

Planning and Optimization

F3. Landmarks: Orderings & LM-Count Heuristic

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F3.1 Landmark Orderings

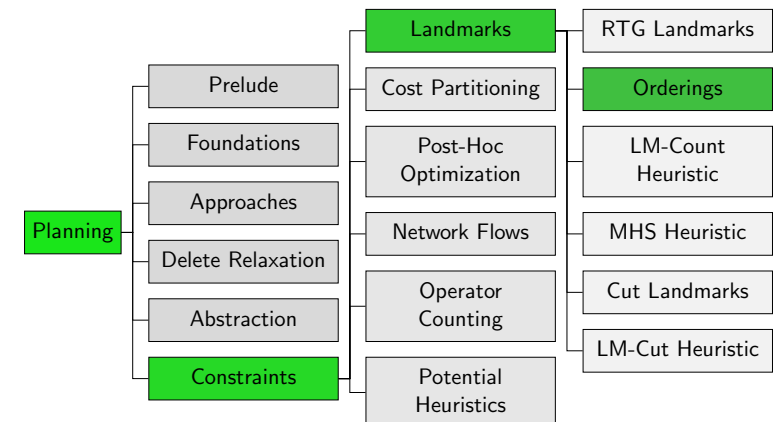
F3.2 Landmark Propagation

F3.3 Landmark-count Heuristic

F3.4 Summary

F3.1 Landmark Orderings

Content of the Course



Why Landmark Orderings?

- ▶ To compute a landmark heuristic estimate for state s we need landmarks for s .
- ▶ We could invest the time to compute them **for every state from scratch**.
- ▶ Alternatively, we can **compute landmarks once** and **propagate** them over operator applications.
- ▶ **Landmark orderings** are used to detect landmarks that should be further considered because they (again) need to be satisfied later.
- ▶ (We will later see yet another approach, where heuristic computation and landmark computation are integrated \rightsquigarrow LM-Cut.)

Example

Consider task $\langle \{a, b, c, d\}, I, \{o_1, o_2, \dots, o_n\}, d \rangle$ with

- ▶ $I(v) = \perp$ for $v \in \{a, b, c, d\}$,
- ▶ $o_1 = \langle \top, a \wedge b \rangle$, and
- ▶ $o_2 = \langle a, c \wedge \neg a \wedge \neg b \rangle$ (plus some more operators).

You know that a, b, c and d are all fact landmarks for I .

- ▶ What landmarks are still required to be made true in state $I[\langle o_1, o_2 \rangle]$?
- ▶ You get the additional information that variable a must be true immediately before d is first made true. Any changes?

Terminology

Let $\pi = \langle o_1, \dots, o_n \rangle$ be a sequence of operators applicable in state I and let φ be a formula over the state variables.

- ▶ φ is **true at time i** if $I[\langle o_1, \dots, o_i \rangle] \models \varphi$.
- ▶ Also special case $i = 0$: φ is **true at time 0** if $I \models \varphi$.
- ▶ No formula is true at time $i < 0$.
- ▶ φ is **added at time i** if it is **true at time i but not at time $i - 1$** .
- ▶ φ is **first added at time i** if it is **true at time i but not at any time $j < i$** .
We denote this i by **$first(\varphi, \pi)$** .
- ▶ **$last(\varphi, \pi)$** denotes the last time in which φ is added in π .

Landmark Orderings

Definition (Landmark Orderings)

Let φ and ψ be formula landmarks. There is

- ▶ a **natural ordering** between φ and ψ (written $\varphi \rightarrow \psi$) if in each plan π it holds that $first(\varphi, \pi) < first(\psi, \pi)$.
“ φ must be true some time strictly before ψ is first added.”
- ▶ a **greedy-necessary ordering** between φ and ψ (written $\varphi \rightarrow_{gn} \psi$) if for every plan $\pi = \langle o_1, \dots, o_n \rangle$ it holds that $S[\langle o_1, \dots, o_{first(\psi, \pi)-1} \rangle] \models \varphi$.
“ φ must be true immediately before ψ is first added.”
- ▶ a **weak ordering** between φ and ψ (written $\varphi \rightarrow_w \psi$) if in each plan π it holds that $first(\varphi, \pi) < last(\psi, \pi)$.
“ φ must be true some time before ψ is last added.”

Not covered: reasonable orderings, which generalize weak orderings

Natural Orderings

Definition

There is a **natural ordering** between φ and ψ (written $\varphi \rightarrow \psi$) if in each plan π it holds that $\text{first}(\varphi, \pi) < \text{first}(\psi, \pi)$.

- ▶ We can directly determine natural orderings from the *LM* sets computed from the simplified relaxed task graph.
- ▶ For fact landmarks v, v' with $v \neq v'$, if $n_{v'} \in LM(n_v)$ then $v' \rightarrow v$.

Greedy-necessary Orderings

Definition

There is a **greedy-necessary ordering** between φ and ψ (written $\varphi \rightarrow_{\text{gn}} \psi$) if in each plan where ψ is first added at time i , φ is true at time $i - 1$.

- ▶ We can again determine such orderings from the sRTG.
- ▶ For an OR node n_v , we define the set of **first achievers** as $FA(n_v) = \{n_o \mid n_o \in \text{succ}(n_v) \text{ and } n_v \notin LM(n_o)\}$.
- ▶ Then $v' \rightarrow_{\text{gn}} v$ if $n_{v'} \in \text{succ}(n_o)$ for all $n_o \in FA(n_v)$.

