Planning and Optimization

F5. Landmarks: Cut Landmarks & LM-Cut Heuristic

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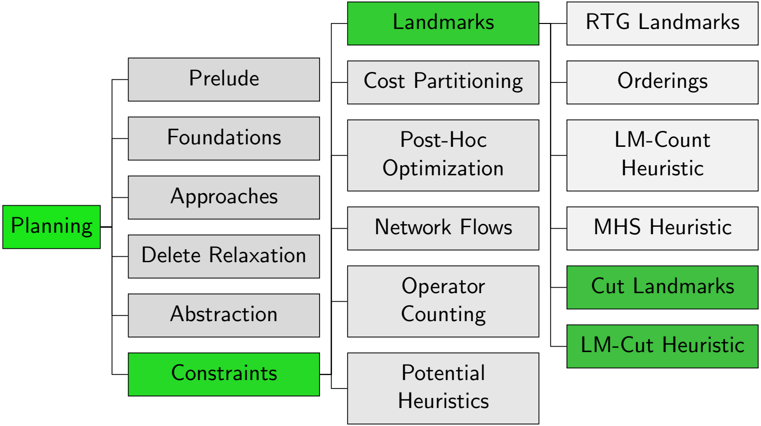
Content of the Course



Roadmap for this Chapter

- We first introduce a new normal form for delete-free STRIPS tasks that simplifies later definitions.
- We then present a method that computes disjunctive action landmarks for such tasks.
- We conclude with the LM-cut heuristic that builds on this method.

F5.1 i-g Form

Delete-Free STRIPS Planning Task in i-g Form (1)

In this chapter, we only consider delete-free STRIPS tasks in a special form:

Definition (i-g Form for Delete-free STRIPS)

A delete-free STRIPS planning task $\langle V, I, O, \gamma \rangle$ is in i-g form if

- V contains atoms i and g
- Initially exactly i is true: $I(v) = \mathbf{T}$ iff $v = i$
- g is the only goal atom: $\gamma = \{g\}$
- Every action has at least one precondition.

Transformation to i-g Form

Every delete-free STRIPS task $\Pi = \langle V, I, O, \gamma \rangle$ can easily be transformed into an analogous task in i-g form.

- If i or g are in V already, rename them everywhere.
- Add i and g to V.
- Add an operator $\langle \{i\}, \{v \in V \mid I(v) = \mathbf{T}\}, \{\}, 0 \rangle$.
- Add an operator $\langle \gamma, \{g\}, \{\}, 0 \rangle$.
- Replace all operator preconditions $\top$ with i.
- Replace initial state and goal.

For the remainder of this chapter, we assume tasks in i-g form.

Example: Delete-Free Planning Task in i-g Form

Example

Consider a delete-relaxed STRIPS planning $\langle V, I, O, \gamma \rangle$ with $V = \{i, a, b, c, d, g\}$, $I = \{i \mapsto \mathbf{T}\} \cup \{v \mapsto \mathbf{F} \mid v \in V \setminus \{i\}\}$, $\gamma = g$ and operators

- $o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$,
- $o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$,
- $o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$,
- $o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 2 \rangle$, and
- $o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$.

optimal solution to reach g from i:

- plan: $\langle o_{\text{blue}}, o_{\text{black}}, o_{\text{red}}, o_{\text{orange}} \rangle$
- cost: $4 + 3 + 2 + 0 = 9$ $(= h^+(I)$ because plan is optimal$)$

F5.2 Cut Landmarks

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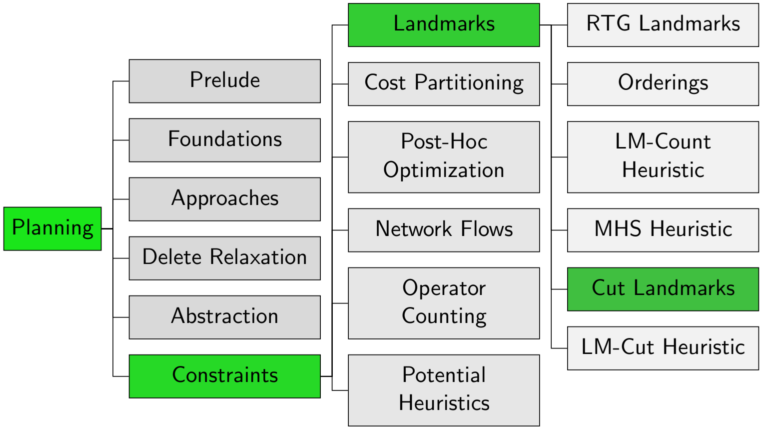


