Compact Representatives of Potential Heuristics

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HSDIP ICAPS 2026

Slide from Helmert, Sievers, Rovner and Corrêa
On the Complexity of Heuristic Synthesis for Satisficing Classical Planning: Potential Heuristics and Beyond ICAPS 2022:

OK, Time to 'Fess Up!

I've been lying to you: we did not actually prove these results.

- proved something less satisfying:
  some results need "compact" heuristics
  (polynomial number of bits for feature weights)
- required for "guess-and-check" to be in NP

working on it $\rightsquigarrow$ stay tuned!

Background

Potential Heuristics

Potential Heuristic

A **potential heuristic** is a heuristic that is computed with a weighted count of the partial states that agree with the given state.

$$h^{pot}(s) = \sum_{p \in \mathcal{P}} w(p) \cdot [p \subseteq s]$$

A **feature** of h^{pot} is a partial state with non-zero weight.
The **dimension** of h^{pot} is the size of the largest feature.

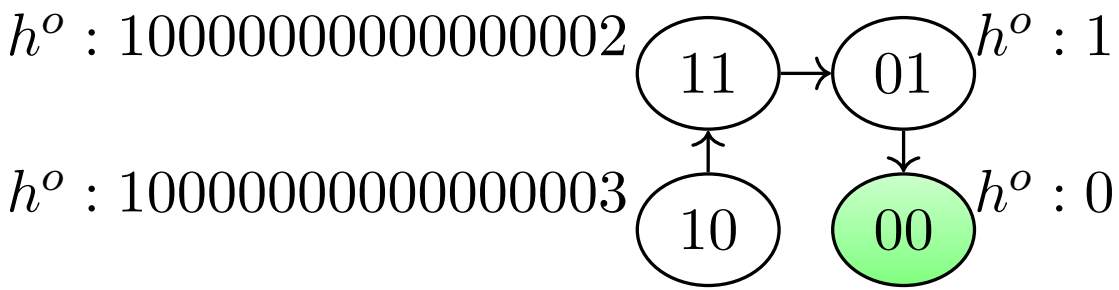
For satisficing planning we would like to have a "nice" heuristic i.e. one that guides greedy best first search without backtracking to a goal state.

UDDA:

- each non-goal state has a successor with lower h value

Different concrete formulations are captured in the work:
DDA, SDDA, UDDA, ∞DDA and PDDA

Example



- $w^o(\{x \mapsto 1, y \mapsto 1\}) = -2$
- $w^o(\{x \mapsto 1\}) = 10000000000000003$
- $w^o(\{y \mapsto 1\}) = 1$

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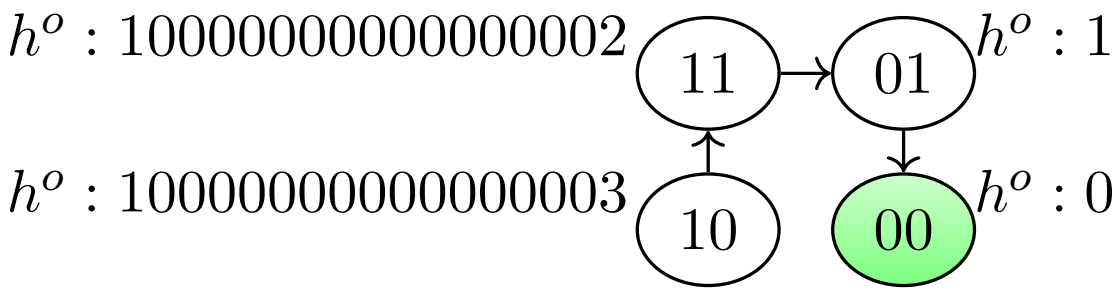

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Is UDDA but writing this took a lot of space.

Can we do it in a compact form (with the same features)?

