On the Complexity of Heuristic Synthesis for Satisficing Classical Planning: Potential Heuristics and Beyond
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Motivation & Background
SAS Example
▶ state variables: $x, y, z \in \{0, 1, 2\}$
▶ initial state: $\{x \mapsto 0, y \mapsto 0, z \mapsto 0\}$
▶ goal: $\{x \mapsto 2, z \mapsto 1\}$
▶ actions:
  $a_1: x \mapsto 0, y \mapsto 0 \rightarrow y := 1, z := 2$
  $a_2: x \mapsto 0, z \mapsto 1 \rightarrow z := 0$

Potential Heuristics
weighted sum of features
$h_0: 7$
$h_1: 3 [x = 1] - 3 [y = 0] + 2 [z = 1]$ dimension 0
$h_2: 3 [x = 0 \land y = 0] + 3 [z = 1] - 2$ dimension 1
$h_1(\{x \mapsto 1, y \mapsto 1, z \mapsto 1\}) = 3 + 2 = 5$

Verification and Synthesis
motivation: find heuristic of particular form with desirable property
▶ Verification: does a given heuristic have the property?
▶ Synthesis: does a heuristic with this property exist?

How difficult is this?

Main Results for Different Properties
DDA
▶ descending: all alive non-goal states
have successor with strictly lower heuristic value
▶ dead-end-avoiding: dead (non-alive) successors of alive states
never have strictly lower heuristic value

Solvable tasks
unsolvable tasks

DDA verification/synthesis for dimension 0

Verification: PSPACE-complete, even for dimension 0.
Synthesis: In P for unrestricted potential heuristics,
PSPACE-complete for restricted cases,
such as dimension 0.

DDA combines "solvable without backtracking" with "unsolvable for any reason"
▶ we need better properties
▶ handle unsolvable tasks differently

sDDA (solvable DDA)
▶ heuristic is DDA
▶ initial state is alive

Solvable tasks
unsolvable tasks

SDDA verification/synthesis for dimension 1

Verification: In P for dimension 0,
PSPACE-complete for dimension 1 or higher.
Synthesis: Same results as for verification

nice:
▶ dimension-1 heuristics can solve PSPACE-complete problems
not so nice:
▶ everything interesting is hard
reason:
▶ SDDA property essentially requires perfect dead-end detection

VDDA (variant DDA)
▶ UDDA (unrestricted DDA): replace "alive state" with "any state"
▶ $\infty$DDA: replace "alive state" with "state with finite $h$ value"

VDDA: collective term for UDDA/$\infty$DDA (spoiler: same properties)

Solvable tasks
unsolvable tasks

VDDA verification/synthesis for dimension 1

Verification: coNP-complete for dimension 1 or higher
Synthesis: $\Sigma_p^2$-complete for dimension 1 or higher

good news:
▶ well below PSPACE, yet still very expressive
▶ interesting connection to $\exists\forall$QBF
bad news:
▶ no tractability for low dimension
  (unlike case of admissibility and consistency)