Planning and Optimization
F5. Landmarks: Cut Landmarks & LM-Cut Heuristic

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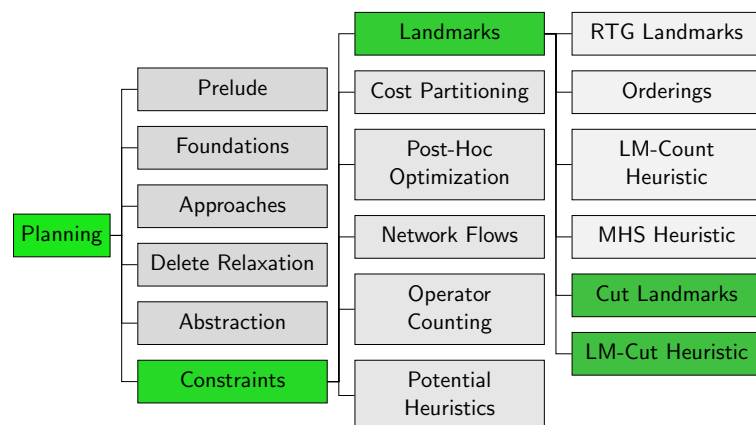Universität Basel

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F5.1 i-g Form

F5.2 Cut Landmarks

F5.3 The LM-Cut Heuristic

F5.4 Summary

Content of the Course

Roadmap for this Chapter

- ▶ We first introduce a new normal form for delete-free STRIPS tasks that simplifies later definitions.
- ▶ We then present a method that computes disjunctive action landmarks for such tasks.
- ▶ We conclude with the LM-cut heuristic that builds on this method.

F5.1 i-g Form

Delete-Free STRIPS Planning Task in i-g Form (1)

In this chapter, we only consider delete-free STRIPS tasks in a special form:

> **Definition (i-g Form for Delete-free STRIPS)**
> A delete-free STRIPS planning task $\langle V, I, O, \gamma \rangle$ is in i-g form if
> - V contains atoms i and g
> - Initially exactly i is true: $I(v) = \mathbf{T}$ iff $v = i$
> - g is the only goal atom: $\gamma = \{g\}$
> - Every action has at least one precondition.

Transformation to i-g Form

Every delete-free STRIPS task $\Pi = \langle V, I, O, \gamma \rangle$ can easily be transformed into an analogous task in i-g form.
- If i or g are in V already, rename them everywhere.
- Add i and g to V.
- Add an operator $\langle \{i\}, \{v \in V \mid I(v) = \mathbf{T}\}, \{\}, 0 \rangle$.
- Add an operator $\langle \gamma, \{g\}, \{\}, 0 \rangle$.
- Replace all operator preconditions $\top$ with i.
- Replace initial state and goal.

For the remainder of this chapter, we assume tasks in i-g form.

Example: Delete-Free Planning Task in i-g Form

> **Example**
> Consider a delete-relaxed STRIPS planning $\langle V, I, O, \gamma \rangle$ with
> $V = \{i, a, b, c, d, g\}$, $I = \{i \mapsto \mathbf{T}\} \cup \{v \mapsto \mathbf{F} \mid v \in V \setminus \{i\}\}$, $\gamma = g$
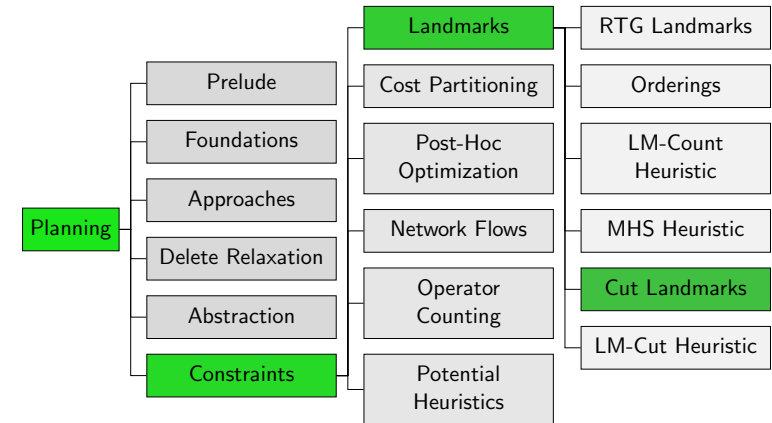> and operators
> - $o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$,
> - $o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$,
> - $o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$,
> - $o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 2 \rangle$, and
> - $o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$.

