Exploiting Cyclic Dependencies in Landmark Heuristics

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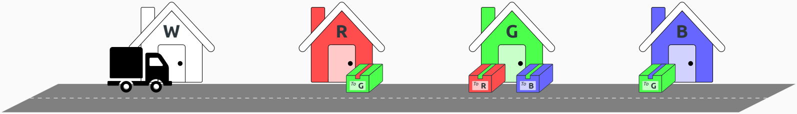
University of Basel

Setting

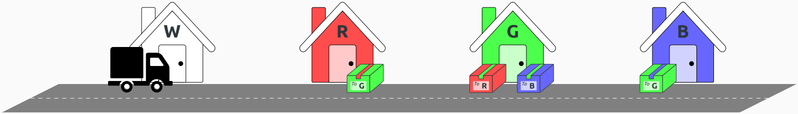
- Classical Planning
- Heuristic Search
- Landmarks



Landmark Heuristic h^{LM}

$$
\begin{aligned}
\min \sum_{a \in \mathcal{A}} \mathrm{Y}_a \cdot cost(a) \quad & \text{s.t.} \\
\mathrm{Y}_a \geq 0 \quad & \text{for all actions } a \in \mathcal{A} \\
\mathrm{Y}_L := \sum_{a \in L} \mathrm{Y}_a \geq 1 \quad & \text{for all landmarks } L \in \mathcal{L}
\end{aligned}
$$

Example: h^{LM}

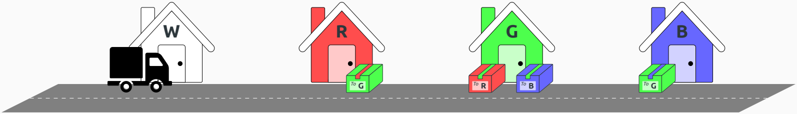


$$\min \; Y_W + Y_R + Y_G + Y_B \quad \text{s.t.}$$

$$Y_W, Y_R, Y_G, Y_B \geq 0$$

landmarks: $$Y_R, Y_G, Y_B \geq 1$$

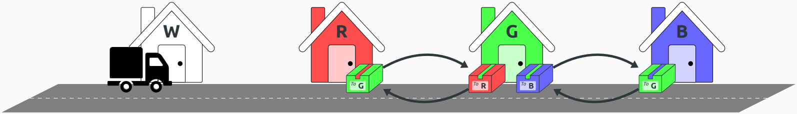
Example: h^{LM}



$Y_W = 0$ $\quad Y_R = 1$ $\quad Y_G = 1$ $\quad Y_B = 1$

$$\left.\begin{array}{rr} \min\ Y_W + Y_R + Y_G + Y_B & \text{s.t.} \\ Y_W, Y_R, Y_G, Y_B & \geq 0 \\ \text{landmarks:} \quad Y_R, Y_G, Y_B & \geq 1 \end{array}\right\} \Rightarrow h^{\mathrm{LM}} = 3$$

Example: h^{LM}



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Landmark Heuristic

$$\begin{aligned} \min \sum_{a \in \mathcal{A}} Y_a \cdot \mathit{cost}(a) \quad & \text{s.t.} \\ Y_a \geq 0 \quad & \text{for all actions } a \in \mathcal{A} \\ Y_L := \sum_{a \in L} Y_a \geq 1 \quad & \text{for all landmarks } L \in \mathcal{L} \end{aligned}$$

Cyclic Landmark Heuristic h^{cycle}

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\mathrm{Y}_L := \sum_{a \in L} \mathrm{Y}_a \geq 1 &\quad \text{for all landmarks } L \in \mathcal{L} \\
\sum_{L \in \mathcal{L}(c)} \mathrm{Y}_L \geq |\mathcal{L}(c)| + 1 &\quad \text{for all cycles } c \in \mathcal{C}
\end{aligned}
$$