Justification Graphs

Definition (Precondition Choice Function)

A precondition choice function (pcf) $P : O \to V$ for a delete-free STRIPS task $\Pi = \langle V, I, O, \gamma \rangle$ in i-g form maps each operator to one of its preconditions (i.e. $P(o) \in pre(o)$ for all $o \in O$).

Definition (Justification Graphs)

Let P be a pcf for $\langle V, I, O, \gamma \rangle$ in i-g form. The justification graph for P is the directed, edge-labeled graph $J = \langle V, E \rangle$, where

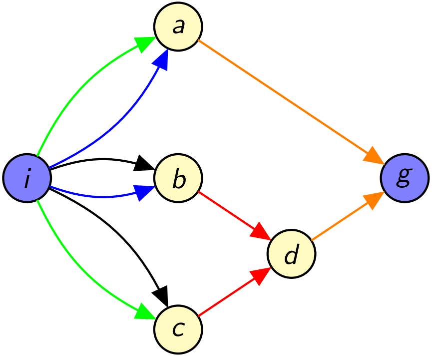
- the vertices are the variables from V, and
- E contains an edge $P(o) \xrightarrow{o} a$ for each $o \in O$, $a \in add(o)$.

Example: Justification Graph

Example (Precondition Choice Function)

$P(o_{\text{blue}}) = P(o_{\text{green}}) = P(o_{\text{black}}) = i,\ P(o_{\text{red}}) = b,\ P(o_{\text{orange}}) = a$

$P'(o_{\text{blue}}) = P'(o_{\text{green}}) = P'(o_{\text{black}}) = i,\ P'(o_{\text{red}}) = c,\ P'(o_{\text{orange}}) = d$

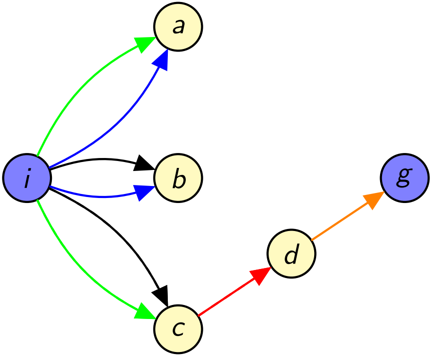


$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 2 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

Cuts

Definition (Cut)

A cut in a justification graph is a subset C of its edges such that all paths from i to g contain an edge from C.



$$
\begin{aligned}
o_{\text{blue}} &= \langle \{i\}, \{a, b\}, \{\}, 4\rangle \\
o_{\text{green}} &= \langle \{i\}, \{a, c\}, \{\}, 5\rangle \\
o_{\text{black}} &= \langle \{i\}, \{b, c\}, \{\}, 3\rangle \\
o_{\text{red}} &= \langle \{b, c\}, \{d\}, \{\}, 2\rangle \\
o_{\text{orange}} &= \langle \{a, d\}, \{g\}, \{\}, 0\rangle
\end{aligned}
$$

Cuts are Disjunctive Action Landmarks

Theorem (Cuts are Disjunctive Action Landmarks)

Let P be a pcf for $\langle V, I, O, \gamma \rangle$ (in i-g form) and C be a cut in the justification graph for P.

The set of edge labels from C (formally $\{o \mid \langle v, o, v' \rangle \in C\}$) is a disjunctive action landmark for I.

