

On the Complexity of Heuristic Synthesis for Satisficing Classical Planning: Potential Heuristics and Beyond

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Motivation & Background

SAS⁺ Example

- ▶ **state variables:** $x, y, z \in \{0, 1, 2\}$
- ▶ **initial state:** $\{x \mapsto 0, y \mapsto 0, z \mapsto 0\}$
- ▶ **goal:** $\{x \mapsto 2, z \mapsto 1\}$
- ▶ **actions:**
 - $a_1: x \mapsto 0, y \mapsto 0 \rightarrow y := 1, z := 2$
 - $a_2: x \mapsto 0, z \mapsto 1 \rightarrow z := 0$

Potential Heuristics

weighted sum of features

$$\begin{aligned} h_0 &: 7 && \text{dimension 0} \\ h_1 &: 3[x = 1] - 3[y = 0] + 2[z = 1] && \text{dimension 1} \\ h_2 &: 3[x = 0 \wedge y = 0] + 3[z = 1] - 2 && \text{dimension 2} \end{aligned}$$

$$h_1(\{x \mapsto 1, y \mapsto 1, z \mapsto 1\}) = 3 + 2 = 5$$

Verification and Synthesis

motivation: find heuristic of particular form with desirable property

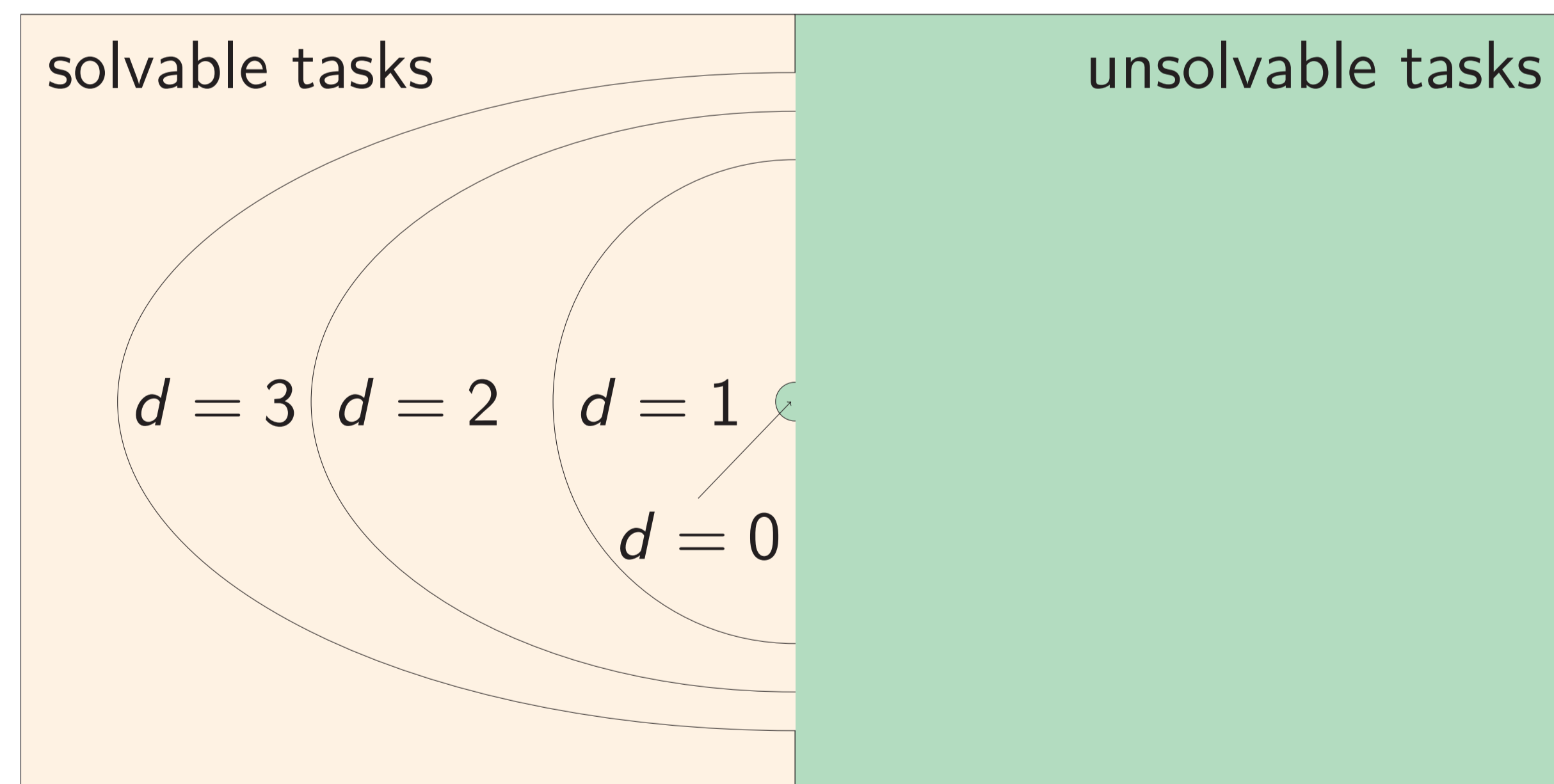
- ▶ **Verification:** does a given heuristic have the property?
- ▶ **Synthesis:** does a heuristic with this property exist?

