

# LP-based Heuristics for Cost-optimal Classical Planning

## 4. Potential Heuristics

Florian Pommerening   Gabriele Röger   Malte Helmert

ICAPS 2015 Tutorial

June 7, 2015

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# Overview

# Potential Heuristics

## Reminder:

### Idea 3: Potential Heuristics

Heuristic design as an optimization problem:

- Define simple numerical **state features**  $f_1, \dots, f_n$ .
- Consider heuristics that are **linear combinations** of features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (**potentials**)  $w_i \in \mathbb{R}$

- Find potentials for which  $h$  is admissible and well-informed.

## Motivation:

- **declarative approach** to heuristic design
- heuristic **very fast to compute** if features are

# Comparison to Previous Parts (1)

What is the same as in operator-counting constraints:

- We again use LPs to compute (admissible) heuristic values  
(spoiler alert!)

## Comparison to Previous Parts (2)

What is different from operator-counting constraints (computationally):

- With potential heuristics, solving one LP defines the **entire heuristic function**, not just the estimate for a single state.
- Hence we only need **one LP solver call**, making LP solving much less time-critical.

# Comparison to Previous Parts (3)

What is different from operator-counting constraints (conceptually):

- **axiomatic approach** for defining heuristics:
  - What should a heuristic look like mathematically?
  - Which properties should it have?
- define a **space of interesting heuristics**
- use **optimization** to pick a good representative

# Literature on Potential Heuristics: The Story So Far

## Papers studying potential heuristics:

- introduced by Pommerening et al. (AAAI 2015)
  - ~~> main focus of this presentation
- studied in more depth by Seipp et al. (ICAPS 2015)
  - ~~> presentation: Thursday, joint ICAPS/SoCS session (last session of conference)
- sufficient to consider **transition normal form** (Pommerening and Helmert, ICAPS 2015)
  - ~~> presentation: Tuesday, first afternoon session

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# Potential Heuristics

# Features

## Definition (feature)

A (state) **feature** for a planning task is a numerical function defined on the states of the task:  $f : S \rightarrow \mathbb{R}$ .

# Potential Heuristics

## Definition (potential heuristic)

A **potential heuristic** for a set of features  $\mathcal{F} = \{f_1, \dots, f_n\}$  is a heuristic function  $h$  defined as a **linear combination** of the features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (**potentials**)  $w_i \in \mathbb{R}$ .

~~> cf. **evaluation functions** for board games like chess

# Atomic Potential Heuristics

Atomic features test if some proposition is true in a state:

## Definition (atomic feature)

Let  $X = x$  be an atomic proposition of a planning task.

The atomic feature  $f_{X=x}$  is defined as:

$$f_{X=x}(s) = \begin{cases} 1 & \text{if variable } X \text{ has value } x \text{ in state } s \\ 0 & \text{otherwise} \end{cases}$$

- We only consider atomic potential heuristics, which are based on the set of all atomic features.
- Example for a task with state variables  $X$  and  $Y$ :

$$h(s) = 3f_{X=a} + \frac{1}{2}f_{X=b} - 2f_{X=c} + \frac{5}{2}f_{Y=d}$$

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# Finding Good Potential Heuristics

# How to Set the Weights?

We want to find **good** atomic potential heuristics:

- admissible
- consistent
- well-informed

How to achieve this? **Linear programming to the rescue!**

# Admissible and Consistent Potential Heuristics

Constraints on potentials **characterize** (= are necessary and sufficient for) admissible and consistent atomic potential heuristics:

Goal-awareness (i.e.,  $h(s) = 0$  for goal states)

$$\sum_{\text{goal facts } f} w_f = 0$$

Consistency

$$\sum_{\substack{f \text{ consumed} \\ \text{by } o}} w_f - \sum_{\substack{f \text{ produced} \\ \text{by } o}} w_f \leq \text{cost}(o) \quad \text{for all operators } o$$

Remarks:

- assumes transition normal form (not a limitation)
- goal-aware and consistent = admissible and consistent

# Well-Informed Potential Heuristics

How to find a **well-informed** potential heuristic?

- ~ encode **quality metric** in the **objective function**  
and use LP solver to find a heuristic maximizing it

Examples:

- maximize **heuristic value of a given state** (e.g., initial state)
- maximize average heuristic value of **all states**  
(including unreachable ones)
- maximize average heuristic value of some **sample states**
- minimize **estimated search effort**

~ see Seipp et al. presentation (joint ICAPS/SoCS session)

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So what does this have to do with what we talked about before?

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Theorem (Pommerening et al., AAAI 2015)

For state  $s$ , let  $h^{\max\text{pot}}(s)$  denote the **maximal** heuristic value of all admissible and consistent atomic potential heuristics in  $s$ .

Then  $h^{\max\text{pot}}(s) = h^{\text{SEQ}}(s) = h^{\text{gOCP}}(s)$ .

- $h^{\text{SEQ}}$ : state equation heuristic a.k.a. flow heuristic
- $h^{\text{gOCP}}$ : optimal general cost partitioning of atomic projections

proof idea: compare dual of  $h^{\text{SEQ}}(s)$  LP  
to potential heuristic constraints optimized for state  $s$

# What Do We Take From This?

- general cost partitioning, operator-counting constraints and potential heuristics: **facets of the same phenomenon**
- study of each reinforces understanding of the others
- potential heuristics: **fast admissible approximations** of  $h^{\text{SEQ}}$
- clear path towards **generalization beyond  $h^{\text{SEQ}}$** :  
use non-atomic features

# The End

- ① ~~Introduction and Overview~~
- ② ~~Cost Partitioning~~
- ③ ~~Operator Counting~~
- ④ Potential Heuristics

Thank you for your attention!