Higher-Dimensional Potential Heuristics: Lower Bound Criterion and Connection to Correlation Complexity

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Planning

SAS$^+$ Planning Task $\Pi = \langle V, I, O, \gamma \rangle$

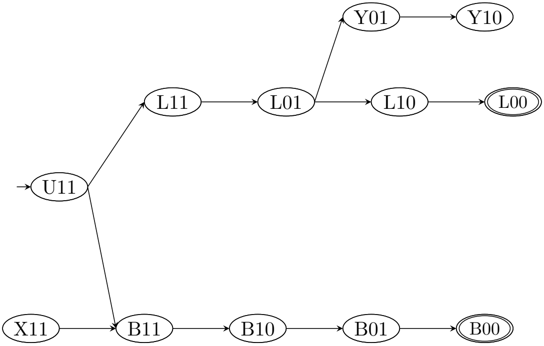
- state variables V with finite domain
- initial state I
- operators O
- goal γ

Satisficing planning ignores plan cost.

Planning

Task induces a directed graph called state space

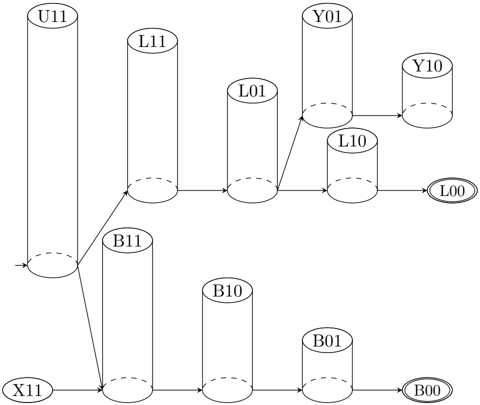
- Nodes correspond to states
- Arcs correspond to operators



Heuristic

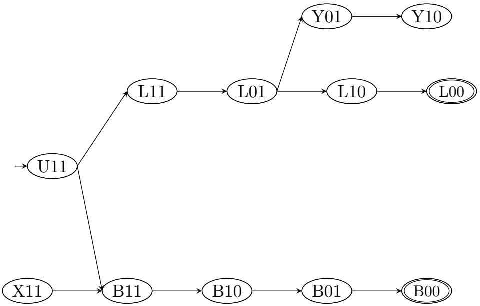
A heuristic h assigns a value to each state.

- Lower values for 'better' states.
- Induces a state space topology



Heuristic Properties

- alive state: reachable and solvable



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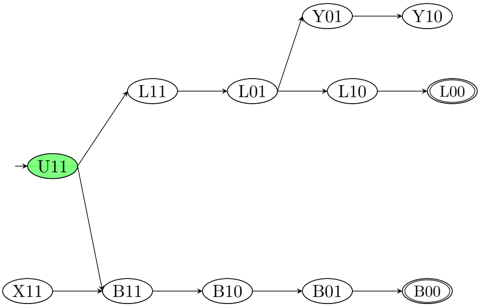

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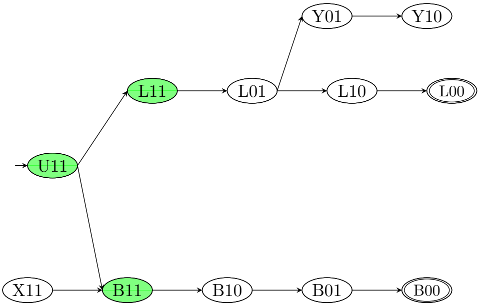

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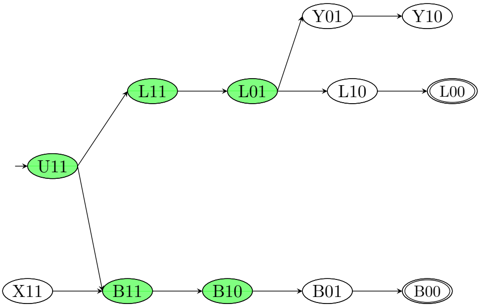

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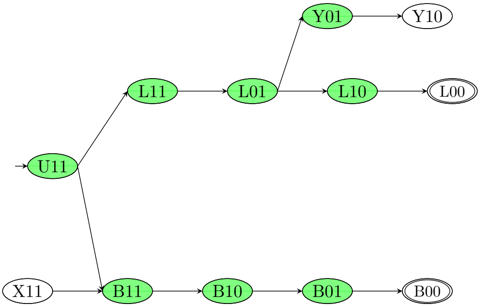

