

# Planning and Optimization

## F8. Optimal and General Cost-Partitioning

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December 8, 2025

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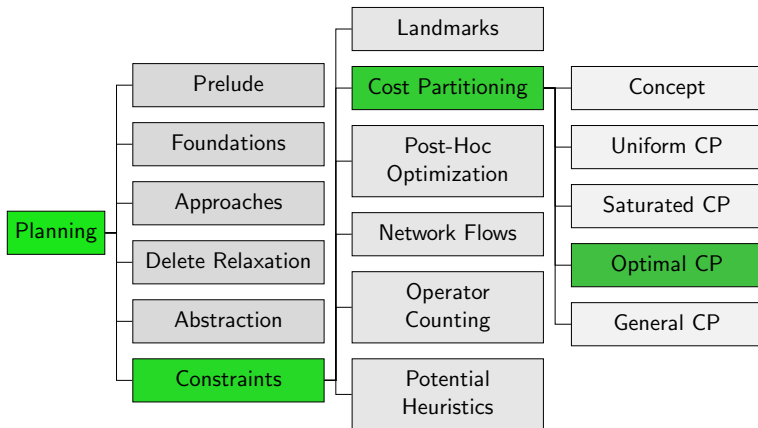
## F8.1 Optimal Cost Partitioning

## F8.2 General Cost Partitioning

## F8.3 Summary

# F8.1 Optimal Cost Partitioning

# Content of the Course



# Optimal Cost Partitioning: General Approach

- ▶ Can we find a better cost partitioning than with the uniform or saturation strategy? Even an **optimal** one?
- ▶ Idea: exploit linear programming
  - ▶ Use variables for cost of each operator in each task copy
  - ▶ Express heuristic values with linear constraints
  - ▶ Maximize sum of heuristic values subject to these constraints

LPs known for

- ▶ abstraction heuristics (not covered in this course)
- ▶ disjunctive action landmarks (now)

# Optimal Cost Partitioning for Landmarks: Basic Version

- ▶ Use an LP that covers the heuristic computation and the cost partitioning.
- ▶ LP variable  $C_{L,o}$  for cost of operator  $o$  in induced task for disjunctive action landmark  $L$  (cost partitioning)
- ▶ LP variable  $Cost_L$  for cost of disjunctive action landmark  $L$  in induced task (value of individual heuristics)

# Optimal Cost Partitioning for Landmarks: Basic LP

## Variables

Non-negative variable  $\text{Cost}_L$  for each disj. action landmark  $L \in \mathcal{L}$

Non-negative variable  $C_{L,o}$  for each  $L \in \mathcal{L}$  and operator  $o$

## Objective

Maximize  $\sum_{L \in \mathcal{L}} \text{Cost}_L$

## Subject to

$$\sum_{L \in \mathcal{L}} C_{L,o} \leq \text{cost}(o) \quad \text{for all operators } o$$

$$\text{Cost}_L \leq C_{L,o} \quad \text{for all } L \in \mathcal{L} \text{ and } o \in L$$

# Optimal Cost Partitioning for Landmarks: Improved

- ▶ **Observation:** Explicit variables for cost partitioning not necessary.
- ▶ Use implicitly  $cost_L(o) = \text{Cost}_L$  for all  $o \in L$  and 0 otherwise.

# Optimal Cost Partitioning for Landmarks: Improved LP

## Variables

Non-negative variable  $\text{Cost}_L$  for each disj. action landmark  $L \in \mathcal{L}$

## Objective

Maximize  $\sum_{L \in \mathcal{L}} \text{Cost}_L$

## Subject to

$$\sum_{L \in \mathcal{L}: o \in L} \text{Cost}_L \leq \text{cost}(o) \quad \text{for all operators } o$$