Cyclic Landmark Heuristic h^{cycle}

$$\min \sum_{a \in \mathcal{A}} Y_a \cdot cost(a) \quad \text{s.t.}$$

$$Y_a \geq 0 \quad \text{for all actions } a \in \mathcal{A}$$

$$Y_L := \sum_{a \in L} Y_a \geq 1 \quad \text{for all landmarks } L \in \mathcal{L}$$

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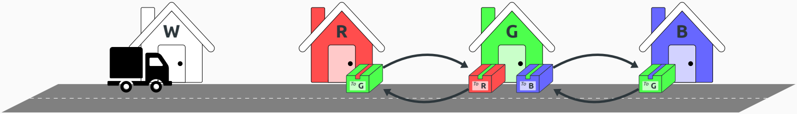
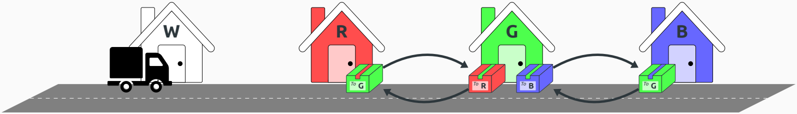
Example: h^{LM}



$Y_W = 0$ $Y_R = 1$ $Y_G = 1$ $Y_B = 1$

$$\left.\begin{array}{rr} \min\ Y_W + Y_R + Y_G + Y_B & \text{s.t.} \\ Y_W, Y_R, Y_G, Y_B \geq 0 & \\ \text{landmarks:} \quad Y_R, Y_G, Y_B \geq 1 & \end{array}\right\} \Rightarrow h^{\mathrm{LM}} = 3$$

Example: $h^{\mathsf{LM}} \rightarrow h^{\mathsf{cycle}}$



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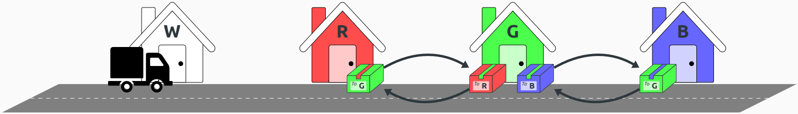
\text{landmarks:} & Y_R, Y_G, Y_B & \geq 1 \\
\text{cycle } R-G\text{:} & Y_R + Y_G & \geq 3 \\
\text{cycle } G-B\text{:} & Y_G + Y_B & \geq 3
\end{array}
\right\}
\quad h^{\mathsf{LM}} = 3
$$

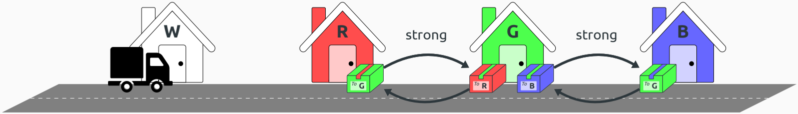
Example: $h^{\mathrm{LM}} \to h^{\mathrm{cycle}}$



$Y_W = 0$ $\quad$ $Y_R = 1$ $\quad$ ~~$Y_G = 1$~~ $Y_G = 2$ $\quad$ $Y_B = 1$

$$\left.\begin{array}{lr} & \min\ Y_W + Y_R + Y_G + Y_B \quad \text{s.t.} \\ & Y_W, Y_R, Y_G, Y_B \geq 0 \\ \text{landmarks:} & Y_R, Y_G, Y_B \geq 1 \\ \text{cycle } R-G\text{:} & Y_R + Y_G \geq 3 \\ \text{cycle } G-B\text{:} & Y_G + Y_B \geq 3 \end{array}\right\} \Rightarrow \begin{array}{l} h^{\mathrm{LM}} = 3 \\ h^{\mathrm{cycle}} = 4 \end{array}$$

