General Idea

Separating functions can capture various unsolvability arguments.

- every move preserves the parity of the number of incorrectly ordered pairs
- initial state has 1 incorrectly ordered pair
- goal state has 0 incorrectly ordered pairs

Separating Function Example

A function \( f : S \to \{E, O\} \) such that
- \( f(s_{\text{init}}) \neq f(s_{\text{goal}}) \)
- \( f(s) = f(s') \) for all transitions \( s \to s' \)

codomain: \( F_2, F_3, R, \ldots \)
relation: \( > \) and \( \leq \), \( \ldots \)
condition: for all transitions \( s \to s' \) reachable from \( s_{\text{init}} \), \( \ldots \)

Potential Functions

Constraints

\[
\sum_{f \in \mathcal{F}} w(f)[s_{\text{init}} = f] \neq \sum_{f \in \mathcal{F}} w(f)[s_{\text{goal}} = f]
\]

\[
\sum_{f \in \mathcal{F}} w(f)[s = f] = \sum_{f \in \mathcal{F}} w(f)[s' = f]
\]
for all transitions \( s \to s' \)

- constraints satisfiable \( \Rightarrow \) task unsolvable
- compact representation for one- and two-dimensional features

Efficient Satisfiability Checks

\( \langle F_2, = \rangle \Rightarrow \) XOR-constraints \( \Rightarrow \) Gaussian elimination
\( \langle R, \leq \rangle \Rightarrow \) linear inequalities \( \Rightarrow \) LP-solver

Results

- detects unsolvability in all instances of sliding-tiles, lights-out, and chessboard-pebbling

Detecting Unsolvability Based on Separating Functions
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