

Planning and Optimization

B5. Positive Normal Form and STRIPS

Malte Helmert and Gabriele Röger

Universität Basel

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B5.1 Motivation

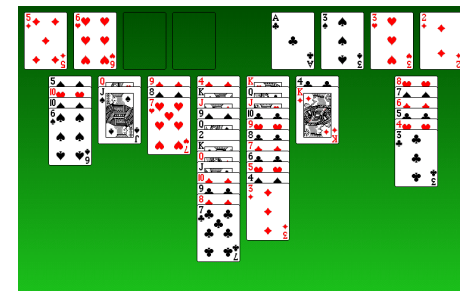
B5.2 Positive Normal Form

B5.3 STRIPS

B5.4 Summary

B5.1 Motivation

Example: Freecell



Example (Good and Bad Effects)

If we move $K♦$ to a free tableau position, the **good effect** is that $4♣$ is now accessible. The **bad effect** is that we lose one free tableau position.

What is a Good or Bad Effect?

Question: Which operator effects are good, and which are bad?

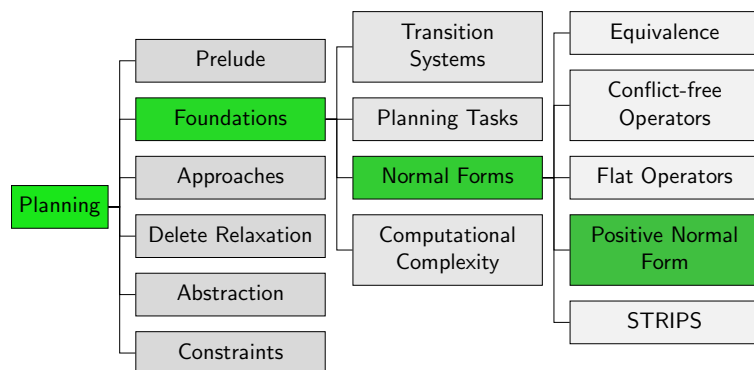
Difficult to answer in general, because it depends on context:

- ▶ Locking our door is **good** if we want to keep burglars out.
- ▶ Locking our door is **bad** if we want to enter.

We now consider a reformulation of propositional planning tasks that makes the distinction between good and bad effects obvious.

B5.2 Positive Normal Form

Content of the Course



Positive Formulas, Operators and Tasks

Definition (Positive Formula)

A logical formula φ is **positive** if no negation symbols appear in φ .

Note: This includes the negation symbols implied by $\rightarrow$ and $\leftrightarrow$.

Definition (Positive Operator)

An operator o is **positive** if $pre(o)$ and all effect conditions in $eff(o)$ are positive.

Definition (Positive Propositional Planning Task)

A propositional planning task $\langle V, I, O, \gamma \rangle$ is **positive** if all operators in O and the goal γ are positive.

Positive Normal Form

Definition (Operator in Positive Normal Form)

An operator o is in **positive normal form** if it is positive and flat.

Definition (Planning Task in Positive Normal Form)

A propositional planning task $\langle V, I, O, \gamma \rangle$ is in **positive normal form** if all operators in O are in positive normal form and the goal γ is positive.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$V = \{home, uni, lecture, bike, bike-locked\}$$

$$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\ uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}\}$$

$$O = \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\ \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\}$$

$$\gamma = lecture \wedge bike$$

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$$\gamma = lecture \wedge bike$$

Identify state variable v occurring negatively in conditions.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$V = \{home, uni, lecture, bike, bike-locked, bike-unlocked\}$$

$$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\ uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\}$$

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$$\gamma = lecture \wedge bike$$

Introduce new variable $\hat{v}$ with complementary initial value.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\} \\
 O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$

Identify effects on variable v .

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\} \\
 O &= \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\
 &\quad \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \wedge \neg bike-unlocked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$

Introduce complementary effects for $\hat{v}$.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
 I &= \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T}, \\
 &\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\} \\
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 &\quad \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\
 &\quad \langle bike \wedge \neg bike-locked, bike-locked \wedge \neg bike-unlocked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$

Identify negative conditions for v .

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$$\begin{aligned}
 V &= \{home, uni, lecture, bike, bike-locked, bike-unlocked\} \\
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 &\quad \langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle, \\
 &\quad \langle bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked \rangle, \\
 &\quad \langle uni, lecture \wedge ((bike \wedge bike-unlocked) \triangleright \neg bike) \rangle\} \\
 \gamma &= lecture \wedge bike
 \end{aligned}$$

Replace by positive condition $\hat{v}$.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$V = \{home, uni, lecture, bike, bike-locked, bike-unlocked\}$
 $I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike-locked \mapsto \mathbf{T},$
 $uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike-unlocked \mapsto \mathbf{F}\}$
 $O = \{\langle home \wedge bike \wedge bike-unlocked, \neg home \wedge uni \rangle,$
 $\langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle,$
 $\langle bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked \rangle,$
 $\langle uni, lecture \wedge ((bike \wedge bike-unlocked) \triangleright \neg bike) \rangle\}$
 $\gamma = lecture \wedge bike$

Positive Normal Form: Existence

Theorem (Positive Normal Form)

For every propositional planning task Π , there is an equivalent propositional planning task Π' in positive normal form. Moreover, Π' can be computed from Π in polynomial time.

Note: Equivalence here means that the transition systems induced by Π and Π' , **restricted to the reachable states**, are isomorphic.

We prove the theorem by describing a suitable algorithm. (However, we do not prove its correctness or complexity.)