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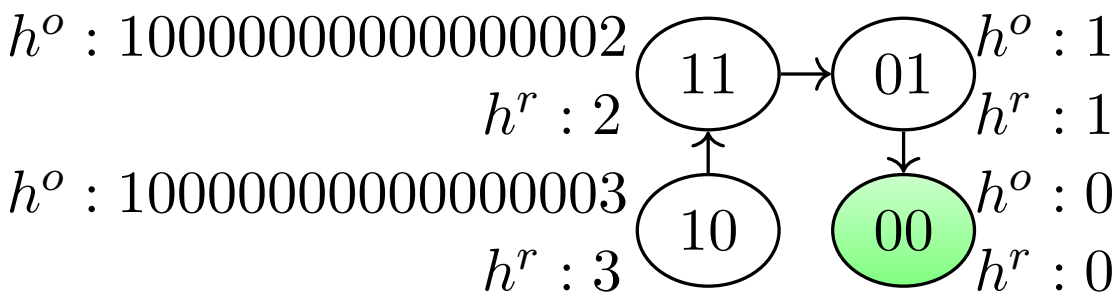

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Can we do it in a compact form (with the same features)?

- $w^r(\{x \mapsto 1, y \mapsto 1\}) = -2$
- $w^r(\{x \mapsto 1\}) = 3$
- $w^r(\{y \mapsto 1\}) = 1$

our Approach

Qualitative Equivalent

A heuristic h^r is **qualitative equivalent** to heuristic h^o if

- they are weak-order equivalent i.e. $h^r(s) > h^r(s')$ iff $h^o(s) > h^o(s')$
- they are sign equivalent i.e. $h^r(s) > 0$ iff $h^o(s) > 0$
- they are infinity equivalent i.e. $h^r(s) = \infty$ iff $h^o(s) = \infty$

Equipotent Reduction

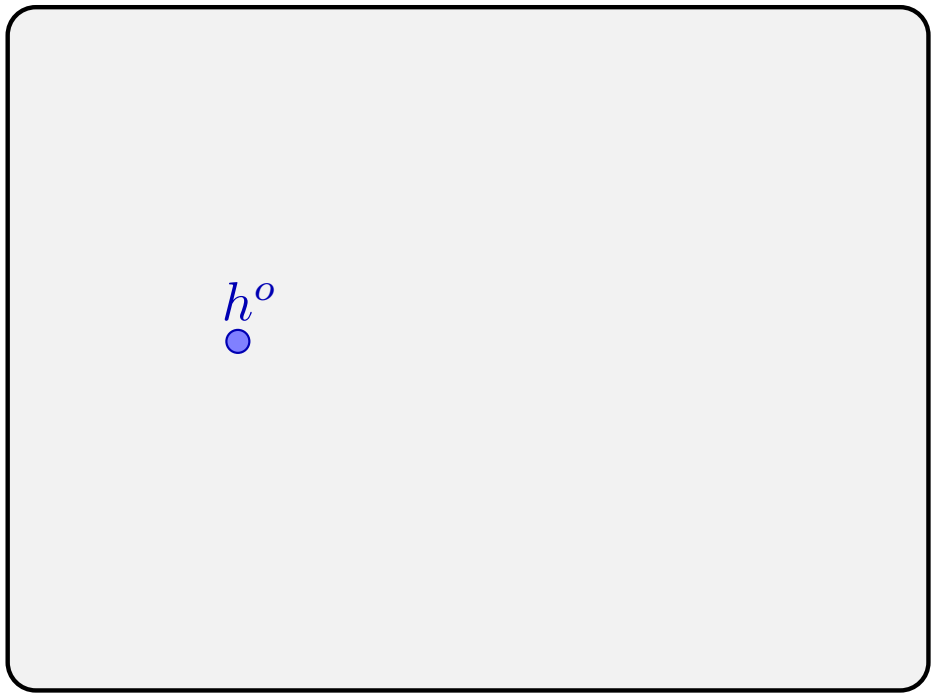
A potential heuristic h^r is an **equipotent reduction** of a potential heuristic h^o if

- all features of h^r are features of h^o
- they are qualitative equivalent

$\rightsquigarrow$ finding weights for an h^r that is an equipotent reduction of h^o can be expressed as linear programming constraints with solutions in $\mathbb{R}^{F(h^o)}$.