optimal solution to reach g from i:
- plan: $\langle o_{\text{blue}}, o_{\text{black}}, o_{\text{red}}, o_{\text{orange}} \rangle$
- cost: $4 + 3 + 2 + 0 = 9$ ($= h^+(I)$ because plan is optimal)

F5.2 Cut Landmarks

Content of the Course

Justification Graphs

Definition (Precondition Choice Function)

A precondition choice function (pcf) $P : O \to V$ for a delete-free STRIPS task $\Pi = \langle V, I, O, \gamma \rangle$ in i-g form maps each operator to one of its preconditions (i.e. $P(o) \in pre(o)$ for all $o \in O$).

Definition (Justification Graphs)

Let P be a pcf for $\langle V, I, O, \gamma \rangle$ in i-g form. The justification graph for P is the directed, edge-labeled graph $J = \langle V, E \rangle$, where
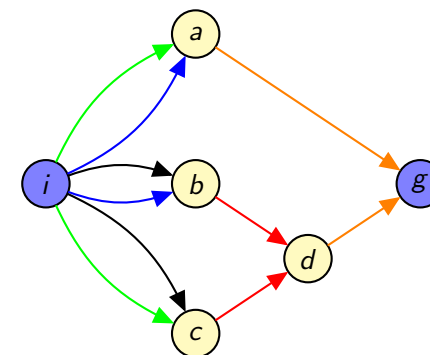
- the vertices are the variables from V, and
- E contains an edge $P(o) \xrightarrow{o} a$ for each $o \in O$, $a \in add(o)$.

Example: Justification Graph

Example (Precondition Choice Function)

$P(o_{\text{blue}}) = P(o_{\text{green}}) = P(o_{\text{black}}) = i$, $P(o_{\text{red}}) = b$, $P(o_{\text{orange}}) = a$
$P'(o_{\text{blue}}) = P'(o_{\text{green}}) = P'(o_{\text{black}}) = i$, $P'(o_{\text{red}}) = c$, $P'(o_{\text{orange}}) = d$
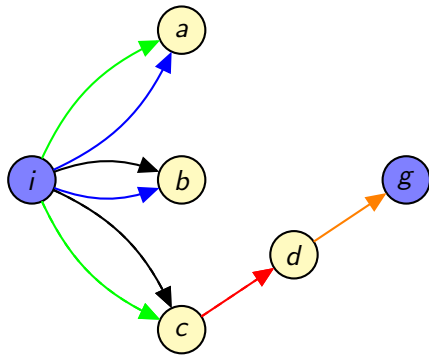

$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 2 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

Cuts

Definition (Cut)
A cut in a justification graph is a subset C of its edges such that all paths from i to g contain an edge from C.

$o_{blue} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$
$o_{green} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$
$o_{black} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{red} = \langle \{b, c\}, \{d\}, \{\}, 2 \rangle$
$o_{orange} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

Cuts are Disjunctive Action Landmarks

Theorem (Cuts are Disjunctive Action Landmarks)
Let P be a pcf for $\langle V, I, O, \gamma \rangle$ (in i-g form) and C be a cut in the justification graph for P.

The set of edge labels from C (formally $\{ o \mid \langle v, o, v' \rangle \in C \}$) is a disjunctive action landmark for I.

Proof idea:
▶ The justification graph corresponds to a simpler problem where some preconditions (those not picked by the pcf) are ignored.
▶ Cuts are landmarks for this simplified problem.
▶ Hence they are also landmarks for the original problem.

