Planning and Optimization

F12. Potential Heuristics

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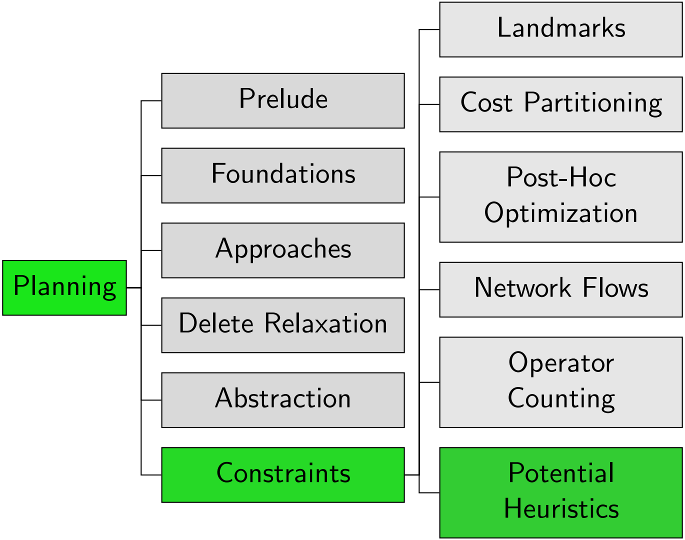
December 15, 2025

Content of the Course



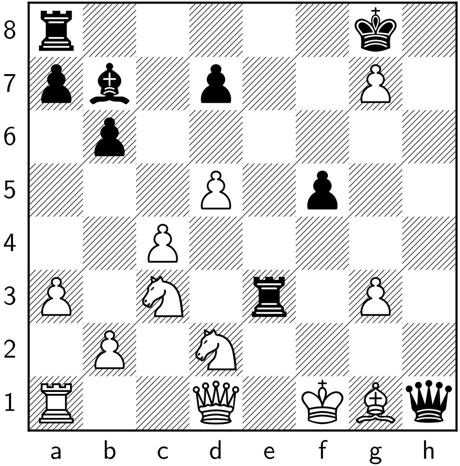
F12.1 Introduction

Reminder: Transition Normal Form

In this chapter, we consider SAS^+ tasks in transition normal form.

- A TNF operator mentions the same variables in the precondition and in the effect.
- A TNF goal specifies a value for every variable.

Material Value of a Chess Position



Material value for white:

$$
\begin{array}{lll}
+ & 1 \cdot 6 & \text{(white pawns)} \\
- & 1 \cdot 4 & \text{(black pawns)} \\
+ & 3 \cdot 2 & \text{(white knights)} \\
- & 3 \cdot 0 & \text{(black knights)} \\
+ & 3 \cdot 1 & \text{(white bishops)} \\
- & 3 \cdot 1 & \text{(black bishops)} \\
+ & 5 \cdot 1 & \text{(white rooks)} \\
- & 5 \cdot 2 & \text{(black rooks)} \\
+ & 9 \cdot 1 & \text{(white queen)} \\
- & 9 \cdot 1 & \text{(black queen)} \\
= & 3 &
\end{array}
$$

Idea

- Define simple numerical state features $f_1, \dots, f_n$.
- Consider heuristics that are linear combinations of features:

  $$h(s) = w_1 f_1(s) + \cdots + w_n f_n(s)$$

  with weights (potentials) $w_i \in \mathbb{R}$
- heuristic very fast to compute if feature values are

F12.2 Potential Heuristics

Definition

Definition (Feature)

A (state) feature for a planning task is a numerical function defined on the states of the task: $f : S \to \mathbb{R}$.

Definition (Potential Heuristic)

A potential heuristic for a set of features $\mathcal{F} = \{f_1, \dots, f_n\}$ is a heuristic function h defined as a linear combination of the features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (potentials) $w_i \in \mathbb{R}$.

Many possibilities $\Rightarrow$ need some restrictions

Features for SAS$^+$ Planning Tasks

Which features are good for planning?

Atomic features test if some atom is true in a state:

Definition (Atomic Feature)

Let $v = d$ be an atom of a FDR planning task.

