

Discrete Structures

B1. Tuples & Cartesian Product

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B1.1 Tuples and the Cartesian Product

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Motivation

- ▶ A **set** is an **unordered collection** of **distinct** objects.
- ▶ We often need a more structured way of representation.
 - ▶ A person is associated with a name, address, phone number.
 - ▶ A set of persons makes sense in many contexts.
 - ▶ Representing the associated data as a set rather not.
- ▶ We could for example want to
 - ▶ directly access the name of a person, or
 - ▶ have a separate billing and delivery address for some order, but in general, these can be the same.
- ▶ **Tuples** are mathematical building blocks that support this.

Sets vs. Tuples

- ▶ A **set** is an **unordered collection** of **distinct** objects.
- ▶ A **tuple** is an **ordered sequence** of objects.

Tuples

- ▶ **k -tuple**: ordered sequence of k objects ($k \in \mathbb{N}_0$)
- ▶ written $(o_1, \dots, o_k)$ or $\langle o_1, \dots, o_k \rangle$
- ▶ unlike sets, **order matters** ($\langle 1, 2 \rangle \neq \langle 2, 1 \rangle$)
- ▶ objects may occur multiple times in a tuple
- ▶ objects contained in tuples are called **components**
- ▶ terminology:
 - ▶ $k = 2$: (ordered) pair
 - ▶ $k = 3$: triple
 - ▶ more rarely: quadruple, quintuple, sextuple, septuple, ...
- ▶ if k is clear from context (or does not matter), often just called **tuple**

German: k -Tupel, Komponente, (geordnetes) Paar, Tripel, Quadrupel

Equality of Tuples

Definition (Equality of Tuples)

Two n -tuples $t = \langle o_1, \dots, o_n \rangle$ and $t' = \langle o'_1, \dots, o'_n \rangle$ are **equal** ($t = t'$) if for $i \in \{1, \dots, n\}$ it holds that $o_i = o'_i$.

Cartesian Product

Definition (Cartesian Product and Cartesian Power)

Let $S_1, \dots, S_n$ be sets. The **Cartesian product** $S_1 \times \dots \times S_n$ is the following set of n -tuples:

$$S_1 \times \dots \times S_n = \{ \langle x_1, \dots, x_n \rangle \mid x_1 \in S_1, x_2 \in S_2, \dots, x_n \in S_n \}.$$

The k -ary **Cartesian power** of a set S (with $k \in \mathbb{N}_1$) is the set $S^k = \{ \langle o_1, \dots, o_k \rangle \mid o_i \in S \text{ for all } i \in \{1, \dots, k\} \} = \underbrace{S \times \dots \times S}_{k \text{ times}}$.

René Descartes: French mathematician and philosopher (1596–1650)

Example: $A = \{a, b\}$, $B = \{1, 2, 3\}$

$$A \times B = \{ \langle a, 1 \rangle, \langle a, 2 \rangle, \langle a, 3 \rangle, \langle b, 1 \rangle, \langle b, 2 \rangle, \langle b, 3 \rangle \}$$

$$A^2 = \{ \langle a, a \rangle, \langle a, b \rangle, \langle b, a \rangle, \langle b, b \rangle \}$$

German: Kartesisches Produkt

(Non-)properties of the Cartesian Product

The Cartesian product is

- ▶ **not commutative**, in most cases $A \times B \neq B \times A$.
- ▶ **not associative**, in most cases $(A \times B) \times C \neq A \times (B \times C)$

Why? Exceptions?

Summary

- ▶ A **k -tuple** is an **ordered sequence** of k objects, called the **components** of the tuple.
- ▶ 2-tuples are also called **pairs** and 3-tuples **triples**.
- ▶ The **Cartesian Product** $S_1 \times \cdots \times S_n$ of set $S_1, \dots, S_n$ is the set of all tuples $\langle o_1, \dots, o_n \rangle$, where for all $i \in \{1, \dots, n\}$ component o_i is an element of S_i .