Explicit-State Abstraction: A New Method for Generating Heuristic Functions

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One-Slide Summary

Abstraction heuristics

Heuristic estimate is goal distance in abstracted state space S' obtained as homomorphism of original state space S.

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Explicit-state abstraction heuristics

You have seen other abstraction heuristics before;
they are called pattern database heuristics.

Ours can do the same and then some.

Outline

Transition Graphs

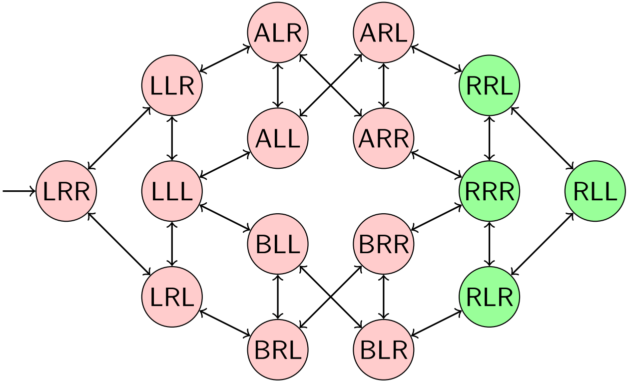
Definition (transition graph)

A transition graph is a 5-tuple $\langle S, L, A, s_0, S_\star \rangle$:

- S: finite set of states
- L: finite set of transition labels
- $A \subseteq S \times L \times S$: labelled transitions
- $s_0 \in S$: initial state
- $S_\star \subseteq S$: goal states

Assumption: States are assignments to a set of state variables.

Running Example



Logistics problem with one package, two trucks, two locations:

- state variable package: $\{L, R, A, B\}$
- state variable truck A: $\{L, R\}$
- state variable truck B: $\{L, R\}$

Abstractions

Definition (abstraction, homomorphism)

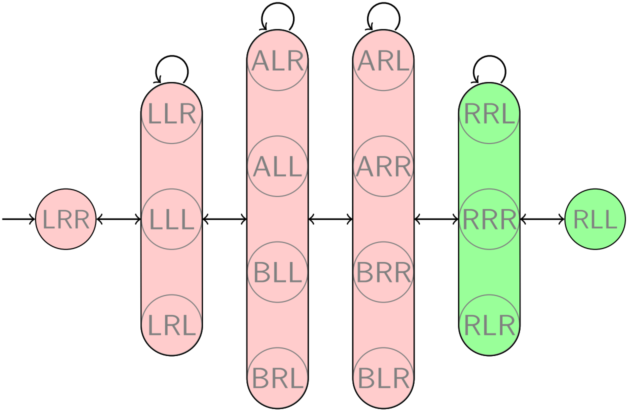
Abstraction of transition graph $\mathcal{T}$: pair $\langle \mathcal{T}', \alpha \rangle$ where

- $\mathcal{T}'$ is a transition graph with the same labels
- α maps states of $\mathcal{T}$ to states of $\mathcal{T}'$ such that
  - initial state maps to initial state
  - goal states map to goal states
  - transitions $\langle s, l, s' \rangle$ map to transitions $\langle \alpha(s), l, \alpha(s') \rangle$

Abstraction (and α) is a homomorphism if $\mathcal{T}'$ only contains necessary goal states and transitions.

Abstraction heuristic: $h(s) = d_\star(\alpha(s))$ admissible, consistent

Example: Perfect Abstraction



⇝ perfect heuristic h^*

Generating Abstractions

Conflicting goals in generating abstractions:

- obtain informative heuristic
- keep representation small

Abstractions have small representations if they have

- few abstract states
- succinct encoding for α

Outline

Projections

One idea to get succinct encodings: projections
$\rightsquigarrow$ map states to abstract states with perfect hash function

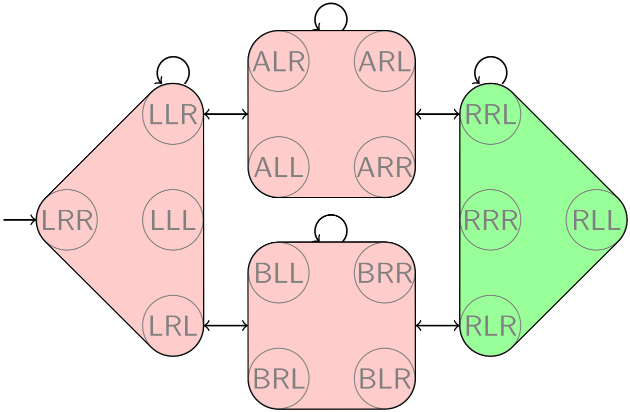
Definition (projection)

Projection $\pi_{\mathcal{V}'}$ to variables $\mathcal{V}' \subseteq \mathcal{V}$: homomorphism α where $\alpha(s) = \alpha(s')$ iff s and s' agree on $\mathcal{V}'$

shorthand for atomic projections: $\pi_v := \pi_{\{v\}}$ $(v \in \mathcal{V})$

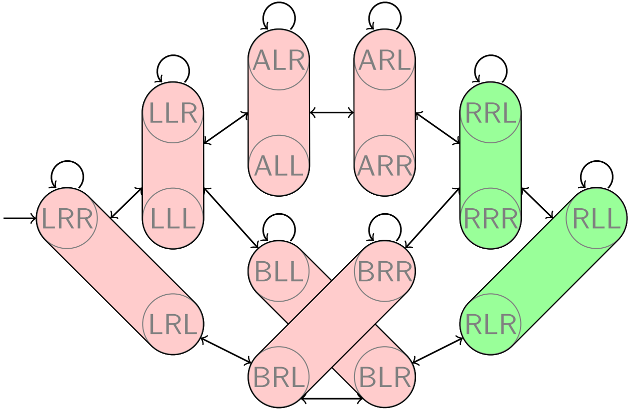
Example: Projection (1)

