Planning and Optimization

F8. Optimal and General Cost-Partitioning

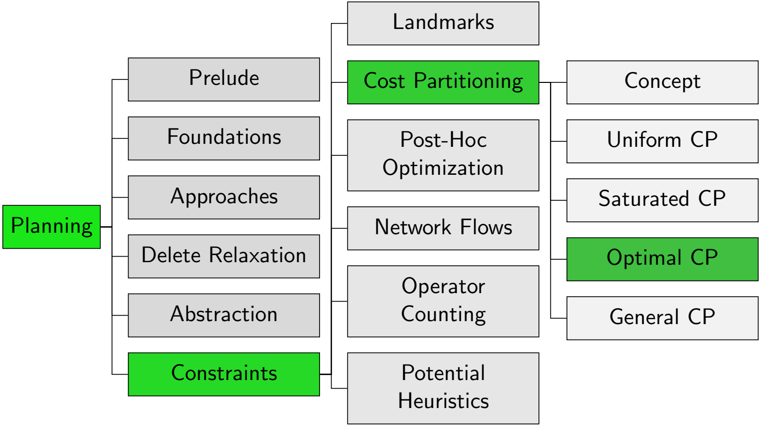
Malte Helmert and Gabriele Röger

Universität Basel

December 8, 2025

Optimal Cost Partitioning

Content of the Course



Optimal Cost Partitioning: General Approach

- Can we find a better cost partitioning than with the uniform or saturation strategy? Even an optimal one?
- Idea: exploit linear programming
  - Use variables for cost of each operator in each task copy
  - Express heuristic values with linear constraints
  - Maximize sum of heuristic values subject to these constraints

Optimal Cost Partitioning: General Approach

- Can we find a better cost partitioning than with the uniform or saturation strategy? Even an optimal one?
- Idea: exploit linear programming
  - Use variables for cost of each operator in each task copy
  - Express heuristic values with linear constraints
  - Maximize sum of heuristic values subject to these constraints

LPs known for

- abstraction heuristics (not covered in this course)
- disjunctive action landmarks (now)

Optimal Cost Partitioning for Landmarks: Basic Version

- Use an LP that covers the heuristic computation and the cost partitioning.
- LP variable $\mathsf{C}_{L,o}$ for cost of operator o in induced task for disjunctive action landmark L (cost partitioning)
- LP variable Cost_L for cost of disjunctive action landmark L in induced task (value of individual heuristics)

Optimal Cost Partitioning for Landmarks: Basic LP

Variables

Non-negative variable Cost_L for each disj. action landmark $L \in \mathcal{L}$
Non-negative variable $C_{L,o}$ for each $L \in \mathcal{L}$ and operator o

Objective

Maximize $\sum_{L \in \mathcal{L}} \mathrm{Cost}_L$

Subject to

$$\sum_{L \in \mathcal{L}} C_{L,o} \leq cost(o) \quad \text{for all operators } o$$

$$\mathrm{Cost}_L \leq C_{L,o} \quad \text{for all } L \in \mathcal{L} \text{ and } o \in L$$

Optimal Cost Partitioning for Landmarks: Improved

- Observation: Explicit variables for cost partitioning not necessary.
- Use implicitly $cost_L(o) = \text{Cost}_L$ for all $o \in L$ and 0 otherwise.

Optimal Cost Partitioning for Landmarks: Improved LP

Variables

Non-negative variable Cost_L for each disj. action landmark $L \in \mathcal{L}$

Objective

Maximize $\sum_{L \in \mathcal{L}} \text{Cost}_L$

Subject to

$$\sum_{L \in \mathcal{L} : o \in L} \text{Cost}_L \leq cost(o) \quad \text{for all operators } o$$

Example (1)

Example

Let Π be a planning task with operators $o_1, \ldots, o_4$ and $cost(o_1) = 3, cost(o_2) = 4, cost(o_3) = 5$ and $cost(o_4) = 0$.
Let the following be disjunctive action landmarks for Π:

$$\begin{aligned}\mathcal{L}_1 &= \{o_4\} \\ \mathcal{L}_2 &= \{o_1, o_2\} \\ \mathcal{L}_3 &= \{o_1, o_3\} \\ \mathcal{L}_4 &= \{o_2, o_3\}\end{aligned}$$

Example (2)