Example: Cuts in Justification Graphs

Example (Landmarks)
▶ $L_1 = \{o_{orange}\}$ (cost = 0)   ▶ $L_2 = \{o_{green}, o_{black}\}$ (cost = 3)
▶ $L_3 = \{o_{red}\}$ (cost = 2)   ▶ $L_4 = \{o_{green}, o_{blue}\}$ (cost = 4)
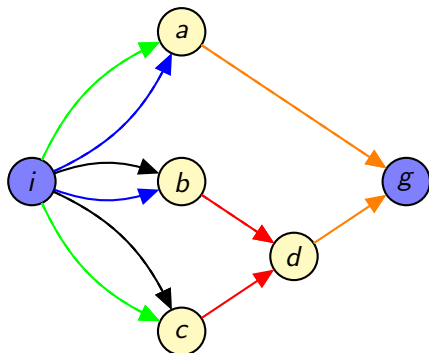
$o_{blue} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$
$o_{green} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$
$o_{black} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{red} = \langle \{b, c\}, \{d\}, \{\}, 2 \rangle$
$o_{orange} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

Power of Cuts in Justification Graphs

▶ Which landmarks can be computed with the cut method?
▶ all interesting ones!

Proposition (perfect hitting set heuristics)
Let $\mathcal{L}$ be the set of all "cut landmarks" of a given planning task with initial state I. Then $h^{MHS}(\mathcal{L}) = h^+(I)$.
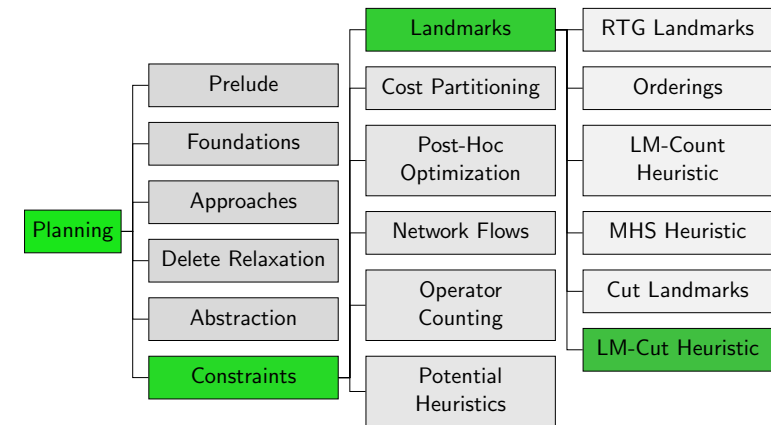
⤳ Hitting set heuristic for $\mathcal{L}$ is perfect.

Proof idea:
▶ Show 1:1 correspondence of hitting sets H for $\mathcal{L}$ and plans, i.e., each hitting set for $\mathcal{L}$ corresponds to a plan, and vice versa.

F5.3 The LM-Cut Heuristic

Content of the Course

LM-Cut Heuristic: Motivation

- In general, there are exponentially many pcfs, hence computing all relevant landmarks is not tractable.
- The LM-cut heuristic is a method that chooses pcfs and computes cuts in a goal-oriented way.
- As a side effect, it computes a
  - a cost partitioning over multiple instances of h^{max} that is also
  - a saturated cost partitioning over disjunctive action landmarks.
    ⇝ next week

LM-Cut Heuristic

$h^{\text{LM-cut}}$: Helmert & Domshlak (2009)

Initialize $h^{\text{LM-cut}}(I) := 0$. Then iterate:

1. Compute h^{max} values of the variables. Stop if $h^{\text{max}}(g) = 0$.
2. Compute justification graph G for the P that chooses preconditions with maximal h^{max} value
3. Determine the goal zone V_g of G that consists of all nodes that have a zero-cost path to g.
4. Compute the cut L that contains the labels of all edges $\langle v, o, v' \rangle$ such that $v \notin V_g$, $v' \in V_g$ and v can be reached from i without traversing a node in V_g.
   It is guaranteed that $cost(L) > 0$.
5. Increase $h^{\text{LM-cut}}(I)$ by $cost(L)$.
6. Decrease $cost(o)$ by $cost(L)$ for all $o \in L$.

