

Planning and Optimization

F4. Landmarks: Minimum Hitting Set Heuristic

Malte Helmert and Gabriele Röger

Universität Basel

December 1, 2025

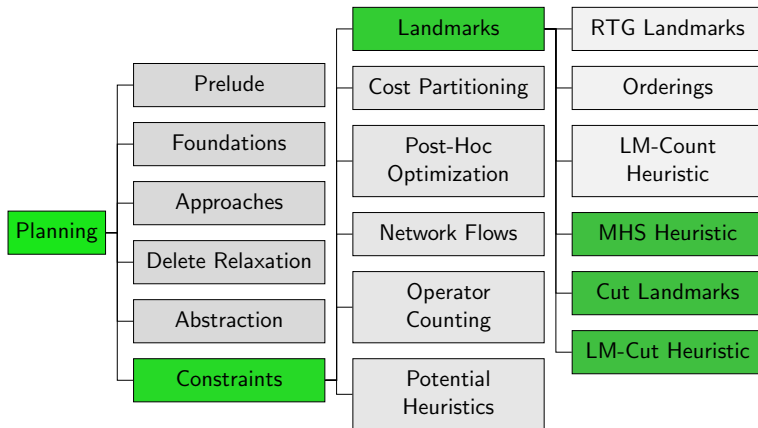
Planning and Optimization

December 1, 2025 — F4. Landmarks: Minimum Hitting Set Heuristic

F4.1 Minimum Hitting Set Heuristic

F4.2 Summary

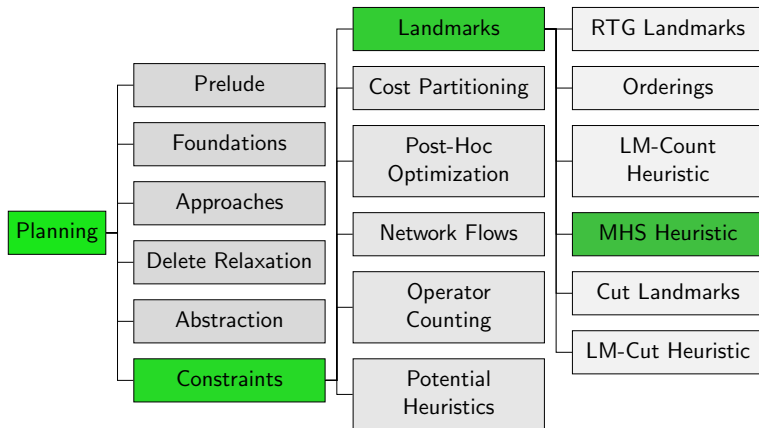
Content of the Course



The remaining landmark topics focus on
disjunctive action landmarks.

F4.1 Minimum Hitting Set Heuristic

Content of the Course



Exploiting Disjunctive Action Landmarks

- ▶ The cost $cost(L)$ of a disjunctive action landmark L is an admissible heuristic, but it is usually not very informative.
- ▶ Landmark heuristics typically aim to combine multiple disjunctive action landmarks.

How can we exploit a given set $\mathcal{L}$ of disjunctive action landmarks?

- ▶ Sum of costs $\sum_{L \in \mathcal{L}} cost(L)$?
 $\rightsquigarrow$ **not admissible!**
- ▶ Maximize costs $\max_{L \in \mathcal{L}} cost(L)$?
 $\rightsquigarrow$ **usually very weak heuristic**
- ▶ **better:** Hitting sets

Hitting Sets

Definition (Hitting Set)

Let X be a set, $\mathcal{F} = \{F_1, \dots, F_n\} \subseteq 2^X$ be a family of subsets of X and $c : X \rightarrow \mathbb{R}_0^+$ be a cost function for X .

A **hitting set** is a subset $H \subseteq X$ that “hits” all subsets in $\mathcal{F}$, i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The **cost** of H is $\sum_{x \in H} c(x)$.

A **minimum hitting set (MHS)** is a hitting set with minimal cost.

MHS is a “classical” NP-complete problem (Karp, 1972)

Example: Hitting Sets

Example

$$X = \{o_1, o_2, o_3, o_4\}$$

$$\mathcal{F} = \{\{o_4\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_2, o_3\}\}$$

$$c(o_1) = 3, \quad c(o_2) = 4, \quad c(o_3) = 5, \quad c(o_4) = 0$$

Specify a minimum hitting set.

Solution: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting Sets for Disjunctive Action Landmarks

Idea: **disjunctive action landmarks** are interpreted as instance of **minimum hitting set**

Definition (Hitting Set Heuristic)

Let $\mathcal{L}$ be a set of disjunctive action landmarks. The **hitting set heuristic** $h^{MHS}(\mathcal{L})$ is defined as the cost of a minimum hitting set for $\mathcal{L}$ with $c(o) = cost(o)$.

Proposition (Hitting Set Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s . Then $h^{MHS}(\mathcal{L})$ is an admissible estimate for s .

Hitting Set Heuristic: Discussion

- ▶ The hitting set heuristic is the **best possible** heuristic that only uses the given information...
- ▶ ...but is NP-hard to compute.
- ▶ $\rightsquigarrow$ Use approximations that can be efficiently computed.
 $\Rightarrow$ **LP-relaxation**, **cost partitioning** (both discussed later)

F4.2 Summary

Summary

- ▶ **Hitting sets** yield the most accurate heuristic for a given set of disjunctive action landmarks.
- ▶ The computation of a **minimal hitting set** is NP-hard.