Example

Maximize $\mathrm{Cost}_{\mathcal{L}_1} + \mathrm{Cost}_{\mathcal{L}_2} + \mathrm{Cost}_{\mathcal{L}_3} + \mathrm{Cost}_{\mathcal{L}_4}$ subject to

$$
\begin{aligned}
&[o_1] \quad & \mathrm{Cost}_{\mathcal{L}_2} + \mathrm{Cost}_{\mathcal{L}_3} &\leq 3 \\
&[o_2] \quad & \mathrm{Cost}_{\mathcal{L}_2} + \mathrm{Cost}_{\mathcal{L}_4} &\leq 4 \\
&[o_3] \quad & \mathrm{Cost}_{\mathcal{L}_3} + \mathrm{Cost}_{\mathcal{L}_4} &\leq 5 \\
&[o_4] \quad & \mathrm{Cost}_{\mathcal{L}_1} &\leq 0 \\
& & \mathrm{Cost}_{\mathcal{L}_i} &\geq 0 \quad \text{for } i \in \{1, 2, 3, 4\}
\end{aligned}
$$

Optimal Cost Partitioning for Landmarks (Dual view)

Variables

Non-negative variable Applied_o for each operator o

Objective

Minimize $\sum_o \text{Applied}_o \cdot cost(o)$

Subject to

$$\sum_{o \in L} \text{Applied}_o \geq 1 \text{ for all landmarks } L$$

Minimize “plan cost” with all landmarks satisfied.

Example: Dual View

Example (Optimal Cost Partitioning: Dual View)

Minimize $3\text{Applied}_{o_1} + 4\text{Applied}_{o_2} + 5\text{Applied}_{o_3}$ subject to

$$
\begin{aligned}
\text{Applied}_{o_4} &\geq 1 \\
\text{Applied}_{o_1} + \text{Applied}_{o_2} &\geq 1 \\
\text{Applied}_{o_1} + \text{Applied}_{o_3} &\geq 1 \\
\text{Applied}_{o_2} + \text{Applied}_{o_3} &\geq 1 \\
\text{Applied}_{o_i} &\geq 0 \quad \text{for } i \in \{1, 2, 3, 4\}
\end{aligned}
$$

Example: Dual View

Example (Optimal Cost Partitioning: Dual View)

Minimize $3\text{Applied}_{o_1} + 4\text{Applied}_{o_2} + 5\text{Applied}_{o_3}$ subject to

$$\begin{aligned}
\text{Applied}_{o_4} &\geq 1 \\
\text{Applied}_{o_1} + \text{Applied}_{o_2} &\geq 1 \\
\text{Applied}_{o_1} + \text{Applied}_{o_3} &\geq 1 \\
\text{Applied}_{o_2} + \text{Applied}_{o_3} &\geq 1 \\
\text{Applied}_{o_i} &\geq 0 \quad \text{for } i \in \{1, 2, 3, 4\}
\end{aligned}$$

This is equal to the LP relaxation of the MHS heuristic

Reminder: LP Relaxation of MHS heuristic

Example (Minimum Hitting Set)

minimize $3X_{o_1} + 4X_{o_2} + 5X_{o_3}$ subject to

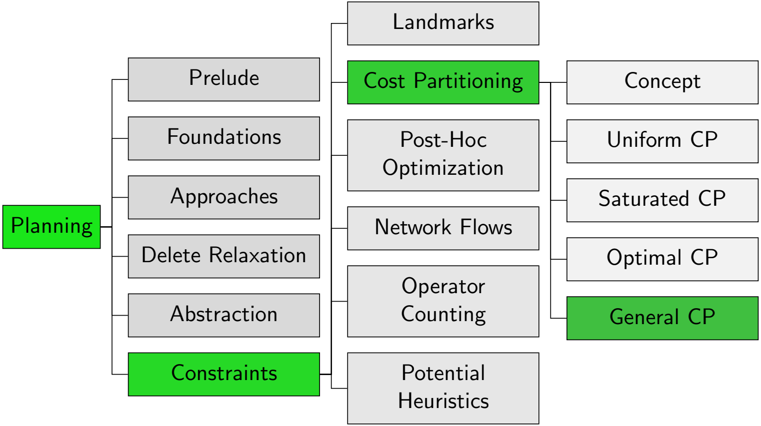
$$X_{o_4} \geq 1$$
$$X_{o_1} + X_{o_2} \geq 1$$
$$X_{o_1} + X_{o_3} \geq 1$$
$$X_{o_2} + X_{o_3} \geq 1$$

$$X_{o_1} \geq 0, \quad X_{o_2} \geq 0, \quad X_{o_3} \geq 0, \quad X_{o_4} \geq 0$$

- ⇝ optimal solution of LP relaxation: $X_{o_4} = 1$ and $X_{o_1} = X_{o_2} = X_{o_3} = 0.5$ with objective value 6
- ⇝ LP relaxation of MHS heuristic is admissible and can be computed polynomial time

General Cost Partitioning

Content of the Course



General Cost Partitioning

Cost functions are usually non-negative.

