

# Planning and Optimization

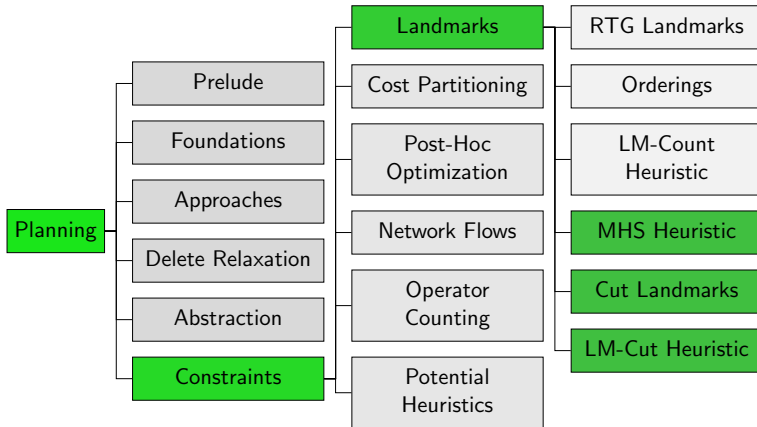
## F4. Landmarks: Minimum Hitting Set Heuristic

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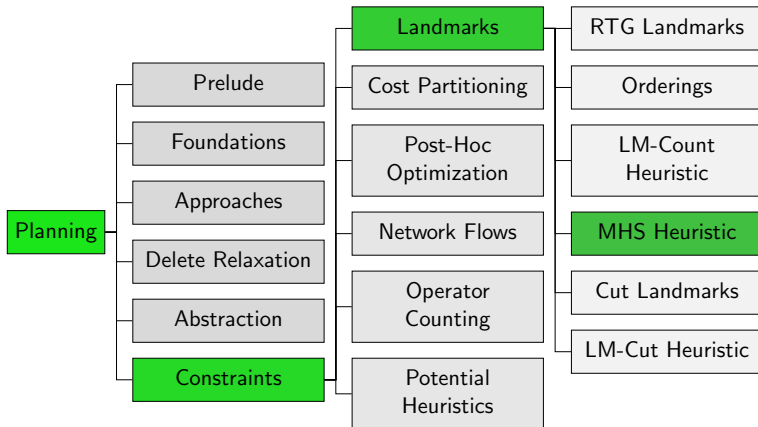
# Content of the Course



The remaining landmark topics focus on  
disjunctive action landmarks.

# Minimum Hitting Set Heuristic

# Content of the Course



# Exploiting Disjunctive Action Landmarks

- The cost  $cost(L)$  of a disjunctive action landmark  $L$  is an admissible heuristic, but it is usually not very informative.
- Landmark heuristics typically aim to combine multiple disjunctive action landmarks.

How can we exploit a given set  $\mathcal{L}$  of disjunctive action landmarks?

- Sum of costs  $\sum_{L \in \mathcal{L}} cost(L)$ ?  
     $\rightsquigarrow$  **not admissible!**
- Maximize costs  $\max_{L \in \mathcal{L}} cost(L)$ ?  
     $\rightsquigarrow$  **usually very weak heuristic**
- **better:** Hitting sets

# Hitting Sets

## Definition (Hitting Set)

Let  $X$  be a set,  $\mathcal{F} = \{F_1, \dots, F_n\} \subseteq 2^X$  be a family of subsets of  $X$  and  $c : X \rightarrow \mathbb{R}_0^+$  be a cost function for  $X$ .

A **hitting set** is a subset  $H \subseteq X$  that “hits” all subsets in  $\mathcal{F}$ , i.e.,  $H \cap F \neq \emptyset$  for all  $F \in \mathcal{F}$ . The **cost** of  $H$  is  $\sum_{x \in H} c(x)$ .

A **minimum hitting set (MHS)** is a hitting set with minimal cost.

MHS is a “classical” NP-complete problem (Karp, 1972)

## Example: Hitting Sets

### Example

$$X = \{o_1, o_2, o_3, o_4\}$$

$$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$$

$$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$$

Specify a minimum hitting set.

## Example: Hitting Sets

### Example

$$X = \{o_1, o_2, o_3, o_4\}$$

$$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$$

$$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$$

Specify a minimum hitting set.

**Solution:**  $\{o_1, o_2, o_4\}$  with cost  $3 + 4 + 0 = 7$

# Hitting Sets for Disjunctive Action Landmarks

Idea: **disjunctive action landmarks** are interpreted as instance of **minimum hitting set**

## Definition (Hitting Set Heuristic)

Let  $\mathcal{L}$  be a set of disjunctive action landmarks. The **hitting set heuristic**  $h^{MHS}(\mathcal{L})$  is defined as the cost of a minimum hitting set for  $\mathcal{L}$  with  $c(o) = cost(o)$ .

## Proposition (Hitting Set Heuristic is Admissible)

*Let  $\mathcal{L}$  be a set of disjunctive action landmarks for state  $s$ . Then  $h^{MHS}(\mathcal{L})$  is an admissible estimate for  $s$ .*

# Hitting Set Heuristic: Discussion

- The hitting set heuristic is the **best possible** heuristic that only uses the given information...
- ...but is NP-hard to compute.
- $\rightsquigarrow$  Use approximations that can be efficiently computed.  
 $\Rightarrow$  LP-relaxation, cost partitioning (both discussed later)

# Summary

# Summary

- **Hitting sets** yield the most accurate heuristic for a given set of disjunctive action landmarks.
- The computation of a **minimal hitting set** is NP-hard.