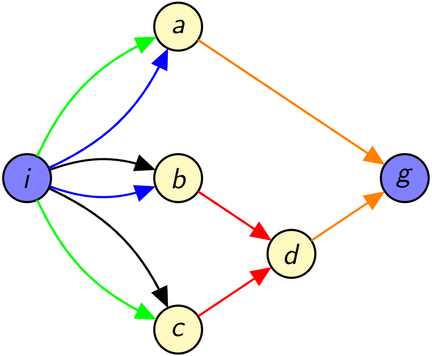
Proof idea:

- The justification graph corresponds to a simpler problem where some preconditions (those not picked by the pcf) are ignored.
- Cuts are landmarks for this simplified problem.
- Hence they are also landmarks for the original problem.

Example: Cuts in Justification Graphs

Example (Landmarks)

- $L_1 = \{o_{\text{orange}}\}$ (cost = 0)
- $L_2 = \{o_{\text{green}}, o_{\text{black}}\}$ (cost = 3)
- $L_3 = \{o_{\text{red}}\}$ (cost = 2)
- $L_4 = \{o_{\text{green}}, o_{\text{blue}}\}$ (cost = 4)



$o_{\text{blue}} = \langle\{i\}, \{a, b\}, \{\}, 4\rangle$
$o_{\text{green}} = \langle\{i\}, \{a, c\}, \{\}, 5\rangle$
$o_{\text{black}} = \langle\{i\}, \{b, c\}, \{\}, 3\rangle$
$o_{\text{red}} = \langle\{b, c\}, \{d\}, \{\}, 2\rangle$
$o_{\text{orange}} = \langle\{a, d\}, \{g\}, \{\}, 0\rangle$

Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?
- all interesting ones!

Proposition (perfect hitting set heuristics)

Let $\mathcal{L}$ be the set of all "cut landmarks" of a given planning task with initial state I. Then $h^{MHS}(\mathcal{L}) = h^{+}(I)$.

$\rightsquigarrow$ Hitting set heuristic for $\mathcal{L}$ is perfect.

Proof idea:

- Show 1:1 correspondence of hitting sets H for $\mathcal{L}$ and plans, i.e., each hitting set for $\mathcal{L}$ corresponds to a plan, and vice versa.

F5.3 The LM-Cut Heuristic

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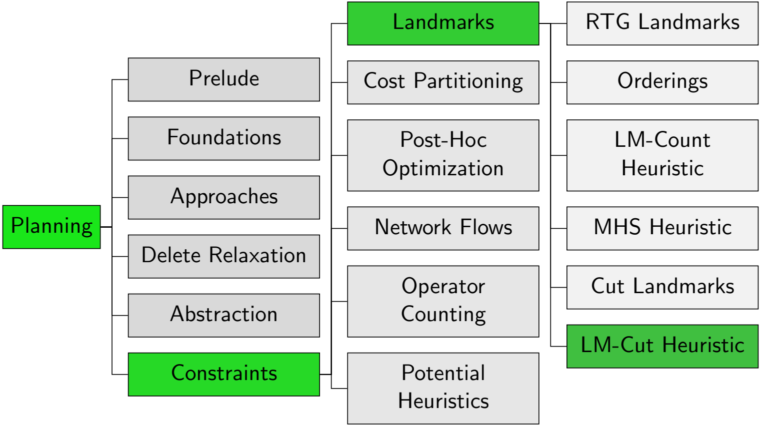


LM-Cut Heuristic: Motivation

- In general, there are exponentially many pcfs, hence computing all relevant landmarks is not tractable.
- The LM-cut heuristic is a method that chooses pcfs and computes cuts in a goal-oriented way.
- As a side effect, it computes a
  - a cost partitioning over multiple instances of $h^{\max}$ that is also
  - a saturated cost partitioning over disjunctive action landmarks.

    ⇝ next week

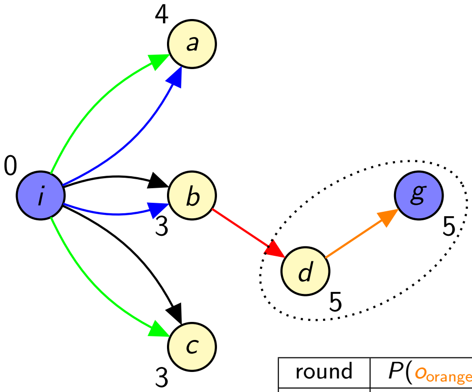
LM-Cut Heuristic

$h^{\text{LM-cut}}$: Helmert & Domshlak (2009)