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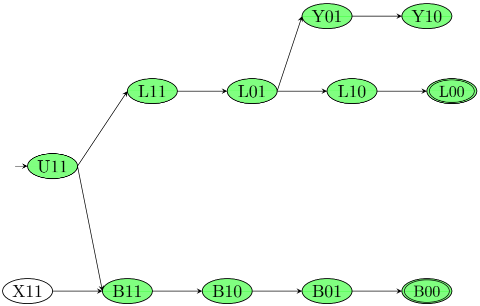

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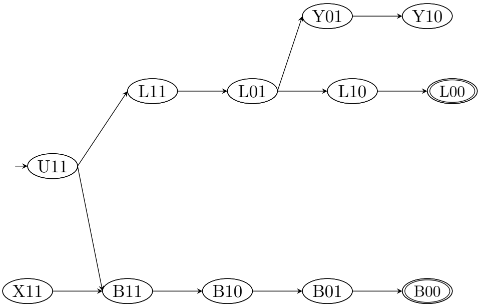

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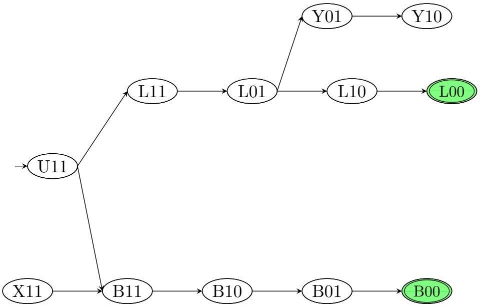

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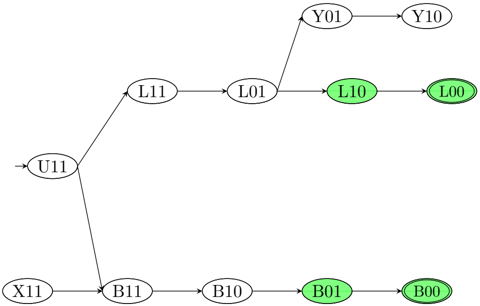

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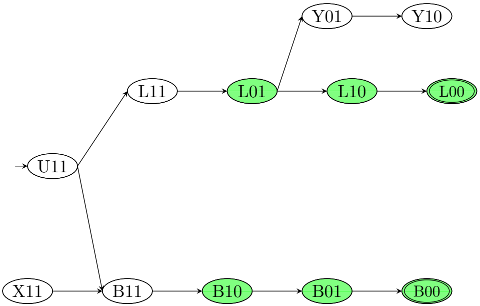

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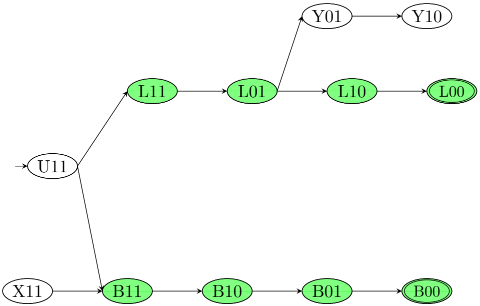

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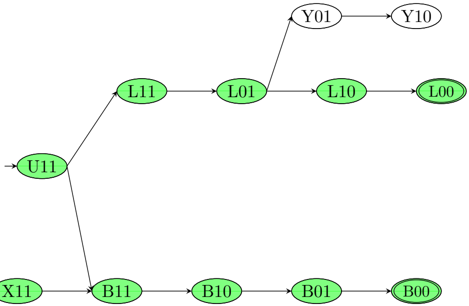
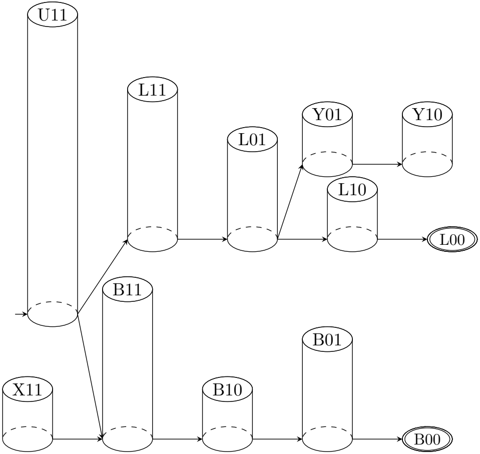


