On Producing Shortest Cost-Optimal Solutions

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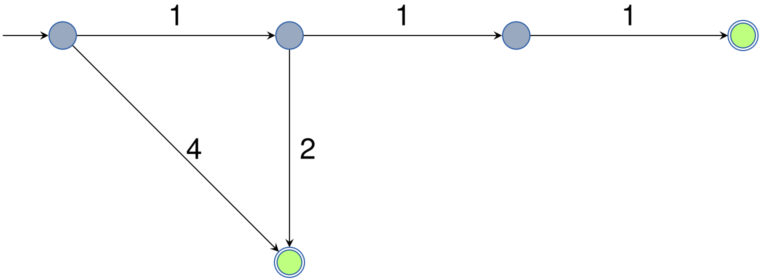
Shortest Cost-optimal Solutions



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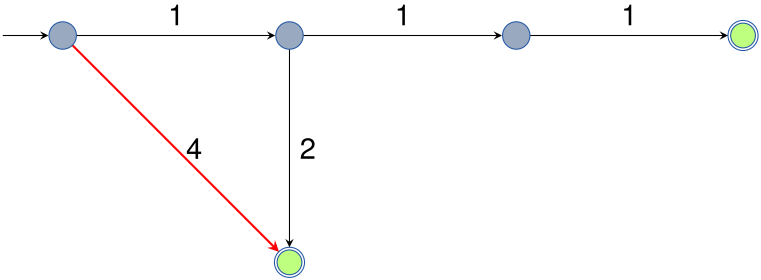

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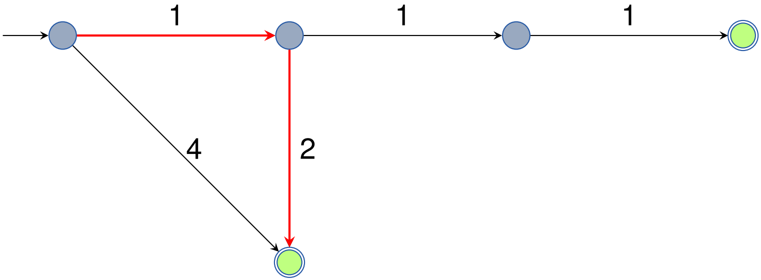

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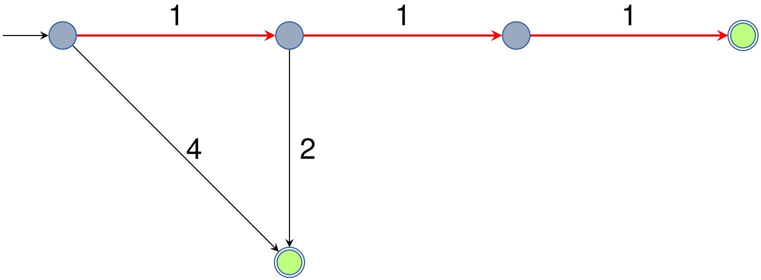

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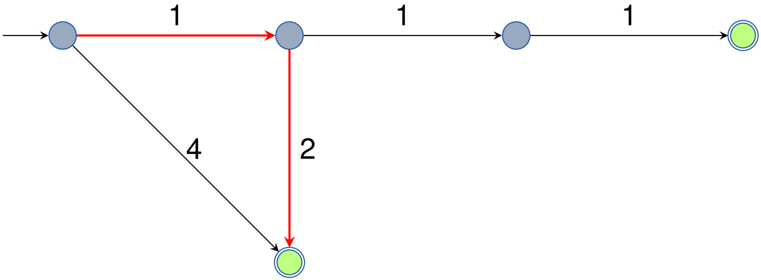

Planning in Labeled Transition Systems

Planning task

- S: finite set of states
- $s_0 \in S$: initial state
- $S_* \subseteq S$: goal states
- A: finite set of actions
- $cost : A \mapsto \mathbb{R}^{0+}$: action cost
- $T \subseteq S \times A \times S$: deterministic transition relation
  $(s, a, t) \in T$: a is applicable in s and t is the resulting state

Plan: a sequence of actions consecutively applicable in s_0 and leading to $s_* \in S_*$