How difficult is this?

Main Results for Different Properties

DDA

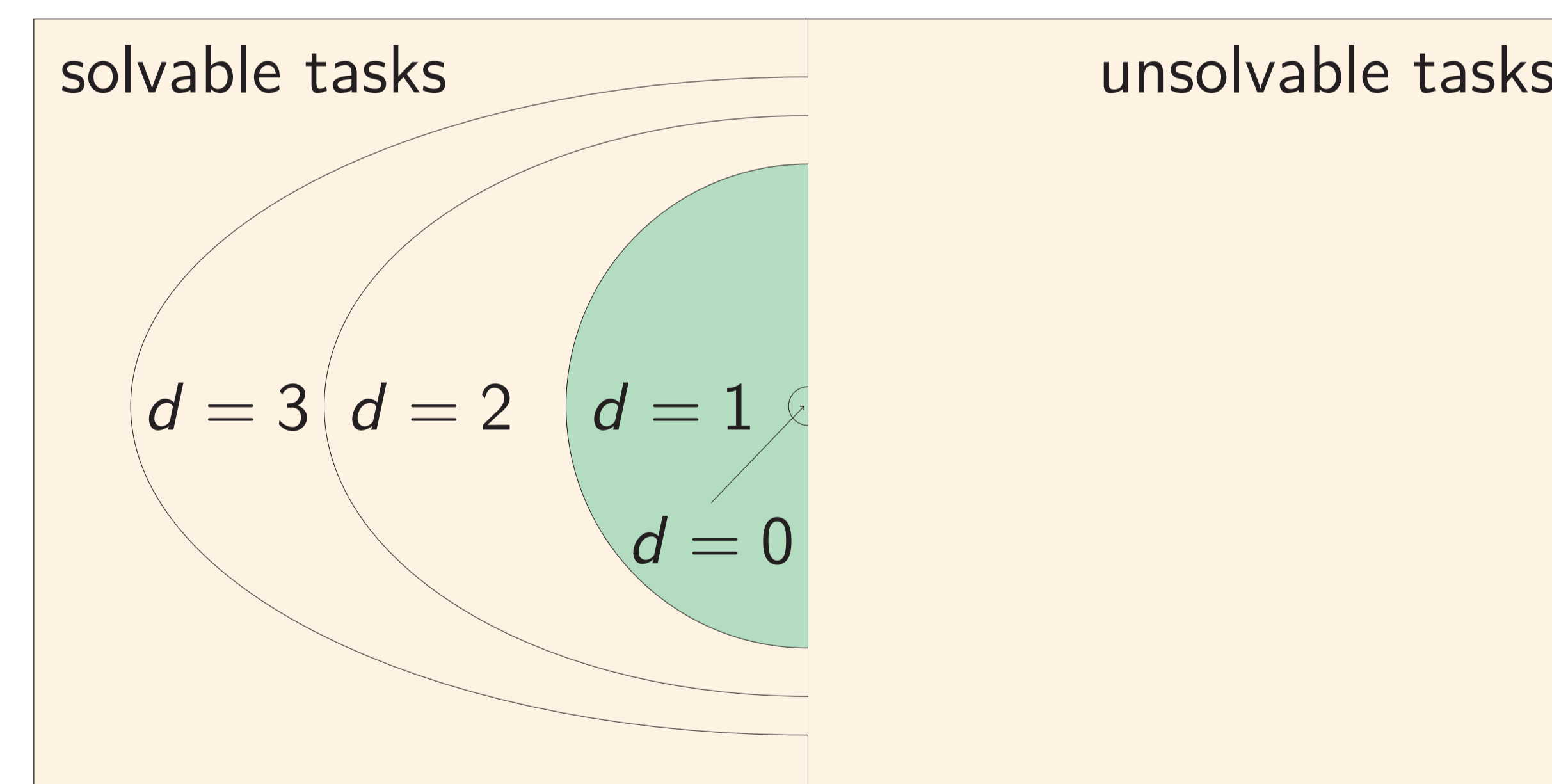
- ▶ **descending:** all **alive** non-goal states have successor with strictly lower heuristic value
- ▶ **dead-end-avoiding:** **dead** (non-alive) successors of alive states never have strictly lower heuristic value



DDA verification/synthesis for dimension 0

SDDA (solvable DDA)