Example: Computation of LM-Cut

$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$
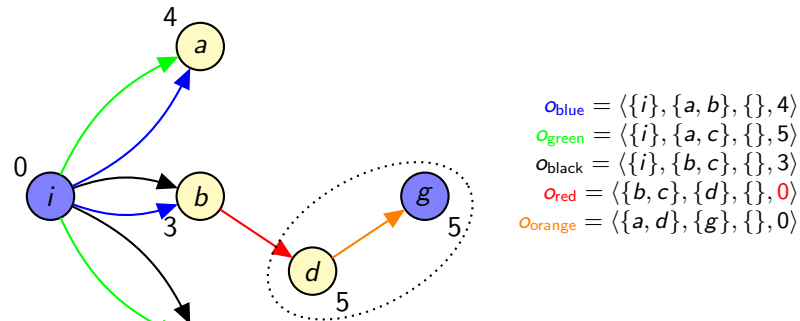
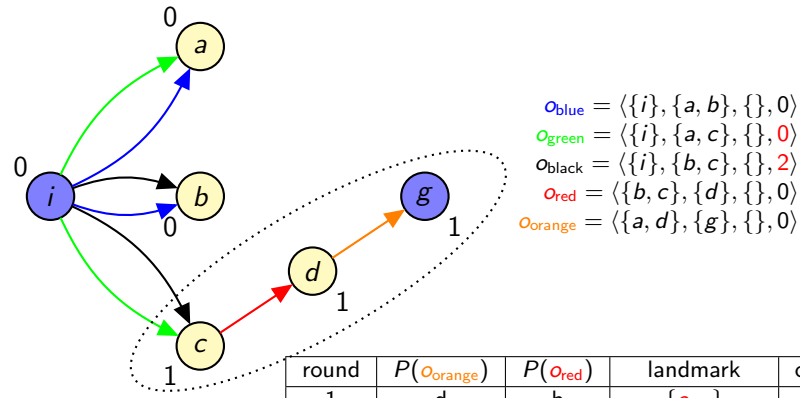| round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
			$h^{\text{LM-cut}}(I)$	2

Example: Computation of LM-Cut

$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 1 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$
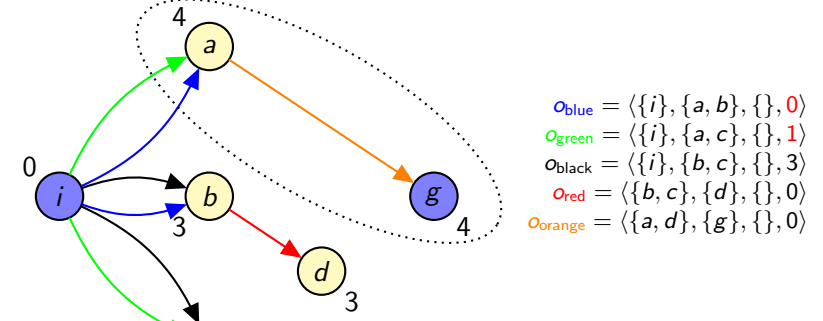
round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
			$h^{\text{LM-cut}}(I)$	6

Example: Computation of LM-Cut

$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 0 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 2 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3	d	c	$\{o_{\text{green}}, o_{\text{black}}\}$	1
			$h^{\text{LM-cut}}(I)$	7

Properties of LM-Cut Heuristic

> **Theorem**
> Let $\langle V, I, O, \gamma \rangle$ be a delete-free STRIPS task in i-g normal form.
> The LM-cut heuristic is admissible: $h^{\text{LM-cut}}(I) \leq h^*(I)$.

Proof omitted.

If Π is not delete-free, we can compute $h^{\text{LM-cut}}$ on Π^+.
Then $h^{\text{LM-cut}}$ is bounded by h^+.

F5.4 Summary

Summary

▶ Cuts in justification graphs are a general method to find disjunctive action landmarks.

▶ The minimum hitting set over all cut landmarks is a perfect heuristic for delete-free planning tasks.

▶ The LM-cut heuristic is an admissible heuristic based on these ideas.