Cost-optimal planning: find a plan minimizing summed action cost

Shortest cost-optimal planning: find a cost-optimal plan minimizing its length

Overview

Present State of Affairs

- 😐 Need for shortest cost-optimal planners (e.g. Katz et. al., AAAI 2020)

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Our Contribution

- Instantiation of cost-algebraic approach for shortest cost-optimal planning
- Alternative: cost-transformation approach
- Formal equivalence between the two in particular settings
- Empirical evaluation of these approaches

Cost-Algebraic Approach (AAAI 2005)

Cost-Algebraic Heuristic Search

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Abstract

Heuristic search is used to efficiently solve the single-node shortest path problem in weighted graphs. In practice, however, one is not only interested in finding a short path, but an *optimal* path, according to a certain cost notion. We propose an algebraic formalism that captures many cost notions, like typical Quality of Service attributes. We thus generalize A*, the popular heuristic search algorithm, for solving optimal-path problem. The paper provides an answer to a fundamental question for AI search, namely to which general notion of cost, heuristic search algorithms can be applied. We proof correctness of the algorithms and provide experimental results that validate the feasibility of the approach.

Introduction

Heuristic search (Pearl 1985) is an efficient solution to ex- applications with QoS (Lluch Lafuente & Montanari 2004; Ferrari & Lluch Lafuente 2004) and soft constraint programming (Bistarelli, Montanari, & Rossi 1997; 2002). However, to the best of our knowledge our work is the first attempt to define an abstract formalism for costs suited for heuristic search algorithms like A*.

We first define a suitable algebraic structure for graphs called *cost algebra*. Then, we generalize classical results of heuristic search specially regarding algorithm A*, according to a general notion of costs. The paper provides an answer to a fundamental question for AI search, namely to which general notion of cost, heuristic search algorithms can be applied. At the practical front, we provide experiments on different cost algebras explained in this paper along with a real-world application in route-planning (Jabbar 2003) where maps are constructed by bicycle tours. Dur-

Cost-Algebraic Approach

Definition

Heuristic for cost-algebraic planning problem: $h(s) = \langle h_c, h_d \rangle \in \mathbb{R}^{0+} \times \mathbb{R}^{0+}$

Admissibility

h is admissible if for a shortest cost-optimal plan π from *s*,
we have $h_c \leq cost(\pi)$ and $h_d \leq |\pi|$.

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Cost-Algebraic A*

1. Open list prioritizes by $\langle f_c, f_d \rangle$, where $f_c = g_c + h_c$ and $f_d = g_d + h_d$, using the lexicographical order.
2. Reopen if $g_c < g'_c$ or if $g_c = g'_c$ and $g_d < g'_d$.

Cost-Algebraic Heuristic

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- h_c: Any admissible heuristic for Π
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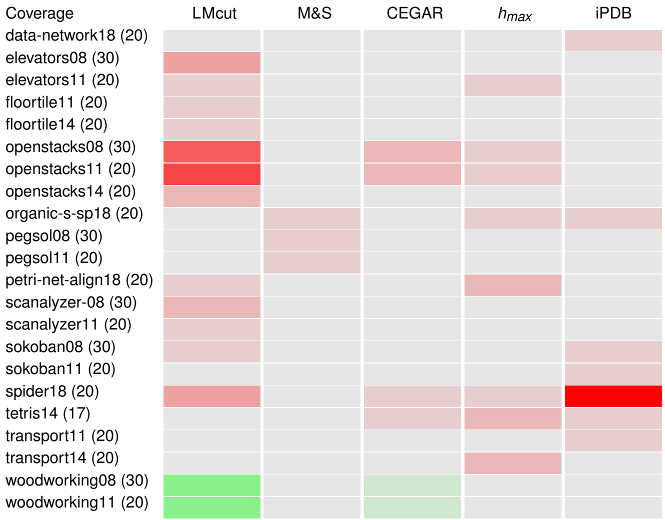
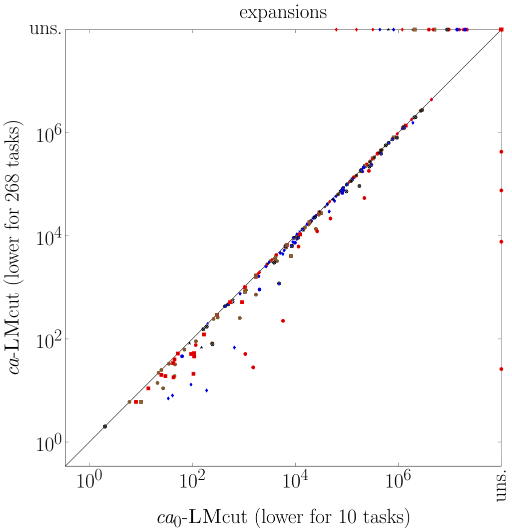
- Might be lower than necessary
- Not clear whether it will pay off. Also possible to use $h_d = 0$.