Initialize $h^{\text{LM-cut}}(I) := 0$. Then iterate:

1. Compute $h^{\max}$ values of the variables. Stop if $h^{\max}(g) = 0$.
2. Compute justification graph G for the P that chooses preconditions with maximal $h^{\max}$ value
3. Determine the goal zone V_g of G that consists of all nodes that have a zero-cost path to g.
4. Compute the cut L that contains the labels of all edges $\langle v, o, v' \rangle$ such that $v \notin V_g$, $v' \in V_g$ and v can be reached from i without traversing a node in V_g.
   It is guaranteed that $cost(L) > 0$.
5. Increase $h^{\text{LM-cut}}(I)$ by $cost(L)$.
6. Decrease $cost(o)$ by $cost(L)$ for all $o \in L$.

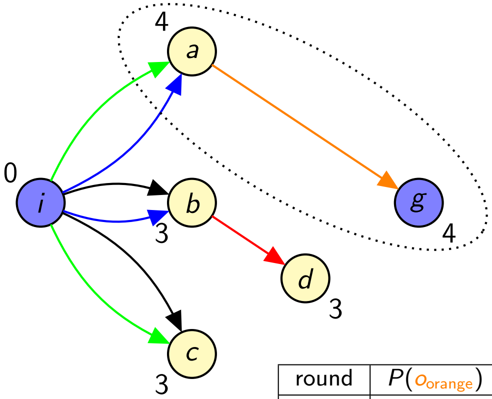
Example: Computation of LM-Cut



$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 4 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 5 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
			$h^{\text{LM-cut}}(I)$	2

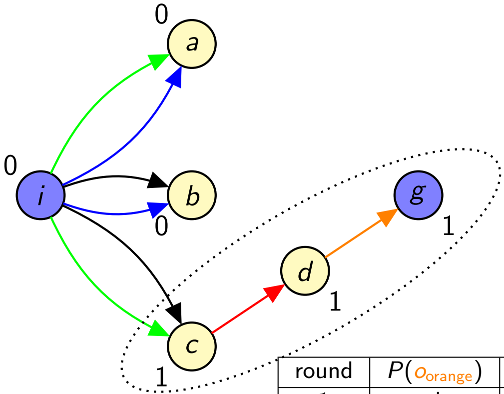
Example: Computation of LM-Cut



$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
$o_{\text{green}} = \langle \{i\}, \{a, c\}, \{\}, 1 \rangle$
$o_{\text{black}} = \langle \{i\}, \{b, c\}, \{\}, 3 \rangle$
$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
			$h^{\text{LM-cut}}(I)$	6

Example: Computation of LM-Cut



$o_{\text{blue}} = \langle \{i\}, \{a, b\}, \{\}, 0 \rangle$
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$o_{\text{red}} = \langle \{b, c\}, \{d\}, \{\}, 0 \rangle$
$o_{\text{orange}} = \langle \{a, d\}, \{g\}, \{\}, 0 \rangle$

round	$P(o_{\text{orange}})$	$P(o_{\text{red}})$	landmark	cost
1	d	b	$\{o_{\text{red}}\}$	2
2	a	b	$\{o_{\text{green}}, o_{\text{blue}}\}$	4
3	d	c	$\{o_{\text{green}}, o_{\text{black}}\}$	1
			$h^{\text{LM-cut}}(I)$	7

Properties of LM-Cut Heuristic

Theorem

Let $\langle V, I, O, \gamma \rangle$ be a delete-free STRIPS task in i-g normal form. The LM-cut heuristic is admissible: $h^{\text{LM-cut}}(I) \leq h^*(I)$.

Proof omitted.

If Π is not delete-free, we can compute $h^{\text{LM-cut}}$ on Π^+.
Then $h^{\text{LM-cut}}$ is bounded by h^+.

F5.4 Summary

Summary

- Cuts in justification graphs are a general method to find disjunctive action landmarks.
- The minimum hitting set over all cut landmarks is a perfect heuristic for delete-free planning tasks.
- The LM-cut heuristic is an admissible heuristic based on these ideas.