Compact Heuristics via Equipotent Reductions

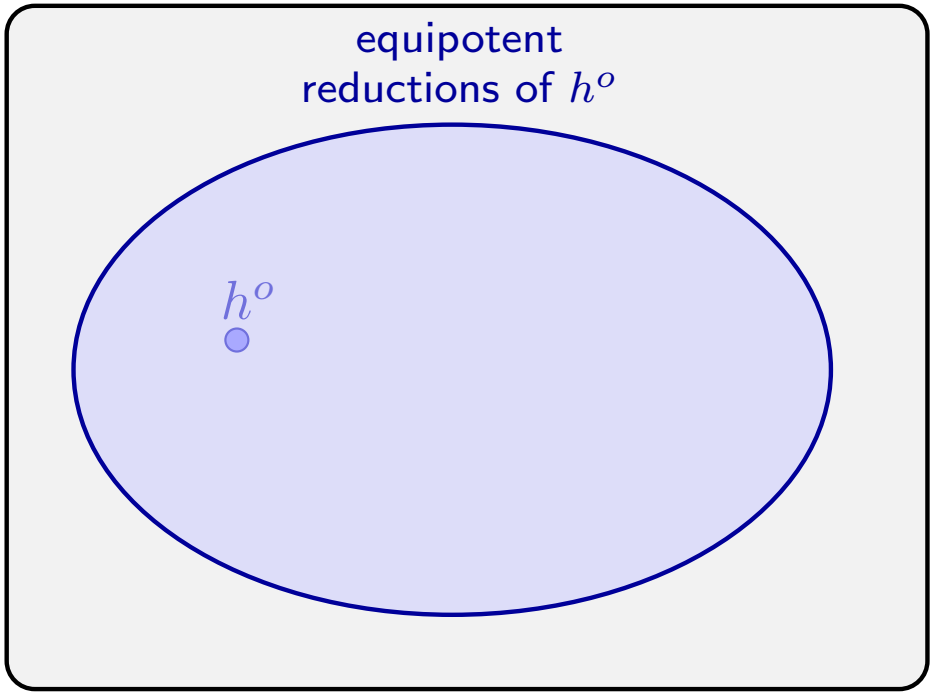
- Given any heuristic h^o



all UDDA potential heuristics with features $F(h^o)$

Compact Heuristics via Equipotent Reductions

- Given any heuristic h^o
- There is a set of heuristics that are equipotent reductions of h^o



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Compact Heuristics via Equipotent Reductions

- Given any heuristic h^o
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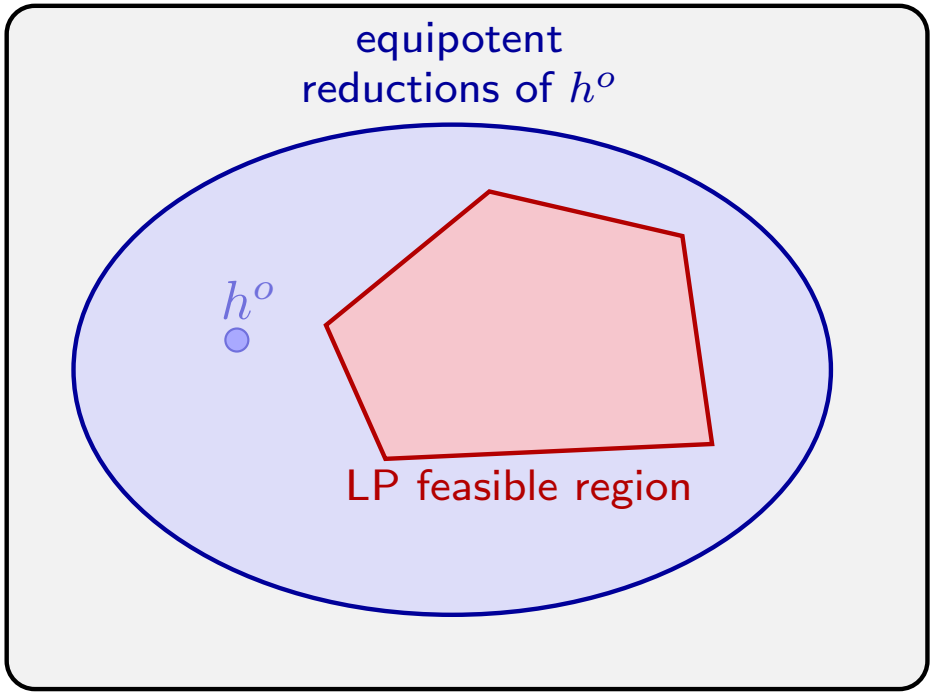
- We can describe a linear program whose feasible region contains only equipotent reductions of h^o