Does the Distance Estimation Effort Pay Off?

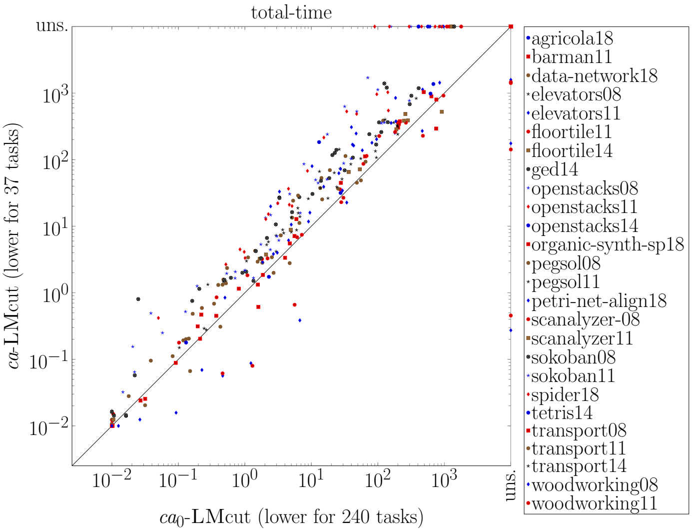


green = informed distance estimate performs better

Zoom into LMcut: Expansions



Zoom into LMcut: Time



Cost Transformation Approach

Theorem

We can transform the action costs of task Π so that cost-optimal plans of the resulting task are shortest cost-optimal plans of Π.

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Idea

- Increase the cost of every action by a small ϵ.

  (cf. Richter & Westphal JAIR 2010, Corrêa et. al., IJCAI 2018)
- Choose ϵ small enough so that for any relevant action sequences π and π', if $\mathit{cost}(\pi) < \mathit{cost}(\pi')$, then $\mathit{cost}'(\pi) < \mathit{cost}'(\pi')$.

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- Large enough M: equivalent integer cost function $\mathit{cost}_M(a) = M \cdot \mathit{cost}(a) + 1$.
  Cost transformed task denoted by Π_M.

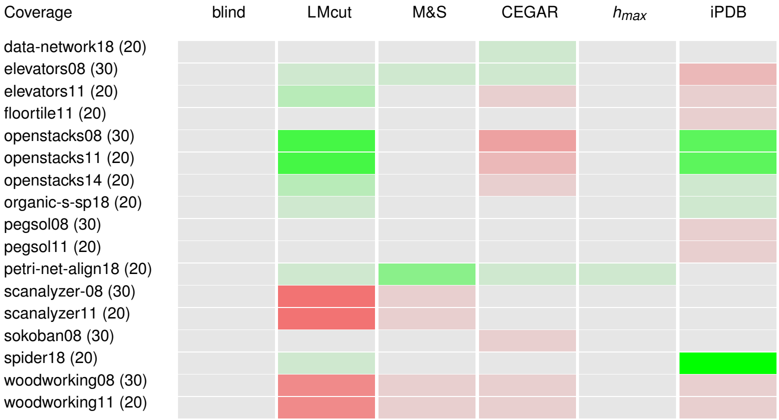
Cost Transformation Heuristics

- Heuristic on cost transformed task (may slow down heuristic computation)
- Heuristic on original task, scaled