- ▶ heuristic is DDA
- ▶ initial state is alive

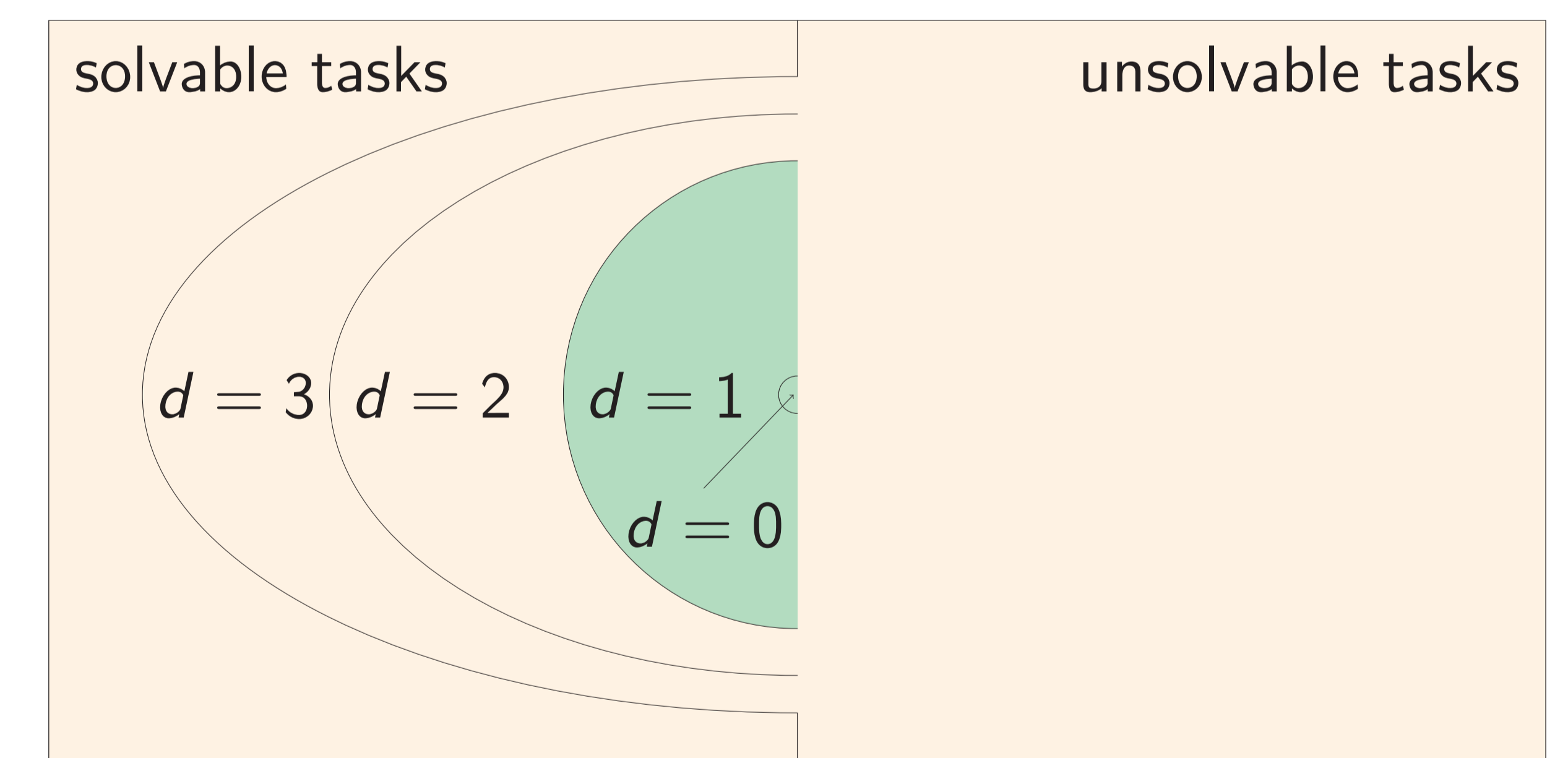


SDDA verification/synthesis for dimension 1

VDDA (variant DDA)

- ▶ **UDDA (unrestricted DDA):** replace "alive state" with "any state"
- ▶ **∞DDA:** replace "alive state" with "state with finite h value"

VDDA: collective term for UDDA/∞DDA (spoiler: same properties)



VDDA verification/synthesis for dimension 1

Verification: PSPACE-complete, even for dimension 0.

Synthesis: In P for unrestricted potential heuristics, PSPACE-complete for restricted cases, such as dimension 0.

DDA combines "solvable without backtracking" with "unsolvable for any reason"

- ▶ we need better properties
- ▶ handle unsolvable tasks differently

Verification: In P for dimension 0, PSPACE-complete for dimension 1 or higher.

Synthesis: Same results as for verification

nice:

- ▶ dimension-1 heuristics can solve PSPACE-complete problems

not so nice:

- ▶ everything interesting is hard

reason:

- ▶ SDDA property essentially requires perfect dead-end detection

Verification: coNP-complete for dimension 1 or higher

Synthesis: Σ₂^P-complete for dimension 1 or higher

good news:

- ▶ well below PSPACE, yet still very expressive
- ▶ interesting connection to ∃∀QBF

bad news:

- ▶ no tractability for low dimension (unlike case of admissibility and consistency)