

# Unsolvability Certificates for Classical Planning

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# Motivation

Validating correctness of planner output:

- Why?
  - ↪ Software bugs, hardware faults, malicious reasons . . .
- How?
  - (a) Planner outputs a plan: VAL/INVAL
  - (b) Planner claims unsolvability: ?

# Proving Unsolvability

## Goal

Generate **unsolvability certificate** which can be verified

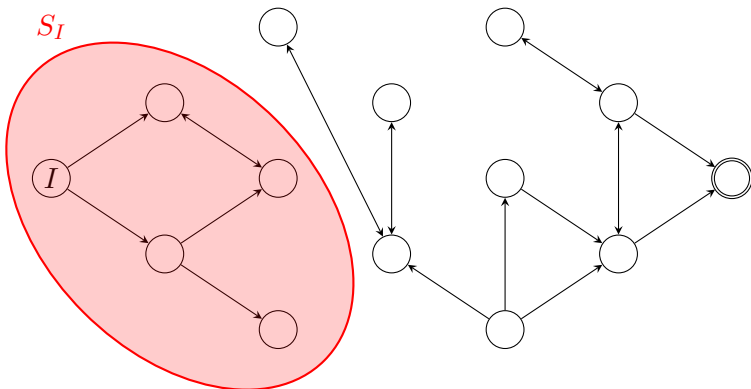
Desired properties:

- Soundness & Completeness
- Efficient generation
- Efficient verification
- Generality



# General Idea

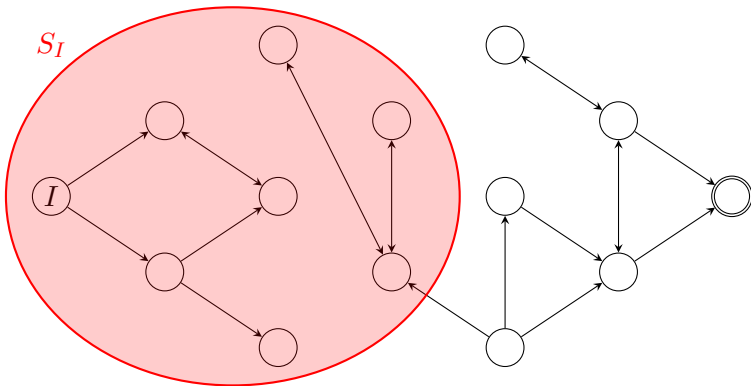
Unsolvable planning problems  $\rightsquigarrow$  No path from  $I$  to goal



Split graph into  $S_I$  and  $S_G (= \overline{S_I})$  s.t. **no outgoing edges from  $S_I$**

# General Idea

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# Inductivity

## Inductive Set

A state set  $S$  is inductive if for all  $s \in S$ ,  
all operator applications lead to a  $s' \in S$ .

## Inductive Certificate

If a state set

- ① contains the initial state
- ② contains no goal state
- ③ is inductive

the planning task is unsolvable.

# Representation of State Sets

Representation of state sets as [logical formulas](#)

We focus on the following representations:

- (RO)BDD
- 2CNF
- Horn Formulas



# Conjunctive/Disjunctive Certificates

Not all state sets compactly representable

## Conjunctive/Disjunctive Certificate

$\mathcal{S} = \{S_1, \dots, S_n\}$  is a

- conjunctive certificate:  $\bigcap_{S_i \in \mathcal{S}} S_i$  is inductive certificate
- disjunctive certificate:  $\bigcup_{S_i \in \mathcal{S}} S_i$  is inductive certificate

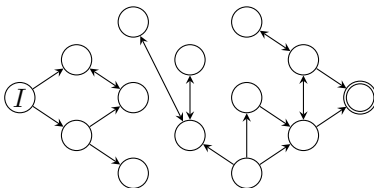
**Efficient Verification?** in general not feasible

↪ only consider up to  $r$  sets at once

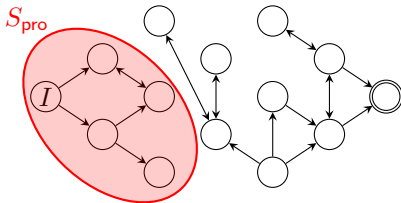
# Suitable Representations

Certificate Type	BDD	2CNF	Horn Formulas
Inductive Certificate	Yes	Yes	Yes
Conjunctive Certificate	No	Yes	Yes
r-conjunctive Certificate	Yes	Yes	Yes
Disjunctive Certificate	No	No	No
r-disjunctive Certificate	Yes	No	No
1-disjunctive Certificate	Yes	Yes	Yes

# Blind Search

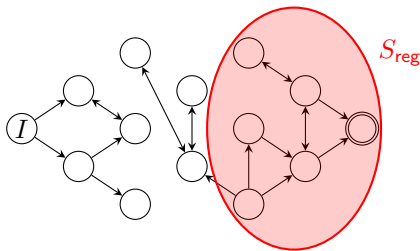


# Blind Search



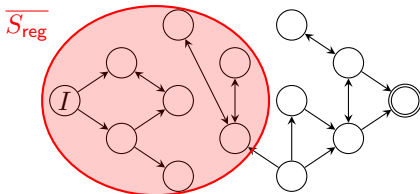
- Progression: expanded = reachable from  $I$   
     $\rightsquigarrow$  inductive certificate