Example: $h^{\mathrm{LM}} \rightarrow h^{\mathrm{cycle}}$



$Y_W = 0$ $\quad$ $Y_R = 1$ $\quad$ ~~$Y_G = 1$~~ $Y_G = 2$ $\quad$ $Y_B = 1$

$$\min\ Y_W + Y_R + Y_G + Y_B \quad \text{s.t.}$$

$$Y_W, Y_R, Y_G, Y_B \geq 0$$

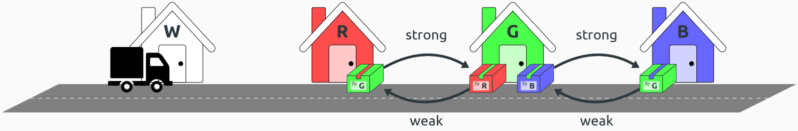
landmarks: $Y_R, Y_G, Y_B \geq 1$

cycle $R-G$: $Y_R + Y_G \geq 3$

cycle $G-B$: $Y_G + Y_B \geq 3$

$\Rightarrow$ $h^{\mathrm{LM}} = 3$, $h^{\mathrm{cycle}} = 4$

Example: $h^{\text{LM}} \to h^{\text{cycle}}$



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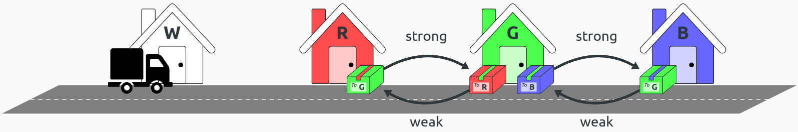
\text{cycle } R - G\text{:} & Y_R + Y_G \geq 3 \\
\text{cycle } G - B\text{:} & Y_G + Y_B \geq 3
\end{array}
\right\}
\Rightarrow
\begin{array}{l}
h^{\text{LM}} = 3 \\
h^{\text{cycle}} = 4
\end{array}
$$



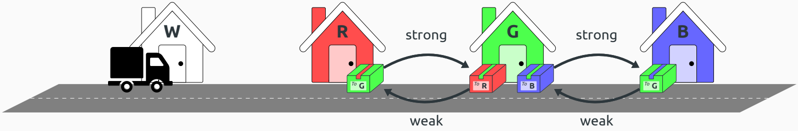
$Y_W = 0$ $Y_R = 1$ $\cancel{Y_G = 1}$ $Y_G = 2$ $Y_B = 1$

$$
\begin{aligned}
& \min \; Y_W + Y_R + Y_G + Y_B \quad \text{s.t.} \\
& Y_W, Y_R, Y_G, Y_B \geq 0 \\
\text{landmarks:} \quad & Y_R, Y_G, Y_B \geq 1 \\
\text{cycle } R-G\text{:} \quad & Y_R + \cancel{Y_G} \geq \cancel{3}\,2 \\
\text{cycle } G-B\text{:} \quad & Y_G + \cancel{Y_B} \geq \cancel{3}\,2
\end{aligned}
$$

$h^{\mathrm{LM}} = 3$

$h^{\mathrm{cycle}} = 4$

Example: $h^{\text{LM}} \rightarrow h^{\text{cycle}} \rightarrow h^{\text{strong}}$



$Y_W = 0$ | ~~$Y_R = 1$~~ $Y_R = 2$ | ~~$Y_G = 1$~~ $Y_G = 2$ | $Y_B = 1$

$$
\begin{aligned}
&\min \; Y_W + Y_R + Y_G + Y_B \quad \text{s.t.} \\
&Y_W, Y_R, Y_G, Y_B \geq 0 \\
\text{landmarks:} \quad &Y_R, Y_G, Y_B \geq 1 \\
\text{cycle } R-G\text{:} \quad &Y_R + \cancel{Y_G} \geq \cancel{3}\, 2 \\
\text{cycle } G-B\text{:} \quad &Y_G + \cancel{Y_B} \geq \cancel{3}\, 2
\end{aligned}
$$

