On Variable Dependencies and Compressed Pattern Databases

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SoCS 2017
Introduction
previous work on compressed pattern databases:

Sturtevant, Felner and Helmert (SoCS 2014)

“This approach worked very well for the 4-peg Towers of Hanoi, for instance, but its success for the sliding tile puzzles was limited and no significant advantage was reported for the Top-Spin domain (Felner et al., 2007).”

this paper: try to understand why
Compressed PDBs

\[ h^*(A) = 6 \\
\] \[ h_{PDB}(A) = 4 \]
\[ h_{compPDB}(A) = 3 \]

\[ \Rightarrow \]

[Diagram of network with nodes labeled A to L]
Compressed PDBs

\[ h^*(A) = 6 \]
Compressed PDBs

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Compressed PDBs

$h^*(A) = 6$

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Compressed PDBs

$h^*(A) = 6$
$h_{PDB}(A) = 4$
$h_{PDB}^{comp}(A) = 3$
Assume we have $N$ units of memory.

Consider three heuristics:

- $h_F$: fine-grained PDB ($M \gg N$ entries)
- $h_F^{\text{comp}}$: compressed fine-grained PDB ($N$ entries)
- $h_C$: coarse-grained PDB ($N$ entries)

Which one should we use, $h_F^{\text{comp}}$ or $h_C$?
### Experimental Results

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- **Hanoi:** 4 pegs and 16 disks; pattern with 15 disks
- **Sliding Tiles A:** $4 \times 4$ puzzle; pattern $\langle\text{blank, 1, 2, 3, 4, 5, 6}\rangle$
- **Sliding Tiles B:** $4 \times 4$ puzzle; pattern $\langle6, 5, 4, 3, 2, 1, \text{blank}\rangle$
- **TopSpin:** 18 tokens and turnstile size 4; pattern with 7 tokens

All use lexicographic ranking
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$h_F^{\text{comp}}$ better than $h_C$ on average

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Good News
Theorem (dominance of compressed PDBs)

Let $h_F$ and $h_C$ be heuristics such that $h_F$ is a refinement of $h_C$. Consider compressed heuristics with a compression regime that is compatible with $h_F$ and $h_C$.

Then

$$h_F^{\text{comp}}(s) \geq h_C(s)$$

for all states $s$.

informally: compression step applies further abstraction on top of the abstraction $h_F$
Dominance of Compressed PDBs: Proof Idea

\[ h^*(A) = 6 \]
\[ h_F(A) = 4 \]
\[ h_F^{\text{comp}}(A) = 3 \]
Dominance of Compressed PDBs: Proof Idea

\[ h^*(A) = 6 \]
\[ h_F(A) = 4 \]
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\[ \begin{array}{c|c|c}
AB & 4 & 3 \\
CD & 3 & \\
EF & 2 & 2 \\
GH & 3 & 2 \\
IJ & 1 & 0 \\
KL & 0 & \\
\end{array} \]

\[ \begin{array}{c|c|c}
AB & 2 & \\
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Dominance of Compressed PDBs: Proof Idea

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$h_{comp}^F(s) \geq h_C(s)$ for all states according to the theorem

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Bad News
States are described in terms of **state variables**.

**Examples:**

- **Towers of Hanoi:** position of one disk
- **sliding tiles:** position of a tile (or blank)
- **TopSpin:** position of a token

PDBs **project** to a subset of variables (the “pattern”).
Variable $u$ depends on variable $v$ if changing $u$ is conditioned in any way on $v$. 
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Towers of Hanoi  
sliding tiles  
TopSpin
**Theorem (no improvements without dependencies)**

Consider the patterns $F \supseteq C$ in an undirected state space. Let $h_F^{\text{comp}}$ be a compressed PDB heuristic with a compression regime compatible with the refinement relation between $F$ and $C$. If no variable in $C$ depends on any variable in $F \setminus C$, then

$$h_F^{\text{comp}}(s) = h_C(s)$$

for all states $s$. 
Improvements vs. Dependencies: Proof Idea

\[ h^*(A) = 4 \]
\[ h_F(A) = 3 \]
\[ h_F^{comp}(A) = 2 \]
\[ h_C(A) = 2 \]
## Improvements vs. Dependencies: Experimental Results

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Related Work in Classical Planning

our result:

- \( h_F^{\text{comp}} = h_C \)
- for \textit{undirected} state spaces
- under certain dependency conditions
Related Work in Classical Planning

our result:

- \( h^{comp}_F = h_C \)
- for undirected state spaces
- under certain dependency conditions

literature (Haslum et al. 2007; Pommerening et al. 2013):

- \( h_F = h_C \)
- for arbitrary state spaces
- under certain (different) dependency conditions

neither result entails the other

\( \rightsquigarrow \) many more details in paper
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**Conclusion**
Conclusion

When is entry compression a good idea?
- never bad when compatible with refinement
- never good when refinement does not capture a dependency

What does this mean for the benchmarks?
- **Towers of Hanoi**: must compress smaller disks away
- **sliding tile**: compressing blank the only useful refinement
- **TopSpin**: no dependencies, hence no gain
  (ditto: **Pancakes, Rubik’s Cube**)


Thank you for your attention!