# Blind Search



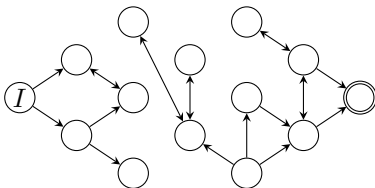
- Progression: expanded = reachable from  $I$   
     $\rightsquigarrow$  inductive certificate
- Regression: expanded = backwards-reachable from goal

# Blind Search



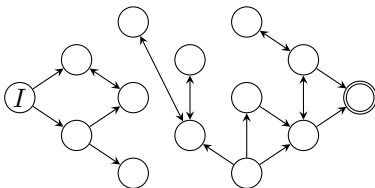
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- Progression: expanded = reachable from  $I$   
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↔ complement is inductive certificate
- Bidirection: whichever direction shows unsolvability

Suitable representation: BDDs (for symbolic search)



# Merge & Shrink

Union of states  $s$  where  $h^{M\&S}(s) = \infty$  is inductive & no goal states  
 $\rightsquigarrow$  If  $h^{M\&S}(I) = \infty$ , this union is **inductive certificate**

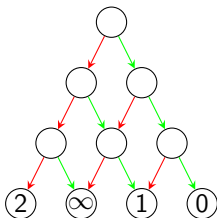
For linear merge strategies:

- 1 Represent cascading tables as ADD
- 2 Compress to BDD: finite  $h$ -values lead to  $\perp$ , infinite to  $\top$

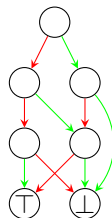
		$v_1$	
		0	1
$v_2$	0	0	1
	1	1	2

		$v_3$	
		0	1
$v_{1,2}$	0	2	$\infty$
	1	$\infty$	1
	2	2	0

Cascading tables



ADD



BDD

# Delete Relaxation Heuristics

$h^+(s) = \infty$  if part of the goal is relaxed unreachable

- $U^+(s)$ : relaxed unreachable variables
- $\varphi_{U^+(s)} = \bigwedge_{v \in U^+(s)} \neg v$  is inductive & no goal states

$\rightsquigarrow$  If  $h^+(I) = \infty$ ,  $\varphi_{U^+(s)}$  represents **inductive certificate**

Covers all delete-relaxation heuristics ( $h^{\max}$ ,  $h^{\text{add}}$ ,  $h^{\text{FF}}$ ,  $h^{\text{LM-Cut}}$ , ...)

Suitable representation: BDDs, Horn Formulas, 2CNF

# $h^m$ -Family

Similar idea to  $h^+$ , but with unreachable conjunctions:

$$\bigwedge_{c \in U^m(I)} \bigvee_{v \in c} \neg v$$

Suitable representation: Horn Formulas, 2CNF (for  $m \leq 2$ ),  
BDDs (as 1-conjunctive Certificate)

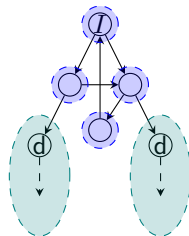
# Heuristic Search

Heuristic certificates sufficient if  $h(I) = \infty$

General heuristic search:

- $\mathcal{S}_{\text{exp}} = \{\{s\} \mid s \in \text{expanded states}\}$
- $\mathcal{S}_{\infty}$ : family of inductive sets covering all detected dead ends

$\rightsquigarrow \mathcal{S}_{\text{exp}} \cup \mathcal{S}_{\infty}$  is 1-disjunctive certificate



Suitable representation: BDDs, Horn Formulas, 2CNF

**Limitation:**

- all sets must have same representation
- sets cannot be conjunctive/disjunctive

# Trapper

Trapper [Lipovetzky et al. 2016]:

- only considers states not violating mutexes  $M$  (based on  $h^2$ )
- no escape from  $\varphi_{\text{trap}} \rightsquigarrow$  **inductive**
- no goal states (in considered states)

Observations:

- $\varphi_{\text{trap}}$  alone no certificate (goal states)
- states not violating mutexes ( $= \varphi_{\neg M}$ ) inductive

$\rightsquigarrow \varphi_{\text{trap}} \wedge \varphi_{\neg M}$  represents **inductive certificate** (even 1-disjunctive)

Suitable representation: 2CNF, Horn Formulas

# Experiments

Proof of concept implementation of

- **FD<sup>cert</sup>**: generates BDD certificates for  $A^* + h^{\max} / h^{\text{M\&S}}$
- **Verifier**: vanilla, r-conjunctive, r-disjunctive BDD certificates

limits: 30 min generation, 4 hours verification

	$h^{\max}$			$h^{\text{M\&S}}$		
	FD	FD <sup>cert</sup>	Ver.	FD	FD <sup>cert</sup>	Ver.
Coverage (702)	212	136	123	223	191	155

all certificates valid

# Conclusion

## unsolvability certificates based on inductive sets

- completeness: yes
- efficient generation: yes/no
- efficient verification: mostly yes (if efficient generation)
- generality: yes/no

## Future Work

- cover more techniques (heuristics, pruning, ...)
- combined certificate with different formalisms