Accuracy of Admissible Heuristic Functions in Selected Planning Domains

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Outline

Motivation

- Goal: Develop efficient optimal planning algorithms
- Subgoal: Find accurate admissible heuristics

How to assess the accuracy of an admissible heuristic?

Most common approach

Run planners on benchmarks and count node expansions.

Drawback: Only comparative statements

Alternative approach

Analytical comparison to optimal heuristic on benchmark domains

Advantage: Absolute statements, theoretical limitations

Scope of our analysis

Considered heuristics

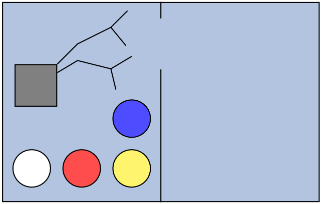
- h^+: optimal plan length for delete relaxation
- h^k: cost of most costly size-k goal subset (roughly)
- h^{PDB}: pattern database heuristics
- $h^{\mathsf{PDB}}_{\mathsf{add}}$: additive pattern database heuristics

Reference point: optimal plan length h^*

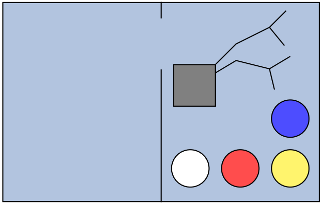
Considered planning domains

GRIPPER, LOGISTICS, BLOCKSWORLD, MICONIC-STRIPS, MICONIC-SIMPLE-ADL, SCHEDULE, SATELLITE

Domains: Gripper

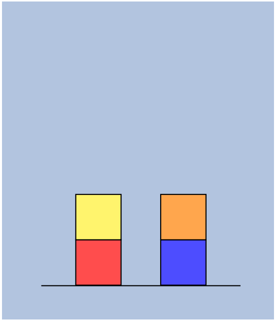


initial state



goal state

Domains: BLOCKSWORLD



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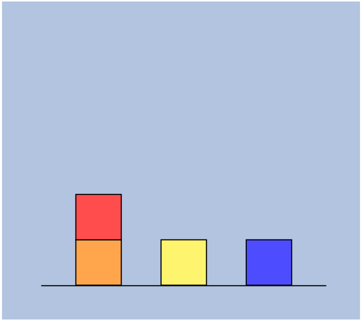
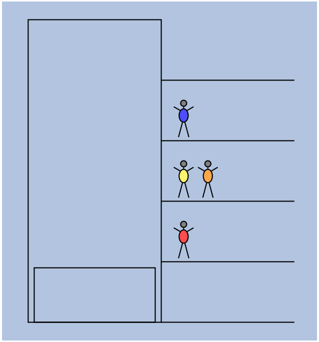
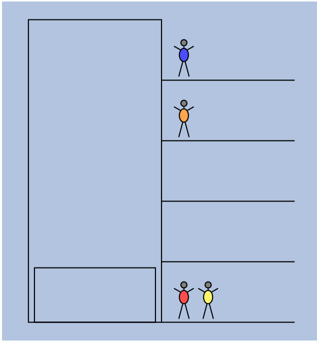


goal state

Domains: Miconic-Strips, Miconic-Simple-Adl


initial state


goal state

Asymptotic accuracy

Definition

Let $\mathcal{D}$ be a planning domain (family of planning tasks).
A heuristic h has asymptotic accuracy $\alpha \in [0, 1]$ on $\mathcal{D}$ iff

- $h(s) \geq \alpha h^*(s) + o(h^*(s))$
  for all initial states s of tasks in $\mathcal{D}$, and
- $h(s) \leq \alpha h^*(s) + o(h^*(s))$
  for all initial states s of an infinite subfamily of $\mathcal{D}$ with unbounded $h^*(s)$

If solution lengths in $\mathcal{D}$ are unbounded, there is exactly one such α for a given heuristic and domain. We write it as $\alpha(h, \mathcal{D})$.

Outline

Delete relaxation

Considered heuristics

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Delete relaxation: BLOCKSWORLD

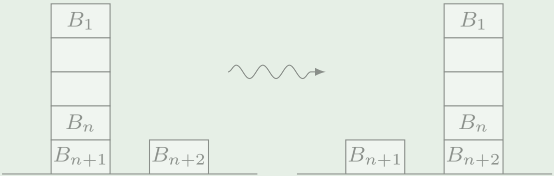
Example (BLOCKSWORLD)

Lower bound:

m = number of blocks touched in optimal plan

$h^*(s) \leq 4m,\ h^+(s) \geq m \Rightarrow \alpha(h^+, \text{BLOCKSWORLD}) \geq 1/4$

Upper bound:



$h^*(s_n) = 4n - 2,\ h^+(s_n) = n + 1 \Rightarrow \alpha(h^+, \text{BLOCKSWORLD}) \leq 1/4$

$\rightsquigarrow \alpha(h^+, \text{BLOCKSWORLD}) = 1/4$

Delete relaxation: Blocksworld

Example (Blocksworld)

