A Beginner's Introduction to Heuristic Search Planning

5. Abstraction Heuristics and Pattern Databases

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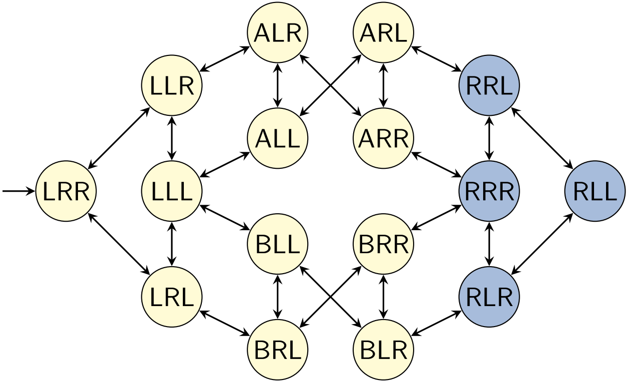
Abstraction Heuristics

Abstraction Heuristics: Idea

- Heuristic estimate = plan cost in simplified state space
- Simplification: Do not distinguish all states

Abstraction: Example

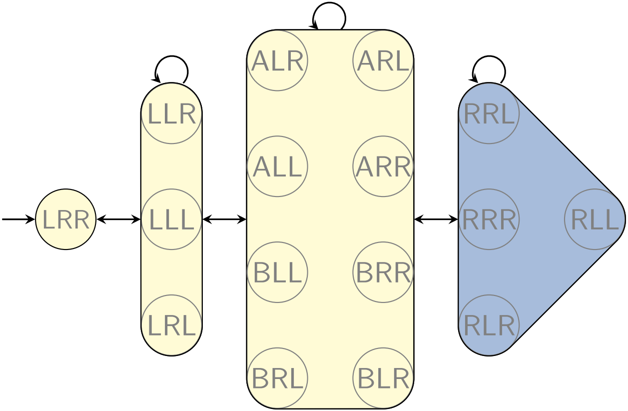
original state space



- state variable *package*: $\{L, R, A, B\}$
- state variable *truck A*: $\{L, R\}$
- state variable *truck B*: $\{L, R\}$

Abstraction: Example

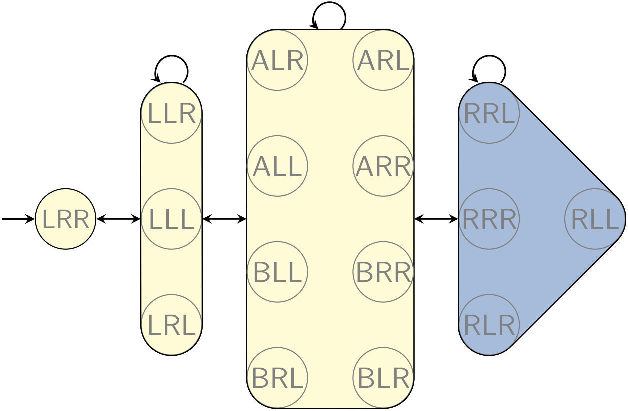
(an) abstract state space



Remark: Most edges correspond to several parallel transitions with different labels.

Abstraction: Example

$$h^{\alpha}(\{p \mapsto \mathsf{L}, t_{\mathsf{A}} \mapsto \mathsf{R}, t_{\mathsf{B}} \mapsto \mathsf{R}\}) = 3$$



Abstraction Heuristics

- Abstract state space is derived from original state space as specified by an abstraction function
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- Abstraction function defines which states should be distinguished
- Preserve all original paths in abstract state space
- Do not relax more than required by abstraction function

Induced Abstraction

Definition (induced abstraction)

Let $\mathcal{S} = \langle S, s_0, S_\star, A, cost, T \rangle$ be a state space and let $\alpha : S \to S'$ be a surjective function.

The abstraction of $\mathcal{S}$ induced by α is the state space $\mathcal{S}^\alpha = \langle S', s'_0, S'_\star, A, cost, T' \rangle$ with:

- $s'_0 = \alpha(s_0)$
- $S'_\star = \{\alpha(s) \mid s \in S_\star\}$
- $T' = \{\langle \alpha(s), a, \alpha(t) \rangle \mid \langle s, a, t \rangle \in T\}$

Abstraction Heuristic

Definition (abstraction heuristic)

For state space $\mathcal{S}$ and abstraction function α, the heuristic estimate $h^\alpha(s)$ for state s is the cost of a cheapest path from $\alpha(s)$ to a goal state in $\mathcal{S}^\alpha$.