## Example (1)

### Example

Let  $\Pi$  be a planning task with operators  $o_1, \dots, o_4$  and  $\text{cost}(o_1) = 3$ ,  $\text{cost}(o_2) = 4$ ,  $\text{cost}(o_3) = 5$  and  $\text{cost}(o_4) = 0$ . Let the following be disjunctive action landmarks for  $\Pi$ :

$$\mathcal{L}_1 = \{o_4\}$$

$$\mathcal{L}_2 = \{o_1, o_2\}$$

$$\mathcal{L}_3 = \{o_1, o_3\}$$

$$\mathcal{L}_4 = \{o_2, o_3\}$$

## Example (2)

### Example

Maximize  $\text{Cost}_{\mathcal{L}_1} + \text{Cost}_{\mathcal{L}_2} + \text{Cost}_{\mathcal{L}_3} + \text{Cost}_{\mathcal{L}_4}$  subject to

$$[o_1] \quad \text{Cost}_{\mathcal{L}_2} + \text{Cost}_{\mathcal{L}_3} \leq 3$$

$$[o_2] \quad \text{Cost}_{\mathcal{L}_2} + \text{Cost}_{\mathcal{L}_4} \leq 4$$

$$[o_3] \quad \text{Cost}_{\mathcal{L}_3} + \text{Cost}_{\mathcal{L}_4} \leq 5$$

$$[o_4] \quad \text{Cost}_{\mathcal{L}_1} \leq 0$$

$$\text{Cost}_{\mathcal{L}_i} \geq 0 \quad \text{for } i \in \{1, 2, 3, 4\}$$

# Optimal Cost Partitioning for Landmarks (Dual view)

## Variables

Non-negative variable  $\text{Applied}_o$  for each operator  $o$

## Objective

Minimize  $\sum_o \text{Applied}_o \cdot \text{cost}(o)$

## Subject to

$$\sum_{o \in L} \text{Applied}_o \geq 1 \text{ for all landmarks } L$$

Minimize “plan cost” with all landmarks satisfied.

## Example: Dual View

### Example (Optimal Cost Partitioning: Dual View)

Minimize  $3\text{Applied}_{o_1} + 4\text{Applied}_{o_2} + 5\text{Applied}_{o_3}$  subject to

$$\text{Applied}_{o_4} \geq 1$$

$$\text{Applied}_{o_1} + \text{Applied}_{o_2} \geq 1$$

$$\text{Applied}_{o_1} + \text{Applied}_{o_3} \geq 1$$

$$\text{Applied}_{o_2} + \text{Applied}_{o_3} \geq 1$$

$$\text{Applied}_{o_i} \geq 0 \quad \text{for } i \in \{1, 2, 3, 4\}$$

This is equal to the LP relaxation of the MHS heuristic

## Reminder: LP Relaxation of MHS heuristic

### Example (Minimum Hitting Set)

minimize  $3X_{o_1} + 4X_{o_2} + 5X_{o_3}$  subject to

$$X_{o_4} \geq 1$$

$$X_{o_1} + X_{o_2} \geq 1$$

$$X_{o_1} + X_{o_3} \geq 1$$

$$X_{o_2} + X_{o_3} \geq 1$$

$$X_{o_1} \geq 0, \quad X_{o_2} \geq 0, \quad X_{o_3} \geq 0, \quad X_{o_4} \geq 0$$

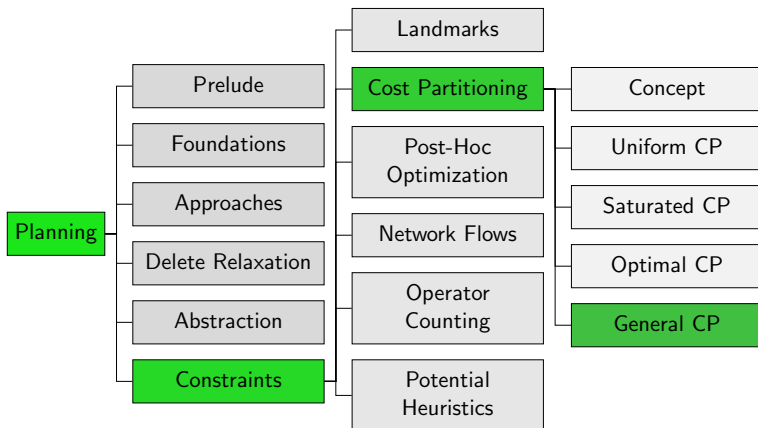
⇒ optimal solution of LP relaxation:

$X_{o_4} = 1$  and  $X_{o_1} = X_{o_2} = X_{o_3} = 0.5$  with objective value 6

⇒ LP relaxation of MHS heuristic is **admissible**  
and can be computed **polynomial time**

## F8.2 General Cost Partitioning

# Content of the Course



# General Cost Partitioning

Cost functions are **usually non-negative**.

