Detecting Unsolvability Based on Separating Functions

Remo Christen, Salomé Eriksson, Florian Pommerening, and Malte Helmert

University of Basel

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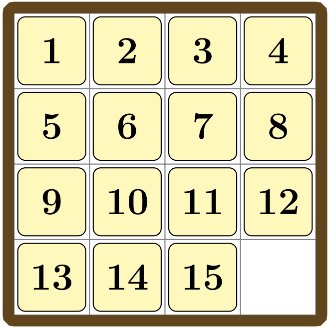
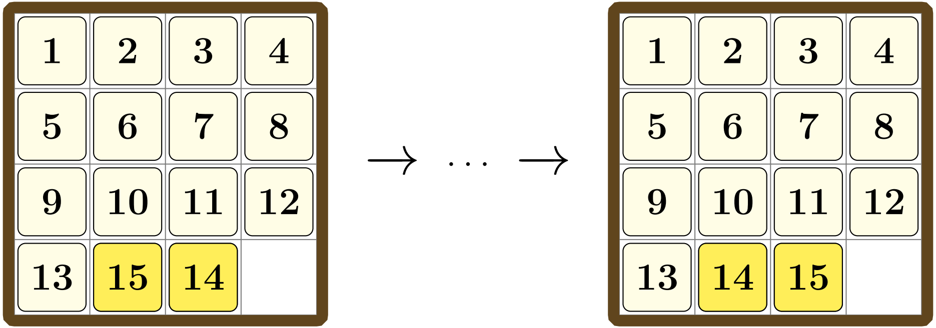
Setting

- classical planning
- unsolvability
- no search
- potential functions



Running Example



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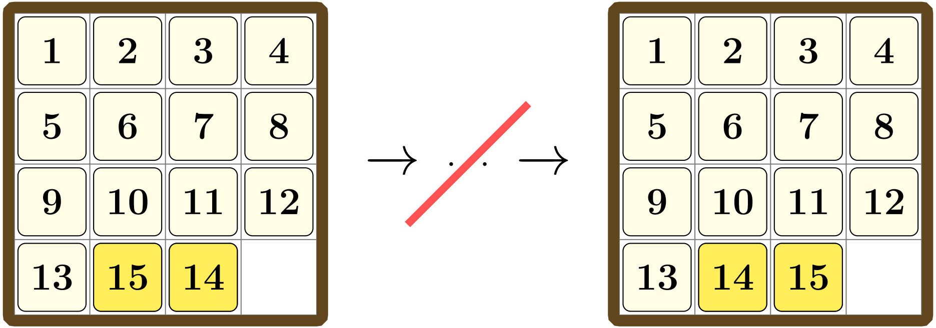
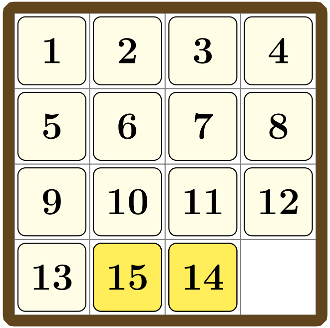
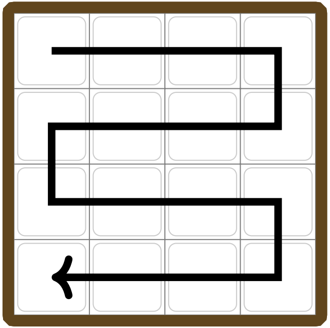


Parity Argument

- Johnson and Story (1879); Archer (1999)





$\langle \ldots, 9, 10, 11, 12, 14, 15, 13 \rangle$ $\Rightarrow$ 1 incorrectly ordered pairs

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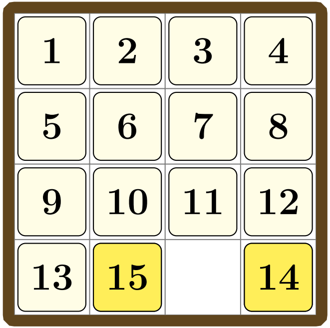

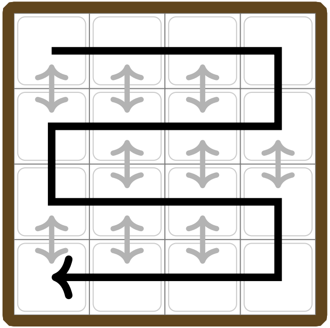

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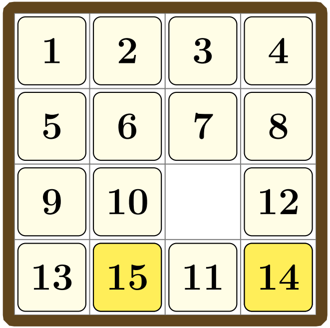

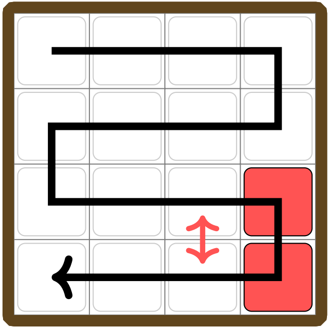

$\langle \ldots, 9, 10, 12, 14, 11, 15, 13 \rangle \Rightarrow 1 + 2$ incorrectly ordered pairs

