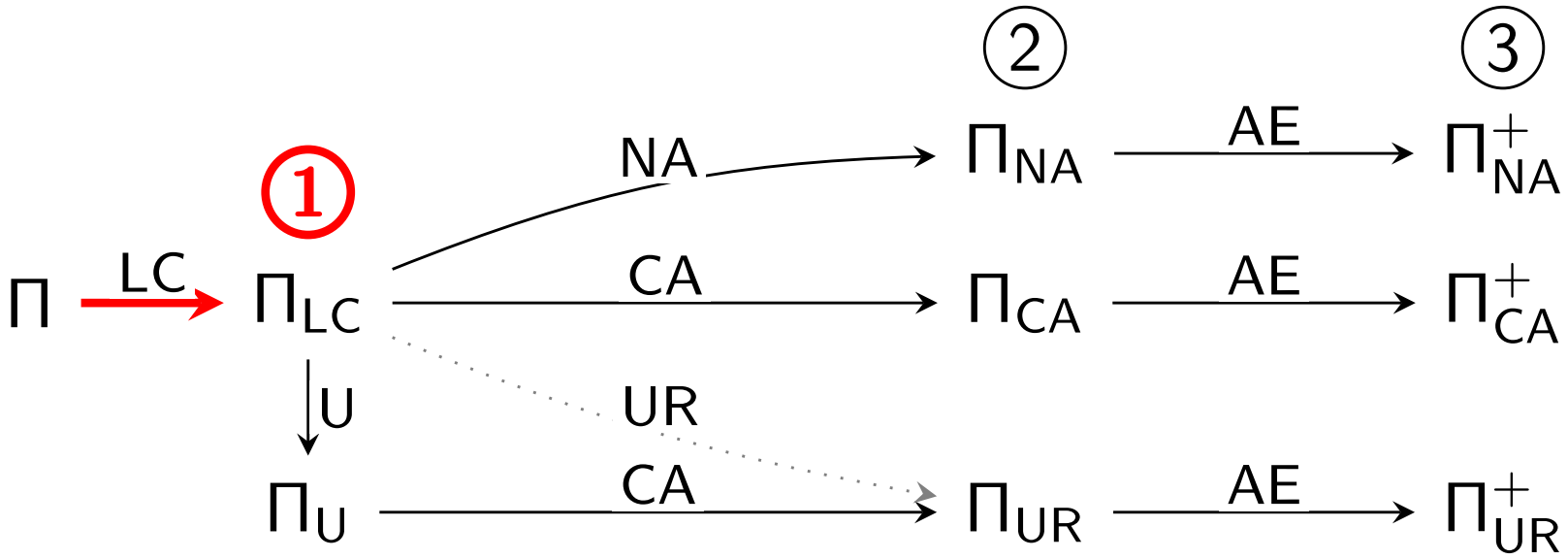
Pipeline



compile away axioms

Literal Conjunctive Form

LC



- Goal: flatten conditions, i.e. remove disjunctions ($\vee$)
- Our approach: Tseitin transformation
- Iteratively replaces each innermost conjunction by a fresh derived variable

LC

operator precondition $\varphi = x \wedge (\neg y \vee z)$

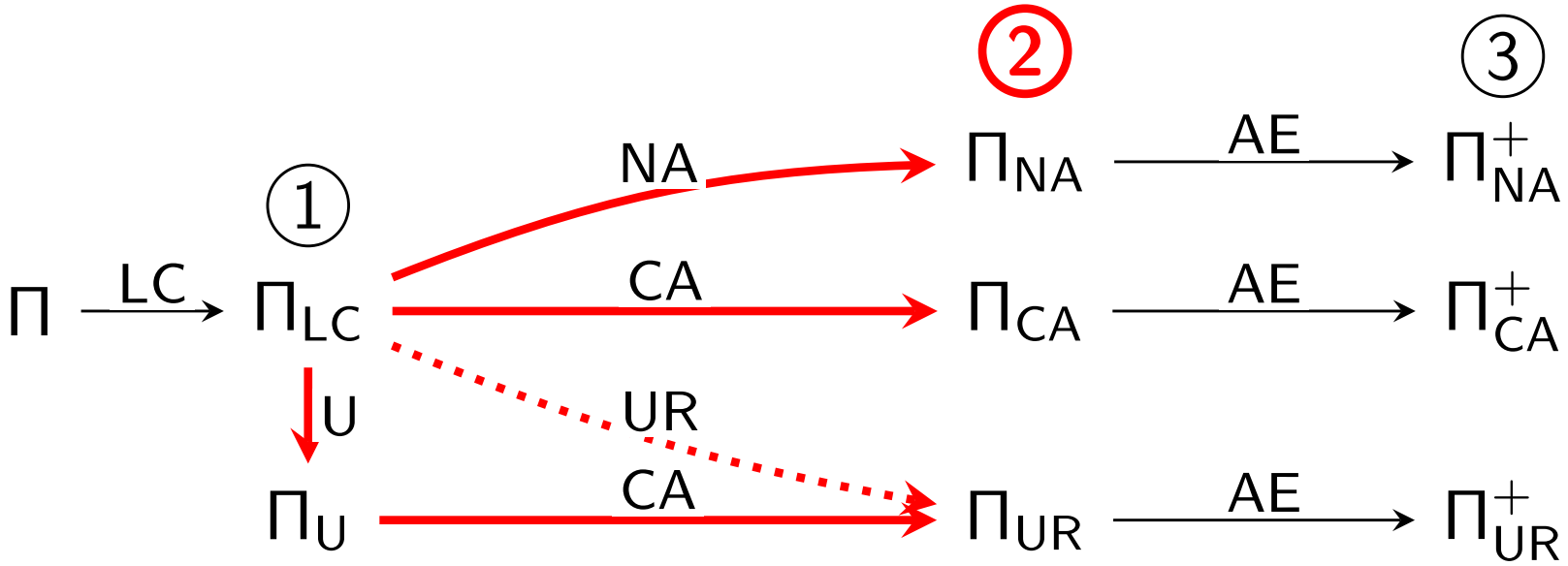
- DNF approach (used in FD):