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Compact Heuristics via Equipotent Reductions

- Given any heuristic h^o
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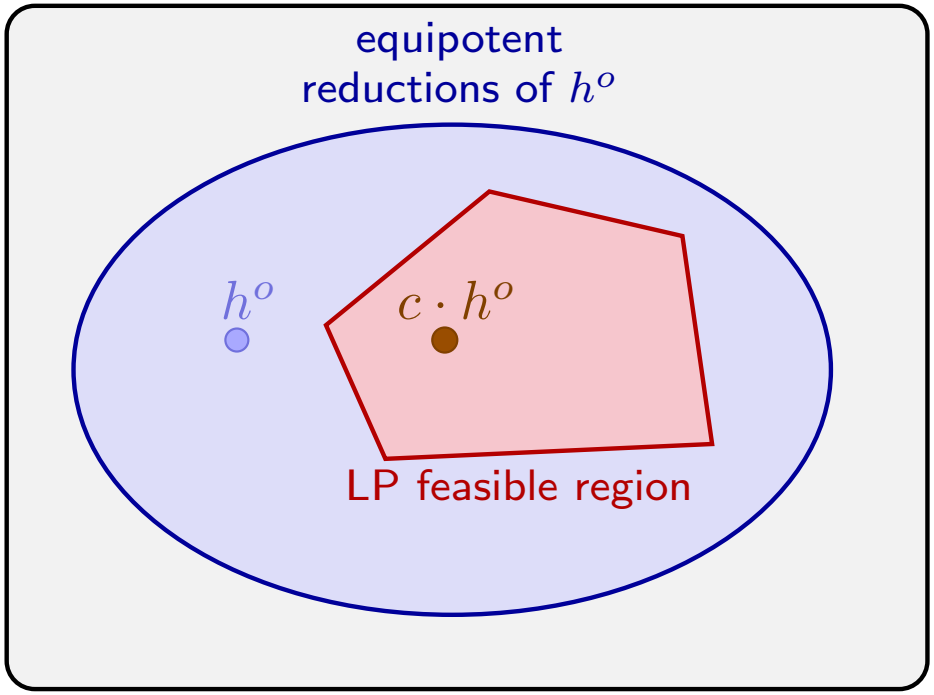
- We can describe a linear program whose feasible region contains only equipotent reductions of h^o
- The region contains at least a scaled verison of h^o



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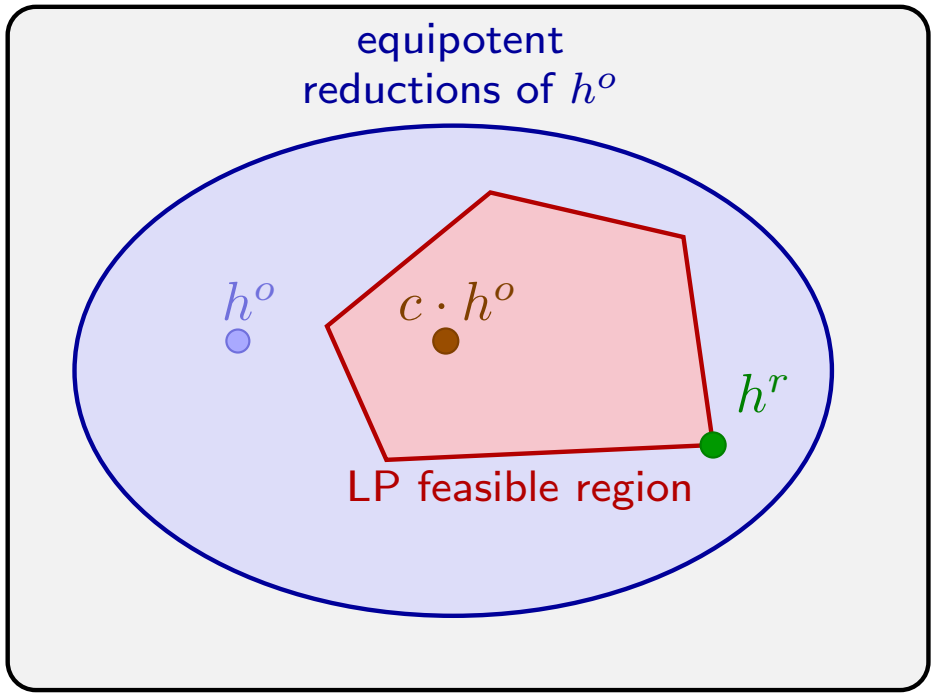
Compact Heuristics via Equipotent Reductions