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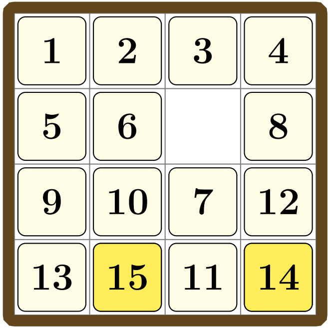
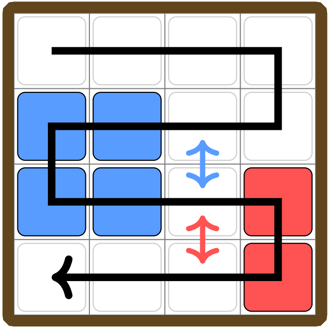
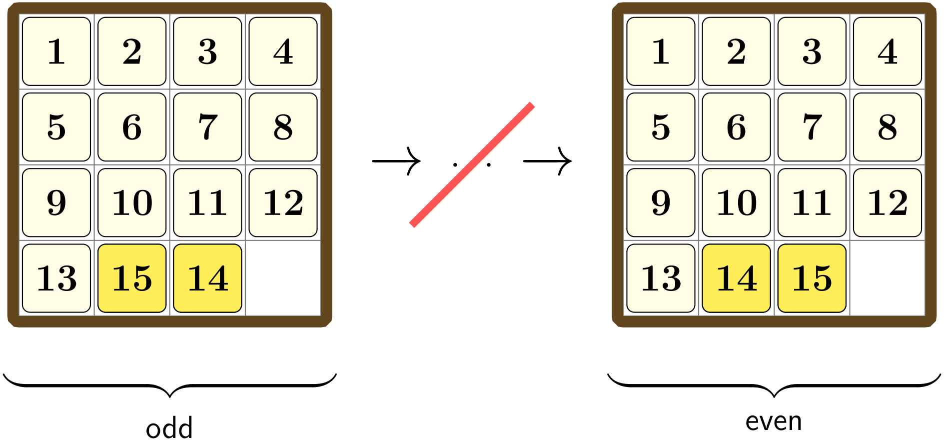




$\langle \ldots, 10, 7, 12, 14, 11, 15, 13 \rangle$ $\Rightarrow 1 + 2 + 4 + \ldots$ incorrectly ordered pairs

Parity Argument

- all reachable states are odd, but goal is even $\Rightarrow$ unsolvable



Parity Function

Formalize as a function:

$$p(s) \begin{cases} \mathbf{E} \text{ if } s \text{ contains an even number of incorrectly ordered pairs} \\ \mathbf{O} \text{ otherwise} \end{cases}$$

Unsolvability is shown by:

- $p(s_{\text{init}}) \neq p(s_{\text{goal}})$
- $p(s) = p(s')$ for all transitions $s \rightarrow s'$

Generalizations

Separating Function Example

A function $f\colon S \to \{\mathbf{E}, \mathbf{O}\}$ such that

- $f(s_{\text{init}}) \neq f(s_{\text{goal}})$
- $f(s) = f(s')$ for all transitions $s \to s'$

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codomain: $\ldots, \mathbb{F}_2, \mathbb{F}_3, \mathbb{R}, \ldots$

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separating function **exists** $\Rightarrow$ task **unsolvable**

Synthesis

Separating Potential Function

A potential function $\varphi\colon S \to F$ given by

$$\varphi(s) = \sum_{f \in \mathcal{F}} w(f)[s \models f]$$

such that the separating conditions hold

Until now:

- define **features**
- define **weight function**
- is separating function $\Rightarrow$ **unsolvable**

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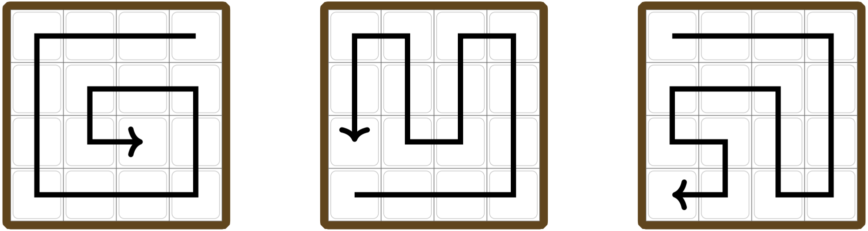
~~Until~~ now:

- define **features**
- ~~define weight function~~
- define **constraints**
- ~~is separating function~~ constraints satisfiable $\Rightarrow$ **unsolvable**

Constraints

- describe **all possible** separating potential functions

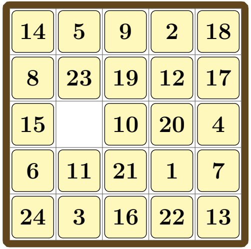


- **compact** for one- and two-dimensional features
- **efficient** satisfiability checks

$\langle \mathbb{F}_2, = \rangle$	$\Rightarrow$	XOR-constraints	$\Rightarrow$	Gaussian elimination
$\langle \mathbb{R}, \leq \rangle$	$\Rightarrow$	linear inequalities	$\Rightarrow$	LP-solver

Results

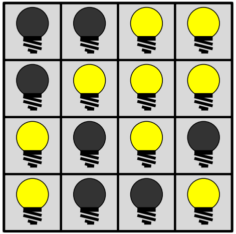
`sliding-tiles`



2-dimensional features

$\langle \mathbb{F}_2, = \rangle$

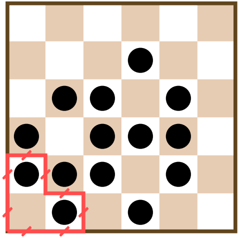
`lights-out`



1-dimensional features

$\langle \mathbb{F}_2, = \rangle$

`chessboard-pebbling`



1-dimensional features

$\langle \mathbb{R}, \leq \rangle$