$$\varphi \overset{DNF}{\rightsquigarrow} (x \wedge \neg y) \vee (x \wedge z) \rightsquigarrow \text{two copies of } o$$

- Tseitin approach:

$$\varphi \rightsquigarrow x \wedge d_{\text{new}}, \mathcal{D} \cup \{d_{\text{new}}\}\mathcal{A} \cup \{d_{\text{new}} \leftarrow \neg y, d_{\text{new}} \leftarrow z\}$$

PNF



- Goal: Remove all negations ($\neg$)
- Three approaches
- Introduce antagonist variables $\hat{x}$ for each $x \in \mathcal{V} \cup \mathcal{D}$

Negation Approximation

Definition (Antagonist Transformation)

For a condition of effect χ let $T(\chi)$ replace each $\neg x$ by $\hat{x}$ in χ.

Set

- $\mathcal{V}_{\mathsf{NA}} := \mathcal{V} \cup \hat{\mathcal{V}}$,
- $\mathcal{D}_{\mathsf{NA}} := \mathcal{D} \cup \hat{\mathcal{D}}$,
- $\mathcal{O}_{\mathsf{NA}} := \{\langle T(\mathsf{pre}(o)), T(\mathsf{eff}(o))\rangle \mid o \in \mathcal{O}\}$,

$$\mathcal{A}_{\mathsf{NA}} := \{d \leftarrow T(\varphi) \mid d \leftarrow \varphi \in \mathcal{A}\} \cup \{\hat{d} \leftarrow \top \mid d \in \mathcal{D}\}$$

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Cycle Approximation

The negation approximation fails to capture the behavior of $\hat{d}$.

Definition (Cyclically Independent)

A variable $d \in \mathcal{D}$ is cyclically independent if there exists a stratification s.t.

$$\forall d' \neq d : \text{level}(d') \neq \text{level}(d)$$

Lemma

Setting

$$\hat{d} \leftarrow \begin{cases} \top & d \text{ is cyclically dependent} \\ T(\text{unsat}(d)^{tseitin}) & d \text{ is cyclically independent} \end{cases}$$

yields a plan-preserving transformation.

Unrolling Relaxation

The cycle relaxation fails to capture the behavior of cyclically dependent $\hat{d}$.