- Given any heuristic h^o
- There is a set of heuristics that are equipotent reductions of h^o
- We can describe a linear program whose feasible region contains only equipotent reductions of h^o
- The region contains at least a scaled verison of h^o
- at this region's corner, we are guaranteed to find a compact heuristic h^r.



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Compact Representative

Theorem

For each potential heuristic h^o with n features there exists an equipotent reduction h^r where each finite weight of h^r is an integer of absolute value at most $n^{n/2}$.

$\rightsquigarrow$ Each potential heuristic has a compact representative.

Complexity

Decision Problem

Does there exists a compact potential heuristic of dimension k that is UDDA for task Π?
(polynomial number of bits for feature weights)

$\rightsquigarrow$ member of Σ_2^p (2nd level of the polynomial hierarchy)

Proof by Helmert et al. (2022) relies on a non-deterministic Turing machine guessing a UDDA heuristic.

More satisfying:
Does there exists any potential heuristic of dimension k that is UDDA for task Π?

$\rightsquigarrow$ member of Σ_2^p

Combining this with other results from Helmert et al. 2022 we get:

Corollary

The decision problem
" Does there exists a potential heuristic of dimension k that is UDDA for task Π? "
is Σ_2^p-complete.

Analogous for ∞-DDA and PDDA.

Thank you for your attention

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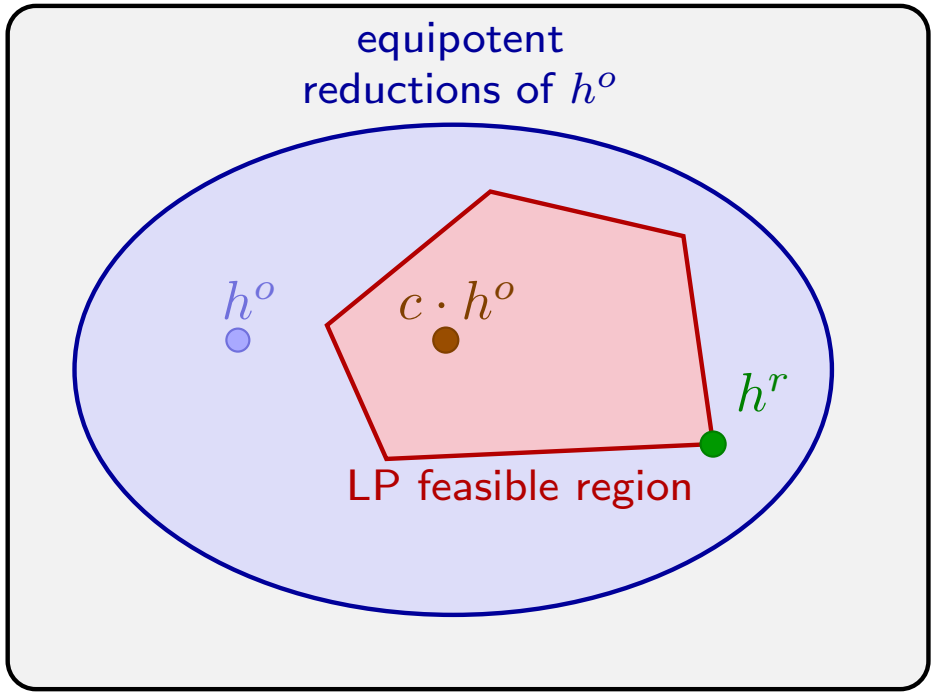

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