The atomic feature $f_{v=d}$ is defined as:

$$f_{v=d}(s) = [(v = d) \in s] = \begin{cases} 1 & \text{if variable } v \text{ has value } d \text{ in state } s \\ 0 & \text{otherwise} \end{cases}$$

Offer good tradeoff between computation time and guidance

Example: Atomic Features

Example

Consider a planning task Π with state variables v_1 and v_2 and $\mathrm{dom}(v_1) = \mathrm{dom}(v_2) = \{d_1, d_2, d_3\}$. The set

$$\mathcal{F} = \{f_{v_i=d_j} \mid i \in \{1,2\}, j \in \{1,2,3\}\}$$

is the set of atomic features of Π and the function

$$h(s) = 3f_{v_1=d_1} + 0.5f_{v_1=d_2} - 2f_{v_1=d_3} + 2.5f_{v_2=d_1}$$

is a potential heuristic for $\mathcal{F}$.
The heuristic estimate for a state $s = \{v_1 \mapsto d_2, v_2 \mapsto d_1\}$ is

$$h(s) = 3 \cdot 0 + 0.5 \cdot 1 - 2 \cdot 0 + 2.5 \cdot 1 = 3.$$

Potentials for Optimal Planning

Which potentials are good for optimal planning
and how can we compute them?

- We seek potentials for which h is admissible and well-informed
$\Rightarrow$ declarative approach to heuristic design
- We derive potentials for atomic features by solving an optimization problem

How to achieve this? Linear programming to the rescue!

Admissible and Consistent Potential Heuristics

We achieve admissibility through goal-awareness and consistency

Goal-awareness

$$\sum_{a \in \gamma} w_a = 0$$

Consistency

$$\sum_{a \in s} w_a - \sum_{a \in s'} w_a \leq cost(o) \quad \text{for all transitions } s \xrightarrow{o} s'$$

One constraint transition per transition.

Can we do this more compactly?

Admissible and Consistent Potential Heuristics

Consistency for a transition $s \xrightarrow{o} s'$

$$
\begin{aligned}
cost(o) &\geq \sum_{a \in s} w_a - \sum_{a \in s'} w_a \\
&= \sum_{a} w_a [a \in s] - \sum_{a} w_a [a \in s'] \\
&= \sum_{a} w_a ([a \in s] - [a \in s']) \\
&= \sum_{a} w_a [a \in s \text{ but } a \notin s'] - \sum_{a} w_a [a \notin s \text{ but } a \in s'] \\
&= \sum_{\substack{a \text{ consumed} \\ \text{by } o}} w_a \; - \sum_{\substack{a \text{ produced} \\ \text{by } o}} w_a
\end{aligned}
$$

Admissible and Consistent Potential Heuristics

Goal-awareness and Consistency independent of s

Goal-awareness

$$\sum_{a \in \gamma} w_a = 0$$

Consistency

$$\sum_{\substack{a \text{ consumed} \\ \text{by } o}} w_a - \sum_{\substack{a \text{ produced} \\ \text{by } o}} w_a \leq cost(o) \quad \text{for all operators } o$$

Potential Heuristics

- All atomic potential heuristics that satisfy these constraints are admissible and consistent
- Furthermore, all admissible and consistent atomic potential heuristics satisfy these constraints

Constraints are a compact characterization of all admissible and consistent atomic potential heuristics.

LP can be used to find the best admissible and consistent potential heuristics by encoding a quality metric in the objective function

Well-Informed Potential Heuristics

What do we mean by the best potential heuristic?
Different possibilities, e.g., the potential heuristic that

- maximizes heuristic value of a given state s (e.g., initial state)
- maximizes average heuristic value of all states (including unreachable ones)
- maximizes average heuristic value of some sample states
- minimizes estimated search effort

Potential and Flow Heuristic

Theorem

For state s, let $h^{\text{maxpot}}(s)$ denote the maximal heuristic value of all admissible and consistent atomic potential heuristics in s.

Then $h^{\text{maxpot}}(s) = h^{\mathit{flow}}(s)$.

Proof idea: compare dual of $h^{\text{flow}}(s)$ LP to potential heuristic constraints optimized for state s.

If we optimize the potentials for a given state then for this state it equals the flow heuristic.

F12.3 Summary

Summary

- Potential heuristics are computed as a weighted sum of state features
- Admissibility and consistency can be encoded compactly in constraints
- With linear programming, we can efficiently compute the best potential heuristic wrt some objective
- Potential heuristics can be used as fast admissible approximations of h^{flow}.