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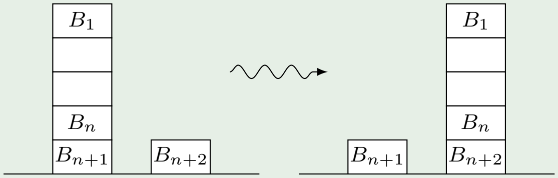

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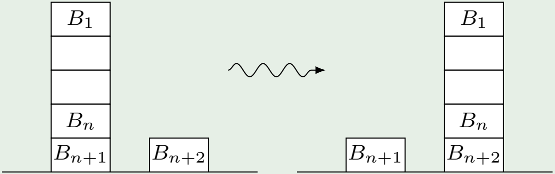

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The h^k heuristic family

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The h^k heuristic family

$\alpha(h^k, \mathcal{D}) = 0$ for all considered domains

Proof idea.

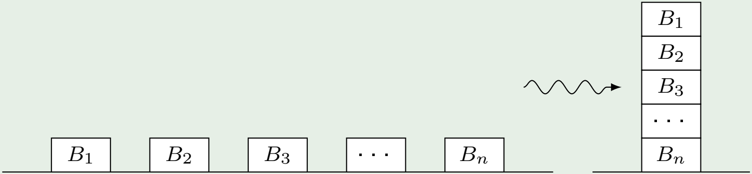
There are families of states $(s_n)_{n \in \mathbb{N}}$ with

- $h^*(s_n) \in \Omega(n)$ and
- $h^k(s_n) \in O(k)$.

□

The h^k heuristic family

Example (Blocksworld)



$h^*(s_n) = 2n - 2,\ h^k(s_n) \leq 2k$

$\rightsquigarrow \alpha(h^k, \textsc{Blocksworld}) = 0$

Non-additive pattern database heuristics

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- h^+: optimal plan length for delete relaxation
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Let n be the problem size.

- Bounded memory: database size limit $O(n^k)$ entries
- Consequently: pattern size limit $O(\log n)$ variables

Non-additive pattern database heuristics

$\alpha(h^{\mathsf{PDB}}, \mathcal{D}) = 0$ for all considered domains

Proof idea.

At most $O(\log n)$ variables in pattern
$\Rightarrow$ at most $O(\log n)$ goals represented in abstraction

There are families of states $(s_n)_{n \in \mathbb{N}}$ with

- $h^*(s_n) \in \Omega(n)$ and
- $h^{\mathsf{PDB}}(s_n) \in O(\log n)$.

□

Additive pattern database heuristics

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Let n be the problem size.

- Bounded memory: overall database size limit $O(n^k)$ entries
- Consequently: size limit $O(\log n)$ variables for each pattern

Additive pattern database heuristics: MICONIC-STRIPS

Example (MICONIC-STRIPS)

Lower bound:

m passengers, singleton pattern for each passenger:

$h^*(s) \leq 4m, \; h^{\text{PDB}}_{\text{add}}(s_n) = 2m$

$\Rightarrow \alpha(h^{\text{PDB}}_{\text{add}}, \text{MICONIC-STRIPS}) \geq 1/2$

Upper bound:

Optimal additive PDB:

- $\{\texttt{elev}, \texttt{pass}_1, \ldots, \texttt{pass}_K\}$ $(K \in O(\log n))$
- $\{\texttt{pass}_{K+1}\}, \ldots, \{\texttt{pass}_n\}$

$\rightsquigarrow h^*(s_n) = 4n, \; h^{\text{PDB}}_{\text{add}}(s_n) = 2n + 2K$

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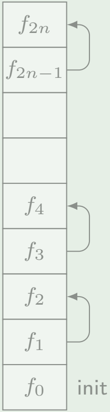

Additive pattern database heuristics: MICONIC-STRIPS

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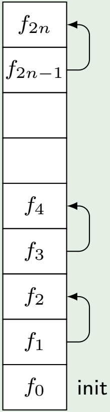

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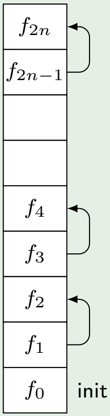

Outline

Summary of results

Asymptotic accuracy

Domain	h^+	h^k	h^{PDB}	$h^{\mathsf{PDB}}_{\mathsf{add}}$
Gripper	2/3	0	0	2/3
Logistics	3/4	0	0	1/2
Blocksworld	1/4	0	0	0
Miconic-Strips	6/7	0	0	1/2
Miconic-Simple-Adl	3/4	0	0	0
Schedule	1/4	0	0	1/2
Satellite	1/2	0	0	1/6

Summary and conclusion

Method:

- Analytical comparison of domain-specific accuracy of the heuristics h^+, h^k, h^{PDB}, $h^{\mathsf{PDB}}_{\mathsf{add}}$

Results:

- h^+: usually most accurate (but NP-hard to compute in general)
- h^k, h^{PDB}: arbitrarily inaccurate
- $h^{\mathsf{PDB}}_{\mathsf{add}}$: good accuracy/effort trade-off (but how to determine a good pattern collection?)

Future work:

- additive h^k
- explicit-state abstraction heuristics

The end

Thank you for your attention!