Project to {package}:



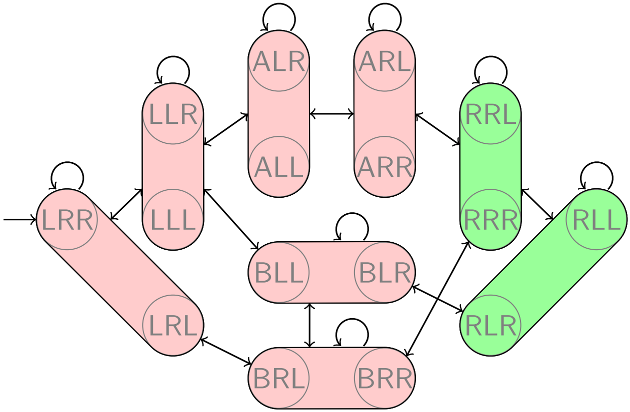
Example: Projection (2)

Project to {package, truck A}:



Example: Projection (2)

Project to {package, truck A}:



Problems of Projections

- abstraction heuristics for projections are pattern database (PDB) heuristics
- must keep number of reflected variables (pattern) small

price in heuristic accuracy:

- consider generalization of running example: N trucks, M locations (still one package)
- consider any pattern that is proper subset of $\mathcal{V}$
- $h(s_0) \leq 2 \rightsquigarrow$ no better than atomic projection to package

(maximizing over patterns or additive patterns do not help either)

Outline

Explicit-State Abstraction Heuristics: Main Idea

Main idea

(due to Dräger, Finkbeiner & Podelski, 2006):

Instead of perfectly reflecting a few state variables,
reflect all state variables, but in a potentially lossy way.

Explicit-State Abstraction Heuristics: Key Insights

Key insights:

1. Information of two abstractions $\mathcal{A}$ and $\mathcal{A}'$ of the same transition system can be composed by a simple graph-theoretic operation (synchronized product $\mathcal{A} \otimes \mathcal{A}'$).
2. Under suitable conditions (factored transition systems), the complete state space can be recovered using only atomic projections:

$$\bigotimes_{v \in \mathcal{V}} \pi_v \text{ is isomorphic to } \pi_{\mathcal{V}}.$$

   $\rightsquigarrow$ build fine-grained abstractions from coarse ones
3. When intermediate results become too big, we can shrink them by aggregating some abstract states.

Computing Explicit-State Abstractions

Generic abstraction computation algorithm

abs := all atomic projections π_v ($v \in \mathcal{V}$).
while abs contains more than one abstraction:
  select $\mathcal{A}_1$, $\mathcal{A}_2$ **from** abs
  shrink $\mathcal{A}_1$ and/or $\mathcal{A}_2$ until *size*$(\mathcal{A}_1) \cdot$ *size*$(\mathcal{A}_2) \leq N$
  abs := abs $\setminus \{\mathcal{A}_1, \mathcal{A}_2\} \cup \{\mathcal{A}_1 \otimes \mathcal{A}_2\}$
return the remaining abstraction

N: parameter bounding number of abstract states

Questions for practical implementation:

- Which abstractions to select? $\rightsquigarrow$ composition strategy
- How to shrink an abstraction? $\rightsquigarrow$ shrinking strategy
- How to choose N?

Outline

Guiding Questions for Evaluation

Comparison to state of the art

Is this competitive with the state of the art?

- Compare scaling behaviour to other heuristics: blind, $h^{\max}$, PDB

⇝ next slide

Comparison to pattern databases

How does this compare to well-chosen PDB heuristics?

- compare to approach of Haslum et al. (2007)
- compare scaling behaviour and runtime
- compare heuristic quality, preprocessing time, search time

⇝ details in the ICAPS 2007 paper

Comparison to State of the Art

Comparison to state of the art

Is this competitive with the state of the art?

- Compare scaling behaviour to other heuristics: blind, $h^{\max}$, PDB

Domain	**abs**	blind	$h^{\max}$	PDB
PIPES-NOTANKAGE	**19**	14	15	15
PIPES-TANKAGE	**13**	10	10	7
SATELLITE	**6**	4	5	6
LOGISTICS	**18**	6	6	16
PSR	**5**	3	4	4
TPP	**7**	5	6	6
total	**68**	42	46	54

Comparison to Pattern Databases: Theory

As powerful as PDBs

PDB heuristics are a special case of our abstraction heuristics, and arise naturally as a side product.

Get additivity for free

If P and P' are additive patterns, then
for all h-preserving abstractions $\mathcal{A}$ of π_P and $\mathcal{A}'$ of $\pi_{P'}$,
the abstraction heuristic for $\mathcal{A} \otimes \mathcal{A}'$ dominates $h^P + h^{P'}$.

Greater representational power

In some planning domains where PDBs have unbounded error (Gripper, Schedule, two Promela variants), we can obtain perfect heuristics in polynomial time with suitable composition/shrinking strategies.

Outline

Conclusion

Summary

- clean, flexible approach to computing heuristics
- works very well for planning and model checking

Future work:

- more theory
- more experiments
- more informed abstraction strategies
- comparison of abstraction strategies
- determine/adjust abstraction size dynamically