$$h^{\text{LM}} = 3$$

$$h^{\text{cycle}} = 4$$

$$\Rightarrow h^{\text{strong}} = 5$$

Finding Cycles

Johnson's algorithm

- finds all elementary cycles
- worst case: $n!$ cycles

Oracle approach

- implicit hitting set
- few cycles sufficient
- same heuristic value

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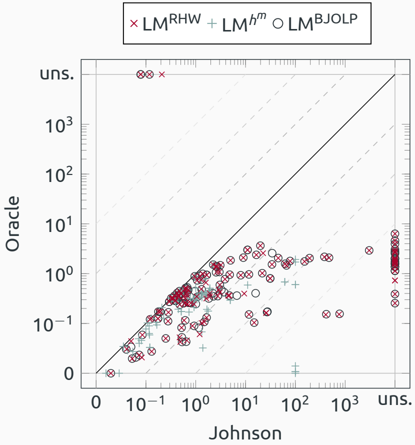

Experiments

Benchmark coverage

		Johnson		Oracle	
	h^{LM}	h^{cycle}	h^{strong}	h^{cycle}	h^{strong}
LM^{RHW}	644	630	643	657	665
LM^{h^1}	652	653	651	659	659
LM^{h^2}	377	383	383	383	383
LM^{BJOLP}	619	611	622	631	639

Summary

Considering *cyclic dependencies* between landmarks *improves heuristics*.

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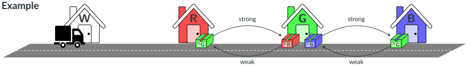
full paper

Example



Landmark: "Visit G to pick up B."
Strong landmark ordering: "Visit R before G because there is no path around it."
Weak landmark ordering: "Visit R after G to deliver R."

Landmark Heuristic

- Every plan must satisfy all landmarks at least once.
- Use ***operator-counting framework*** to estimate cost:

$$\min \sum_{a \in \mathcal{A}} \mathsf{Y}_a \cdot cost(a) \quad \text{s.t.} \qquad (1)$$

$$\mathsf{Y}_a \geq 0 \quad \text{for all actions } a \in \mathcal{A} \qquad (2)$$

$$\mathsf{Y}_L := \sum_{a \in L} \mathsf{Y}_a \geq 1 \quad \text{for all landmarks } L \in \mathcal{L} \qquad (3)$$

- Example: $h^{\mathrm{LM}} = 3$ because R, G, and B must all be visited at least once.

Cyclic Landmark Heuristic

- R must be visited both before and after G.
- ***Cyclic dependency***: one landmark per cycle required twice:

$$\sum_{L \in \mathcal{L}(c)} \mathsf{Y}_L \geq |\mathcal{L}(c)| + 1 \quad \text{for all cycles } c \in \mathcal{C} \qquad (4a)$$

- Example: $h^{\mathrm{cycle}} = 4$ because visiting G twice resolves both cycle constraints.

Strong Cyclic Landmark Heuristics

- R cannot be delivered when first visiting R.
- Only landmarks with ***incoming weak ordering*** can resolve cycles:

$$\sum_{L \in \mathcal{L}^{\mathrm{w}}(c)} \mathsf{Y}_L \geq |\mathcal{L}^{\mathrm{w}}(c)| + 1 \quad \text{for all cycles } c \in \mathcal{C} \qquad (4b)$$

- Example: $h^{\mathrm{strong}} = 5$ because R and G must be visited twice to resolve both cycles.

Finding Cycles in LM Graphs

Johnson's Algorithm

- Finds ***all*** elementary cycles.
- Infeasible in graphs with many cycles.

Oracle Approach

- Few cycles are often sufficient to cover all cycles.
- Use ***implicit hitting set*** algorithm to find a sufficient subset of all cycles iteratively:
  1. Solve LP (initialized using Eq. (1–3)).
  2. Construct weighted graph with $w_{L \to L'} = \mathsf{Y}_{L'} - 1$.
  3. Compute shortest cycles using Floyd-Warshall.
  4. Add constraint (4) of ***most uncovered*** cycle c with minimal $\sum_{L \to L' \in c} w_{L \to L'} < 1$.
  5. Repeat until all cycles are covered.
- Disadvantage: needs multiple LP runs.