When are BDDs **compatible** with a decoupled state?

When is a decoupled state **compatible** with a Merge&Shrink tree?

**Theoretical Results**

**Theorem 1.** Let $A$ be an ADD of a PDB heuristic $h$, $s$ a decoupled state, and $B \in \mathbb{R}^+ \cup \{\infty\}$. It is NP-complete to decide if $h_B(s) < B$.

**Corollary 2.** Let $\sigma$ be an FM that represents a M&S heuristic $h$ and $s$ a decoupled state. It is NP-complete to decide if $h_{\sigma}(s) < B$.

**Compliance of Compact Data Structures**

**Definition 3 (Covered Leaves by FMs, Nestedness).**

- Leaves covered by an FM: $\text{cov}(\sigma) := \{L \in \mathcal{L} | L \cap \text{vars}(\sigma) \neq \emptyset\}$.
- Leaves fully covered by an FM: $\text{fc}(\sigma) := \{L \in \mathcal{L} | L \subseteq \text{vars}(\sigma)\}$.
- Leaves partially covered by an FM: $\text{pc}(\sigma) := \text{cov}(\sigma) \setminus \text{fc}(\sigma)$.
- Leaves exactly covered by an FM: $\text{ec}(\sigma) := \text{fc}(\sigma) \setminus (\text{fc}(\sigma) \cup \text{pc}(\sigma))$.

**Nestedness of an FM:** $N(\sigma) := 0$ for atomic FMs, otherwise $N(\sigma) := \max(N(\sigma_1), N(\sigma_2), \text{pc}(\sigma_1) \cup \text{pc}(\sigma_2))$.

**Definition 4 (Compliant FMs).** An FM is **compliant** with a factoring $\mathcal{F}$ if $N(\sigma) \leq 1$; it is **strongly compliant** with $\mathcal{F}$ if for all $L \in \mathcal{L}$ there exists a descendant $\sigma'$ of $\sigma$ s.t. $\text{cov}(\sigma') = \text{ec}(\sigma') = \{L\}$.

**Definition 5 (Compliant ADDs).** A variable ordering is **compliant** with a factoring $\mathcal{F}$ if:

$$\forall_{v \in \text{vars}}(\mathcal{F}(v) \neq \mathcal{F}(v') \neq \mathcal{F}(v'') \neq \emptyset) \Rightarrow \mathcal{F}(v') \in \{\mathcal{F}(v), \mathcal{F}(v''), \mathcal{F}(v''')\}$$

**Runtime – General vs. Compliant Heuristic**

**Symbolic PDBs (ADDs) vs. M&Ss Factored Mappings**

Represented state sets:
- All states that satisfy formula.
- All states mapped to finite value set.

![Symbolic PDBs (ADDs) vs. M&Ss Factored Mappings Diagram](image)