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Abstraction heuristics are admissible and consistent.

Classes of Abstractions

- Projections
  $\rightarrow$ Simple abstractions used for pattern databases
  (Culberson & Schaeffer, Computational Intelligence 1998; Edelkamp, ECP 2001; Haslum et al., AAAI 2007)
- Merge & Shrink abstractions
  $\rightarrow$ Can represent arbitrary abstractions
  (Dräger et al., SPIN 2006; Helmert et al., ICAPS 2007; Sievers et al. AAAI, 2014)
- Cartesian abstractions
  $\rightarrow$ Generalization of projections
  (Seipp & Helmert, ICAPS 2013; ICAPS 2014)
- Structural patterns
  $\rightarrow$ Easy to solve despite being large
  (Katz & Domshlak, ICAPS 2008)

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Projections and Pattern Database Heuristics

Pattern database heuristics

Pattern database (PDB) heuristics

- represent some aspects (= state variables) perfectly, but
- entirely ignore all other aspects

Example (15-puzzle)

- Choose subset P of tiles (the pattern).
- In the abstract state space...
  - consider the exact position of all tiles in P,
  - assume that all other tiles and the blank position can be everywhere.

Projections

PDB heuristics are abstraction heuristics where the abstraction function is a projection.

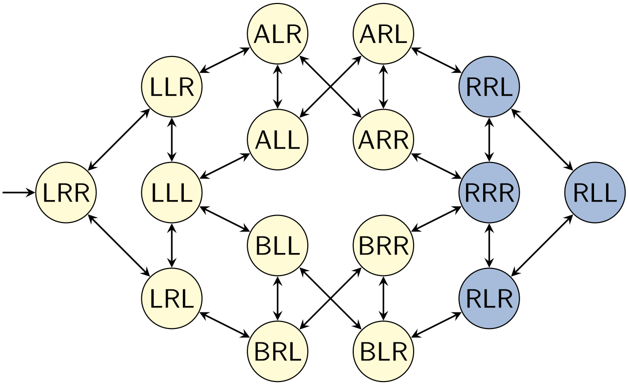
Definition (Projection)

Let Π be a SAS$^+$ planning task with variables V and states S. For $P \subseteq V$ let S'_P be the set of partial variable assignments that are defined exactly on P.

The projection $\pi_P : S \to S'_P$ is defined as $\pi_P(s) := s|_P$ (with $s|_P(v) := s(v)$ for all $v \in P$).

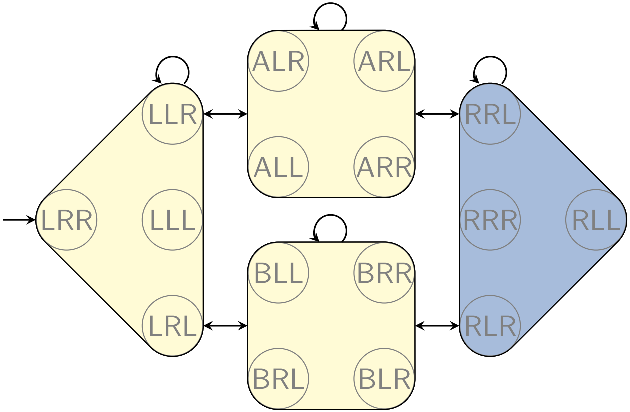
Put differently: π_P maps two concrete states to the same abstract state iff they agree on all variables in P.

Example: concrete state space



Example: Projection (1)

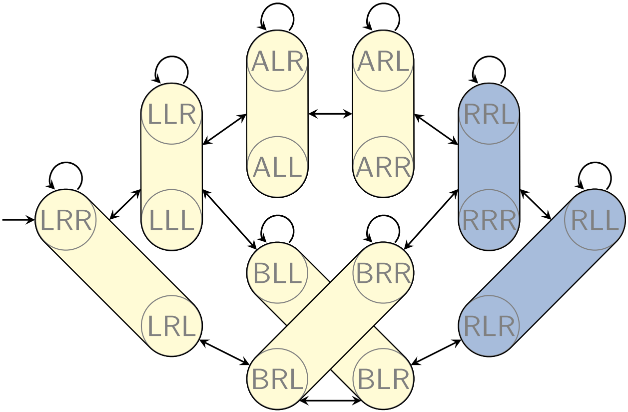
Abstraction induced by $\pi_{\{package\}}$:



$$h^{\{package\}}(\mathsf{LRR}) = 2$$

Example: Projection (2)

Abstraction induced by $\pi_{\{package, truck\ A\}}$:



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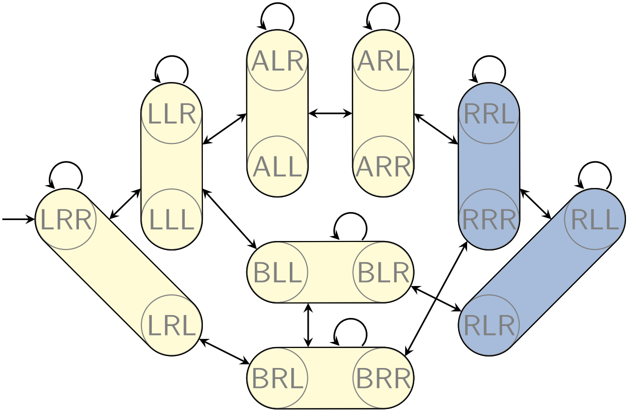
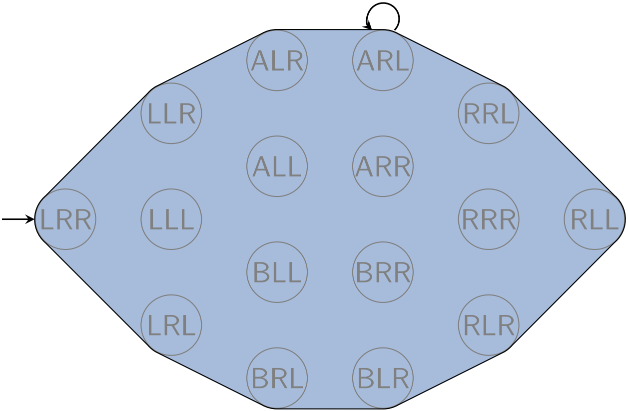


$$h^{\{package, truck\ A\}}(\mathrm{LRR}) = 2$$

Example: Projection (3)

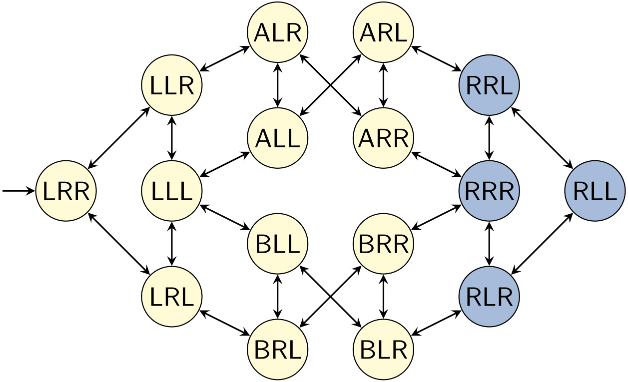
Abstraction induced by $\pi_{\{\}}$:



$h^{\{package, truck\ A\}}(\mathrm{LRR}) = 0$

Example: Projection (4)

Abstraction induced by $\pi_{\{package,truck\ A,truck\ B\}}$:



$$h^{\{package,truck\ A\}}(\mathrm{LRR}) = 4$$

Pattern Collections and iPDB

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- Post-hoc optimization: Solves linear program (LP) to determine admissible weighted sum of individual estimates (Pommerening et al., IJCAI 2013)
- Optimal cost partitioning: Solves very large LP to suitably adjust cost functions of the individual abstractions (Katz & Domshlak, AIJ 2010)

How to Find a Good Pattern Collection for a Task?

Example: iPDB (Haslum et al., AAAI 2007)

- Hill-climbing search in the space of pattern collections
- Optimizing estimates of canonical heuristic
- Initial pattern collection: $\{\{v\} \mid v \text{ is goal variable}\}$
- Search neighborhood: Add one new pattern which
  - extends an already included pattern with one variable...
  - ...that can improve the heuristic estimate according to some relevance criterion, and ...
  - ...the resulting PDBs fit into a prespecified memory limit
- Stop if no successor improves the heuristic estimate

in Fast Downward

```
ipdb(...)
```

Hands on: iPDB in Fast Downward

Hands-On

```
$ cd hands-on
$ ./fd ipc/logistics00/probLOGISTICS-6-0.pddl \
      --search "astar(ipdb())"
```

Summary

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- abstraction heuristics: map state space to smaller space
- projections: abstraction functions that perfectly represent some state variables (given by the pattern) and entirely ignore all others
- pattern database (PDB) heuristic: abstraction heuristic using a projection
- combining PDBs: better heuristic estimates
- pattern selection
  - precomputation time and memory vs. heuristic quality
  - usually task-specific, e.g. with iPDB