Heuristic Properties

- descending heuristic: all (non-goal) alive states have an improving successor



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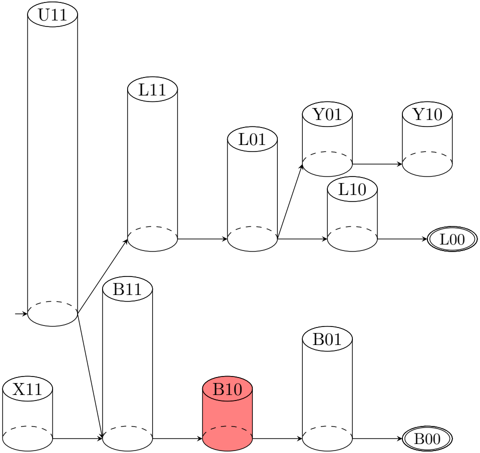

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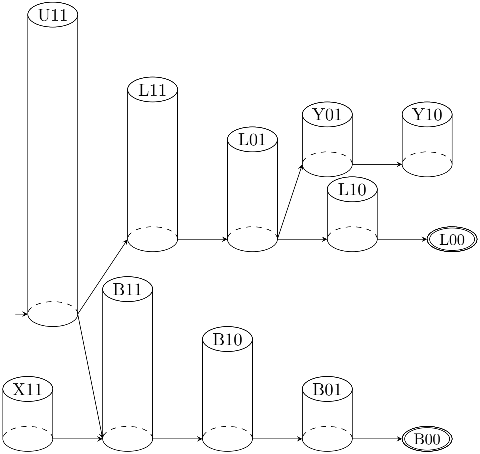
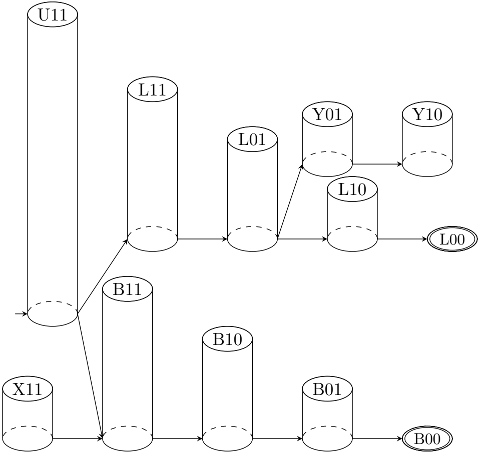


Heuristic Properties

- dead-end avoiding heuristic: all improving successors of (non-goal) alive states are solvable



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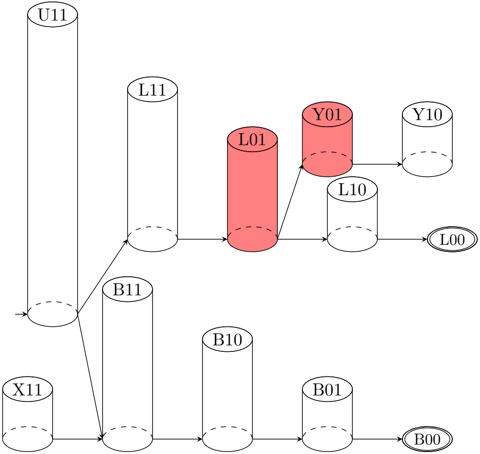

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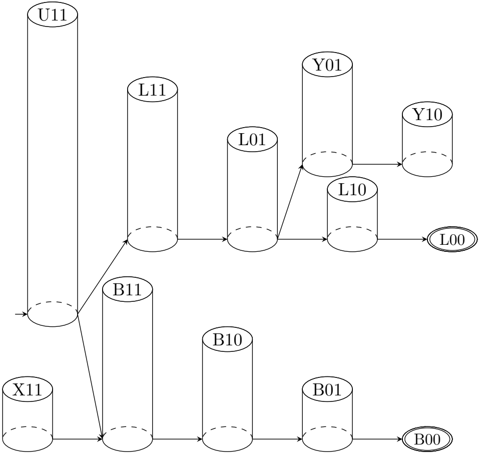

