Best-Case and Worst-Case Behavior of Greedy Best-First Search

Manuel Heusner    Thomas Keller    Malte Helmert

University of Basel

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Motivation

A* [Hart et al., 1968]

- many potentially expanded states on last $f$-layer
- tie-breaking is important
- best case: shortest path
- worst case: all potentially expanded states
- polynomial-time computable in size of state space
**Motivation**

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Greedy best-first search [Doran and Michie, 1966]
- large heuristic plateaus
- tie-breaking assumed to be important
- best case: ?
- worst case: ?
- tractable?
Complexity Results

Given a state space and a heuristic:

- How many states does GBFS expand in its best case?
- How many states does GBFS expand in its worst case?
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NP-complete in general

- overlapping benches and craters that can be reached on different paths
- combinatorial problem
Complexity Results

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Polynomial-time computable

- in size of the state space
- undirected edges
- overlap-free craters and benches
Background

- locally characterized progress states
- based on high-water mark

![Diagram of a directed acyclic graph with benches and search episodes]
• locally characterized progress states
• based on high-water mark
• directed acyclic graph of benches
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- directed acyclic graph of benches
- search run is sequence of episodes
- episode searches on single bench along a bench path
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- directed acyclic graph of benches
- search run is sequence of episodes
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- crater relates to local minimum
Best-Case and Worst-Case Behavior

- **Best case**: minimize along state path including all necessarily expanded crater states.
- **Worst case**: maximize along bench path including all potentially expanded bench states.
- Beware of overlapping benches and craters.
Best-Case and Worst-Case Behavior

- **best case**: minimize along state path including all necessarily expanded crater states

```
best case worst case expansions
```

6

M. Heusner, T. Keller, M. Helmert (Basel)
Best-Case and Worst-Case Behavior

- best case: minimize along state path including all necessarily expanded crater states
- worst case: maximize along bench path including all potentially expanded bench states

3

2

1

0

best case worst case

expansions

6 9
• best case: minimize along state path including all necessarily expanded crater states
• worst case: maximize along bench path including all potentially expanded bench states
• beware of overlapping benches and craters

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- **worst case**: maximize along bench path including all potentially expanded bench states
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Experimental Results

• implemented algorithms for computing best and worst cases
• state spaces of planning tasks from international planning competitions
• Fast Forward heuristic
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- DAG of benches for 764 instances from 78 domains
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- state spaces of planning tasks from international planning competitions
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- DAG of benches for 764 instances from 78 domains
- best cases for 679 instances
- worst cases for 739 instances
Standard Tie-Breaking Strategies

- **crater**
- **crater-free**

![Graphs showing covered instances vs. expansions for crater and crater-free scenarios with various tie-breaking strategies: best, fifo, lifo, rand, worst.](image-url)
Conclusion

- **NP-complete** in general
- computing best and worst cases is often feasible
- large impact of tie-breaking for less informed heuristics
- room for improvement over standard tie-breaking strategies
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Thank you for your attention!