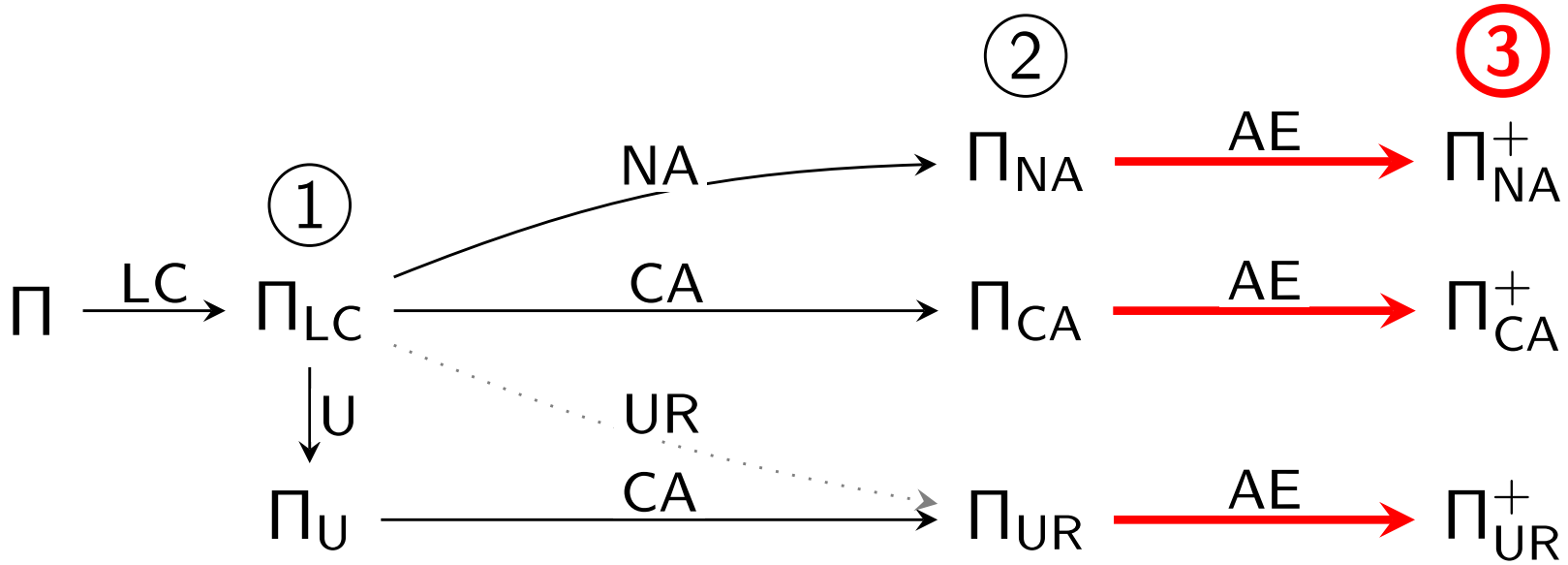
Lemma

Cyclic dependencies can be eliminated in time $O(|\mathcal{A}|^2)$.

Definition (Unrolling Relaxation)

$$\Pi^{\mathsf{UR}} = (\Pi^{U})^{\mathsf{CA}}$$

Axiom Elimination



- Goal: compile away axioms

Theorem

If Π is positive, then axioms can be compiled into zero-cost operators.

Other approaches

Ivankovic & Haslum, 2015

Theorem

$$h^+_{NA} \leq h^+_{CA} \leq h^+_{UR} \leq h^*$$

Theorem

$$h^+_{3VL} \leq h^+_{ASP}$$

Conjecture

$$h^+_{UR} \stackrel{?}{=} h^+_{3VL}$$

Fast Downward, FFX

```
(define (domain ffx-unsolvable)
  (:requirements :strips
     :derived-predicates
     :negative-preconditions)
  (:predicates (v) (p) (q) (r))

  ; not v -> p <-> q <-> r
  (:derived (p) (not (v)))
  (:derived (p) (q))
  (:derived (q) (p))
  (:derived (r) (q))
  (:derived (q) (r))

  (:action set-v
   :effect (v)
  )
)
```

```
(define (problem P)
  (:domain ffx-unsolvable)
  (:init (not (v)))
  (:goal (not (r)))
)
```

A case where $h^+(\mathcal{I}) = \infty$ although a plan exists.

Conclusion

Conclusion

- 3 new admissible delete-relaxation heuristics
- Establish hierarchy
- Address issues with other approaches

Questions?

	Tseitin LC				DNF LC				IH 2015	
Domain	h^{blind}	$h^{\max}_{\text{NA}}$	$h^{\max}_{\text{CA}}$	$h^{\max}_{\text{UR}}$	h^{blind}	$h^{\max}_{\text{NA}}$	$h^{\max}_{\text{CA}}$	$h^{\max}_{\text{UR}}$	$h^{\max}_{\text{3VL}}$	$h^{\max}_{\text{ASP}}$
Blocker (9)	**7**	**7**	6	6	**7**	**7**	**7**	6	6	5
Blocks (105)	**54**	**54**	**54**	**54**	**54**	**54**	**54**	**54**	52	33
Collab (3)	**3**	**3**	**3**	**3**	1	1	1	1	1	1
GED (266)	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	**16**	12
Ghosh-Etal (107)	14	14	14	14	**16**	15	15	15	7	2
Grid-Axioms (5)	1	**3**	**3**	**3**	1	**3**	**3**	**3**	2	1
Horn-DL (271)	226	225	223	**230**	**230**	228	227	**230**	208	98
Miconic-Axioms (150)	55	55	55	55	**60**	**60**	**60**	**60**	55	45
Muddy (12)	**9**	**9**	**9**	**9**	2	2	2	2	2	2
Opt-Telegraphs (48)	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	1
Philosophers (48)	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	3
PSR (100)	48	48	48	43	**50**	49	49	48	46	24
Snowman (51)	27	**29**	**29**	**29**	26	**29**	28	**29**	26	6
Social-Planning (2)	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	1
Sokoban-Axioms (30)	24	**27**	**27**	**27**	24	**27**	**27**	**27**	21	5
Sum (5)	3	**4**	**4**	**4**	1	1	1	1	1	1
Word-Rooms (5)	**5**	**5**	**5**	**5**	2	2	2	2	2	0
Sum (1217)	501	**507**	505	**507**	499	503	501	506	454	240