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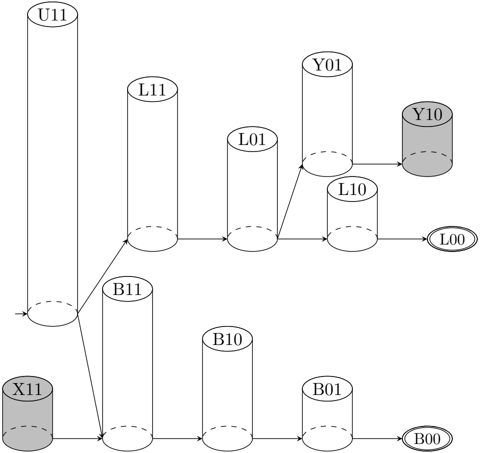


Potential Heuristics

Potential Heuristic

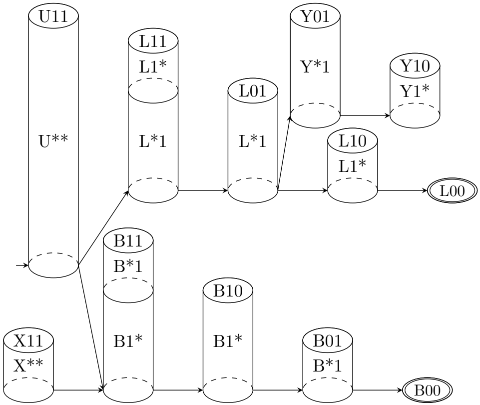
A **potential heuristic** is a heuristic that is computed with a weighted count of the partial assignments that agree with the given state.

$$h^{pot}(s) = \sum_{p \in \mathcal{P}} (w(p) \cdot [p \subseteq s])$$

The **dimension** of h^{pot} is $max_{p \in \mathcal{P}, w(p) \neq 0} |p|$.

Correlation Complexity

Correlation Complexity (Seipp et. al. 2016) asks: "What dimension is required to construct a descending and dead-end avoiding potential heurisitc?"

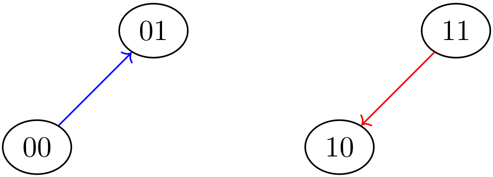


p	$w(p)$
U^{**}	5
$L1^{*}$	1
$L^{*}1$	2
$B1^{*}$	2
$B^{*}1$	1
$Y^{*}1$	2
$Y1^{*}$	1
X^{**}	1

Original Criterion

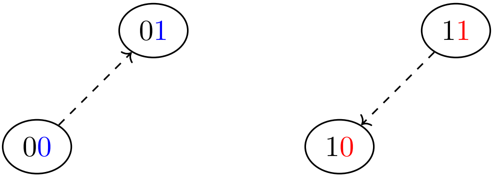
Critical operator: An alive state exists where each plan contains this operator.

Criterion (Seipp et. al. 2016) Let Π be a planning task in normal form, and let o and o' be critical operators of Π that are inverses of each other. Then Π has correlation complexity at least 2.



Shift view

They focus on the critical operators, we focus on the states and differences of heuristic values.



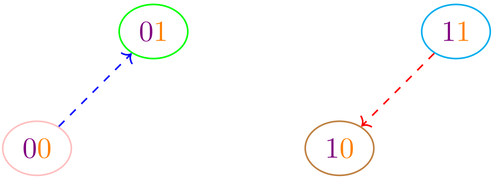
Witness

Witnessing Quartet Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and h a heuristic for Π. We call $\langle [a, b, c, d], [W, M] \rangle$ with a, b, c, d states in Π and $\{W, M\}$ a partition of V such that:

$$h(a) > h(b), \quad a^W = b^W, \quad a^M = d^M,$$

$$h(c) \geq h(d), \quad c^W = d^W, \quad b^M = c^M,$$

a **witnessing quartet** for h.



New Criterion

Theorem If a witnessing quartet for h exists, then the dimension of h is at least 2.

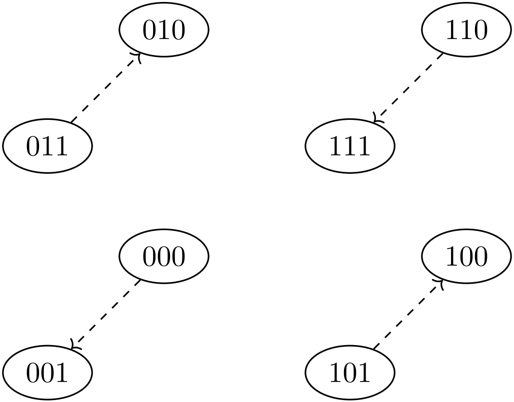
Is a generalization of the critical operators criterion.