- ▶ We tacitly also required this for task copies
- ▶ Makes intuitively sense: original costs are non-negative
- ▶ But: not necessary for cost-partitioning!

# General Cost Partitioning

## Definition (General Cost Partitioning)

Let  $\Pi$  be a planning task with operators  $O$ .

A **general cost partitioning** for  $\Pi$  is a tuple  $\langle cost_1, \dots, cost_n \rangle$ , where

- ▶  $cost_i : O \rightarrow \mathbb{R}$  for  $1 \leq i \leq n$  and
- ▶  $\sum_{i=1}^n cost_i(o) \leq cost(o)$  for all  $o \in O$ .

# General Cost Partitioning: Admissibility

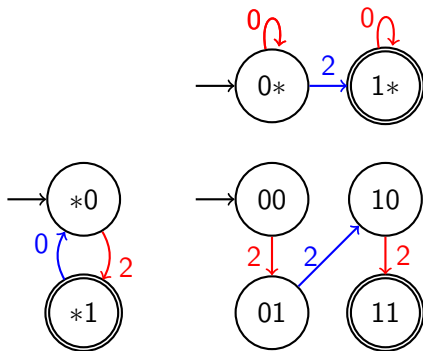
## Theorem (Sum of Solution Costs is Admissible)

*Let  $\Pi$  be a planning task,  $\langle cost_1, \dots, cost_n \rangle$  be a **general** cost partitioning and  $\langle \Pi_1, \dots, \Pi_n \rangle$  be the tuple of induced tasks.*

*Then the sum of the solution costs of the induced tasks is an **admissible heuristic** for  $\Pi$ , i.e.,  $\sum_{i=1}^n h_{\Pi_i}^* \leq h_{\Pi}^*$ .*

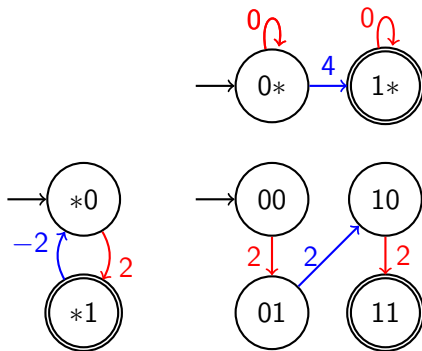
(Proof omitted.)

# General Cost Partitioning: Example



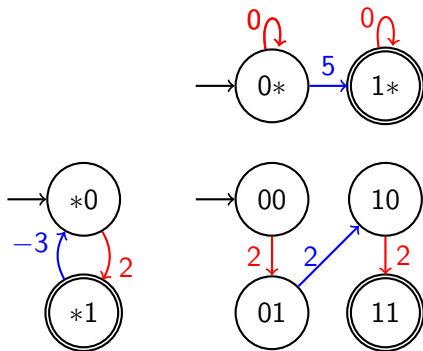
Heuristic value:  $2 + 2 = 4$

# General Cost Partitioning: Example



Heuristic value:  $4 + 2 = 6$

# General Cost Partitioning: Example



Heuristic value:  $-\infty + 5 = -\infty$

## F8.3 Summary

# Summary

- ▶ For abstraction heuristics and disjunctive action landmarks, we know how to determine an **optimal cost partitioning**, using linear programming.
- ▶ Although solving a linear program is possible in polynomial time, the better heuristic guidance often does not outweigh the overhead (in particular for abstraction heuristics).
- ▶ In contrast to standard (non-negative) cost partitioning, **general cost partitioning** allows negative operators costs.
- ▶ General cost partitioning has the same relevant properties as non-negative cost partitioning but is more powerful.