Positive Normal Form: Algorithm

Transformation of $\langle V, I, O, \gamma \rangle$ to Positive Normal Form

Replace all operators with equivalent conflict-free operators.

Convert all conditions to negation normal form (NNF).

while any condition contains a negative literal $\neg v$:

Let v be a variable which occurs negatively in a condition.

$V := V \cup \{\hat{v}\}$ for some new propositional state variable $\hat{v}$

$$I(\hat{v}) := \begin{cases} \mathbf{F} & \text{if } I(v) = \mathbf{T} \\ \mathbf{T} & \text{if } I(v) = \mathbf{F} \end{cases}$$

Replace the effect v by $(v \wedge \neg \hat{v})$ in all operators $o \in O$.

Replace the effect $\neg v$ by $(\neg v \wedge \hat{v})$ in all operators $o \in O$.

Replace $\neg v$ by $\hat{v}$ in all conditions.

Convert all operators $o \in O$ to flat operators.

Here, **all conditions** refers to all operator preconditions, operator effect conditions and the goal.

Why Positive Normal Form is Interesting

In the **absence of conditional effects**, positive normal form allows us to distinguish good and bad effects easily:

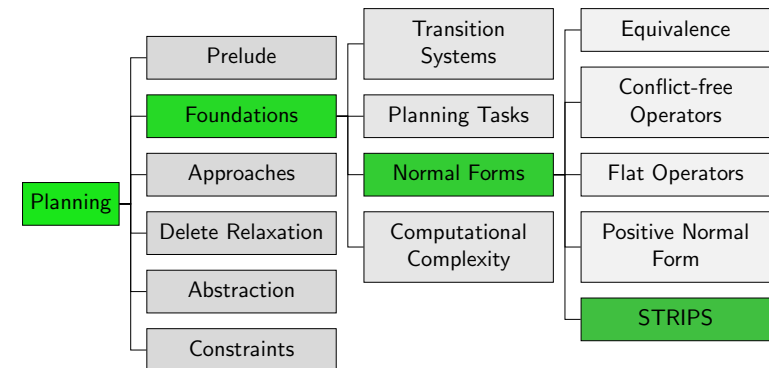
- Effects that make state variables true (**add effects**) are good.
- Effects that make state variables false (**delete effects**) are bad.

This is particularly useful for planning algorithms based on **delete relaxation**, which we will study in Part D.

(Why restriction “in the absence of conditional effects”?)

B5.3 STRIPS

Content of the Course



STRIPS Operators and Planning Tasks

Definition (STRIPS Operator)

An operator o of a prop. planning task is a **STRIPS operator** if

- ▶ $pre(o)$ is a conjunction of state variables, and
- ▶ $eff(o)$ is a conflict-free conjunction of atomic effects.

Definition (STRIPS Planning Task)

A propositional planning task $\langle V, I, O, \gamma \rangle$ is a **STRIPS planning task** if all operators $o \in O$ are STRIPS operators and γ is a conjunction of state variables.

Note: STRIPS operators are conflict-free and flat.
STRIPS is a special case of positive normal form.

STRIPS Operators: Remarks

- ▶ Every STRIPS operator is of the form

$$\langle v_1 \wedge \dots \wedge v_n, \ell_1 \wedge \dots \wedge \ell_m \rangle$$

where v_i are state variables and ℓ_j are atomic effects.

- ▶ Often, STRIPS operators o are described via three **sets of state variables**:
 - ▶ the **preconditions** (state variables occurring in $pre(o)$)
 - ▶ the **add effects** (state variables occurring positively in $eff(o)$)
 - ▶ the **delete effects** (state variables occurring negatively in $eff(o)$)
- ▶ Definitions of STRIPS in the literature often do **not** require conflict-freeness. But it is easy to achieve and makes many things simpler.
- ▶ There exists a variant called **STRIPS with negation** where negative literals are also allowed in conditions.

Why STRIPS is Interesting

- ▶ STRIPS is **particularly simple**, yet expressive enough to capture general planning tasks.
- ▶ In particular, STRIPS planning is **no easier** than planning in general (as we will see in Chapters B6–B7).
- ▶ Many algorithms in the planning literature are **only presented for STRIPS planning tasks** (generalization is often, but not always, obvious).

STRIPS

STanford Research Institute Problem Solver
(Fikes & Nilsson, 1971)

Transformation to STRIPS

- ▶ Not every operator is equivalent to a STRIPS operator.
- ▶ However, each operator can be transformed into a **set** of STRIPS operators whose “combination” is equivalent to the original operator. (How?)
- ▶ However, this transformation may exponentially increase the number of operators. There are planning tasks for which such a blow-up is unavoidable.
- ▶ There are polynomial transformations of propositional planning tasks to STRIPS, but these do not lead to isomorphic transition systems (auxiliary states are needed). (They are, however, equivalent in a weaker sense.)

B5.4 Summary

Summary

- ▶ A **positive** task helps distinguish good and bad effects. The notion of positive tasks only exists for **propositional** tasks.
- ▶ A positive task with flat operators is in **positive normal form**.
- ▶ **STRIPS** is even more restrictive than positive normal form, forbidding complex preconditions and conditional effects.
- ▶ Both forms are expressive enough to capture general propositional planning tasks.
- ▶ Transformation to positive normal form is possible with polynomial size increase.
- ▶ Isomorphic transformations of propositional planning tasks to STRIPS can increase the number of operators exponentially; non-isomorphic polynomial transformations exist.