Further Generalization

Theorem If a witnessing 2^n constellation for h exists, then the dimension of h is at least n.

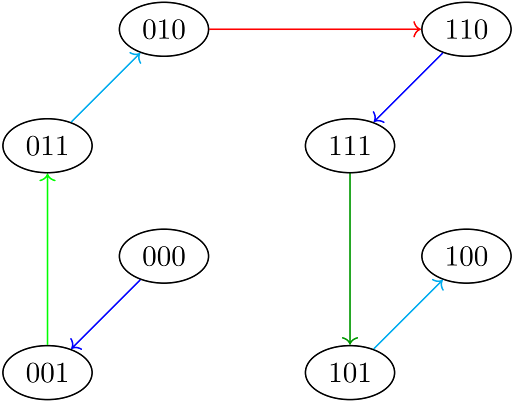
Witness

Witnessing 2^n Constellation (informal) A witnessing 2^2 constellation is basically the same as the witnessing quartet and a witnessing 2^3 constellation kinda looks like this:



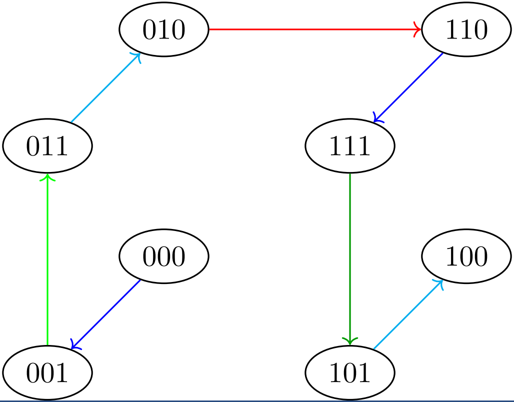
Folded Macro

Shift view back to operators (and sequences of operators i.e. marcos)



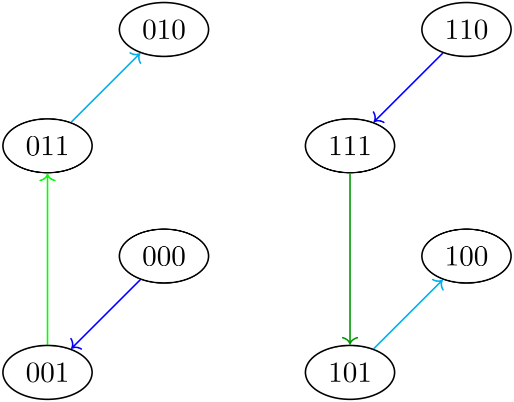
Folded Macro

Folded Macro A folded macro consists of a set-up a main execution and a tear-down. The set-up and tear-down are inverse of each other. If they are equally folded n times the macro is folded $n+1$ times.



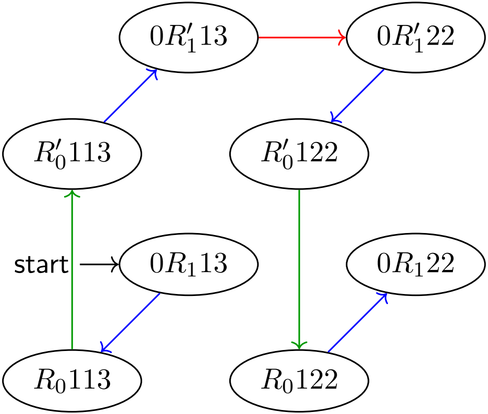
Folded Macro

Folded Macro Criterion Let Π be a planning task in normal form, and let *set-up* and *tear-down* be critical, n times equally folded macros of Π that are inverses of each other. Then Π has correlation complexity at least $n + 2$.



Conclusion

- In theory planning tasks with arbitrary correlation complexity exist.
- With the folded macro criterion we detect that the Termes tasks of the IPC have a correlation complexity of at least 3.



move-down pos-2 n1 pos-1 n0
create-block pos-1
move-up pos-1 n0 pos-2 n1
place-block pos-2 pos-3 n1 n2
move-up pos-2 n1 pos-3 n2
remove-block pos-3 pos-4 n3 n2
move-down pos-3 n2 pos-2 n1
move-down pos-2 n1 pos-1 n0
destroy-block pos-1
move-up pos-1 n0 pos-2 n1