- We tacitly also required this for task copies
- Makes intuitively sense: original costs are non-negative
- But: not necessary for cost-partitioning!

General Cost Partitioning

Definition (General Cost Partitioning)

Let Π be a planning task with operators O.

A general cost partitioning for Π is a tuple $\langle cost_1, \dots, cost_n \rangle$, where

- $cost_i : O \to \mathbb{R}$ for $1 \leq i \leq n$ and
- $\sum_{i=1}^{n} cost_i(o) \leq cost(o)$ for all $o \in O$.

General Cost Partitioning: Admissibility

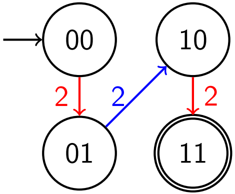
Theorem (Sum of Solution Costs is Admissible)

Let Π be a planning task, $\langle cost_1, \dots, cost_n \rangle$ be a general cost partitioning and $\langle \Pi_1, \dots, \Pi_n \rangle$ be the tuple of induced tasks.

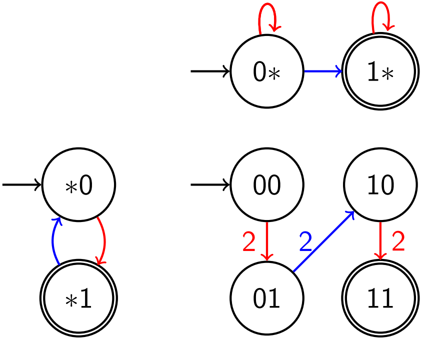
Then the sum of the solution costs of the induced tasks is an admissible heuristic for Π, i.e., $\sum_{i=1}^{n} h^*_{\Pi_i} \leq h^*_{\Pi}$.

(Proof omitted.)

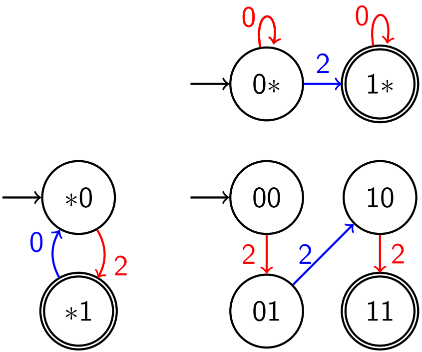
General Cost Partitioning: Example



General Cost Partitioning: Example

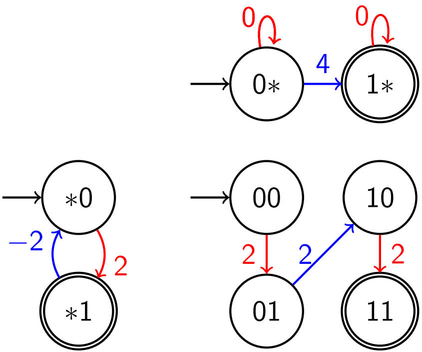


General Cost Partitioning: Example



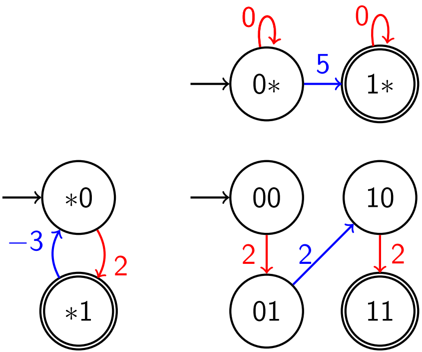
Heuristic value: $2 + 2 = 4$

General Cost Partitioning: Example



Heuristic value: $4 + 2 = 6$

General Cost Partitioning: Example



Heuristic value: $-\infty + 5 = -\infty$

Summary

Summary

- For abstraction heuristics and disjunctive action landmarks, we know how to determine an optimal cost partitioning, using linear programming.
- Although solving a linear program is possible in polynomial time, the better heuristic guidance often does not outweigh the overhead (in particular for abstraction heuristics).
- In constrast to standard (non-negative) cost partitioning, general cost partitioning allows negative operators costs.
- General cost partitioning has the same relevant properties as non-negative cost partitioning but is more powerful.