Theorem

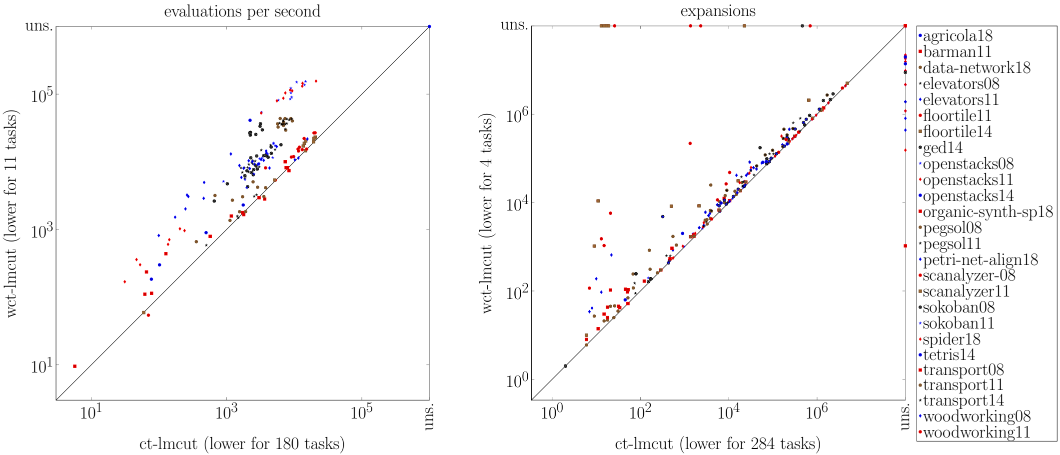
If h is an admissible estimate for state s in Π then Mh is admissible for s in Π_M.

Heuristic: To Transform, or Not to Transform



green = scaled heuristic computed on original task performs better

Zoom into LMcut



Equivalent Configurations

Theorem

With large enough M, two approaches will expand nodes in the same order on tasks with integer costs:

- wct: standard A* on Π_M with heuristic Mh_c,
- ca_0: cost-algebraic A* on Π with heuristics h_c and $h_d = 0$.

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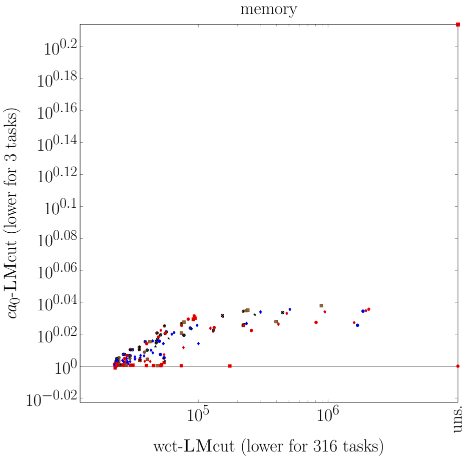
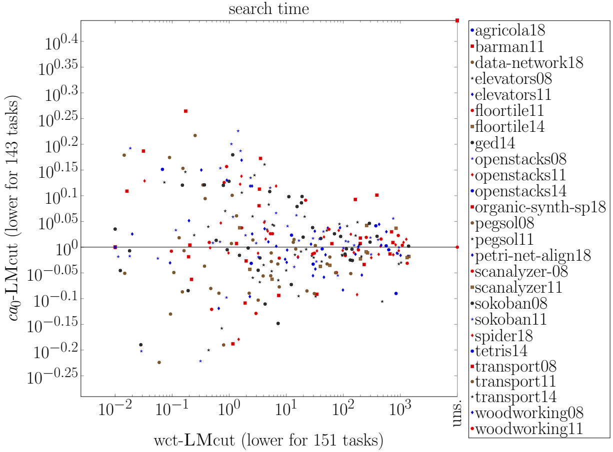
	LMcut		M&S		CEGAR		h_{max}		iPDB	
Coverage	ca_0	wct	ca_0	wct	ca_0	wct	ca_0	wct	ca_0	wct
Sum (587)	320	319	301	302	293	294	313	317	316	316

Equivalent Configurations: LMcut





Summary and Future Work

- Consider new interesting and useful computational problem
- Implement planner based on cost-algebraic A*
- Propose (and implement) transformation based approaches
- Theoretical and practical comparison

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- Efficient cost-algebraic heuristic computation
- What's the right M?