F3.2 Landmark Propagation

Example Revisited

Consider task $\langle \{a, b, c, d\}, I, \{o_1, o_2, \dots, o_n\}, d \rangle$ with

- ▶ $I(v) = \perp$ for $v \in \{a, b, c, d\}$,
- ▶ $o_1 = \langle \top, a \wedge b \rangle$ and $o_2 = \langle a, c \wedge \neg a \wedge \neg b \rangle$ (plus some more).

You know that a, b, c and d are all fact landmarks for I .

- ▶ What landmarks are still required to be made true in state $I[\langle o_1, o_2 \rangle]$? **All not achieved yet on the state path**
- ▶ You get the additional information that variable a must be true immediately before d is first made true. Any changes? **Exploit orderings to determine landmarks that are still required.**
- ▶ There is another path to the same state where b was never true. What now? **Exploit information from multiple paths.**

Past and Future Landmarks

- ▶ In the following, \mathcal{L}_I is always a set of formula landmarks for the initial state with set of orderings \mathcal{O}_I .
- ▶ The set $\mathcal{L}_{\text{past}}^*(s)$ of **past landmarks** of a state s contains all landmarks from \mathcal{L}_I that are **at some point true in every path from the initial state to s** .
- ▶ The set $\mathcal{L}_{\text{fut}}^*(s)$ of **future landmarks** of a state s contains all landmarks from \mathcal{L}_I that are also **landmarks of s but not true in s** .
- ▶ Past landmarks are important for inferring which orderings are still relevant, future landmarks are relevant for the heuristic estimates.
- ▶ Since the exact sets are defined over **all** paths between certain states, we use approximations.

Landmark State

Definition

Let \mathcal{L}_I be a set of formula landmarks for the initial state.

A **landmark state** \mathbb{L} is \perp or a pair $\langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle$ such that $\mathcal{L}_{\text{fut}} \cup \mathcal{L}_{\text{past}} = \mathcal{L}_I$.

\mathbb{L} is **valid** in state s if

- ▶ $\mathbb{L} = \perp$ and Π has no s -plan, or
- ▶ $\mathbb{L} = \langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle$ with $\mathcal{L}_{\text{past}} \supseteq \mathcal{L}_{\text{past}}^*$ and $\mathcal{L}_{\text{fut}} \subseteq \mathcal{L}_{\text{fut}}^*$.

Context in Search: LM-BFS Algorithm

```

 $\mathbb{L}(\text{init}), \mathcal{L}_I, \mathcal{O}_I := \text{compute\_landmark\_info}(\text{init}())$ 
if  $h(\text{init}(), \mathbb{L}(\text{init})) < \infty$  then
   $\text{open.insert}(\langle \text{init}(), 0, h(\text{init}(), \mathbb{L}(\text{init})) \rangle)$ 
while  $\text{open} \neq \emptyset$  do
   $\langle s, g, v \rangle = \text{open.pop}()$ 
  if  $v < h(s, \mathbb{L}(s))$  then
     $\text{open.insert}(\langle s, g, h(s, \mathbb{L}(s)) \rangle)$ 
  else if  $g < \text{distances}(s)$  then
     $\text{distances}(s) := g$ 
    if  $\text{is\_goal}(s)$  then return  $\text{extract\_plan}(s)$ ;
    foreach  $\langle a, s' \rangle \in \text{succ}(s)$  do
       $\mathbb{L}' := \text{progress\_landmark\_state}(\mathbb{L}(s), \langle s, a, s' \rangle)$ 
       $\mathbb{L}(s') := \text{merge\_landmark\_states}(\mathbb{L}(s'), \mathbb{L}')$ 
      if  $\mathbb{L}(s') \neq \perp$  and  $h(s', \mathbb{L}(s')) < \infty$  then
         $\text{open.insert}(\langle s', g + \text{cost}(a), h(s', \mathbb{L}(s')) \rangle)$ 

```

$\mathbb{L}(s) := \langle \mathcal{L}_I, \emptyset \rangle$ and $\text{distances}(s) := \infty$ if read before set.

Context: Exploit Information from Multiple Paths

```

 $\mathbb{L}(\text{init}), \mathcal{L}_I, \mathcal{O}_I := \text{compute\_landmark\_info}(\text{init}())$ 
if  $h(\text{init}(), \mathbb{L}(\text{init})) < \infty$  then
   $\text{open.insert}(\langle \text{init}(), 0, h(\text{init}(), \mathbb{L}(\text{init})) \rangle)$ 
while  $\text{open} \neq \emptyset$  do
   $\langle s, g, v \rangle = \text{open.pop}()$ 
  if  $v < h(s, \mathbb{L}(s))$  then
     $\text{open.insert}(\langle s, g, h(s, \mathbb{L}(s)) \rangle)$ 
  else if  $g < \text{distances}(s)$  then
     $\text{distances}(s) := g$ 
    if  $\text{is\_goal}(s)$  then return  $\text{extract\_plan}(s)$ ;
    foreach  $\langle a, s' \rangle \in \text{succ}(s)$  do
       $\mathbb{L}' := \text{progress\_landmark\_state}(\mathbb{L}(s), \langle s, a, s' \rangle)$ 
       $\mathbb{L}(s') := \text{merge\_landmark\_states}(\mathbb{L}(s'), \mathbb{L}')$ 
      if  $\mathbb{L}(s') \neq \perp$  and  $h(s', \mathbb{L}(s')) < \infty$  then
         $\text{open.insert}(\langle s', g + \text{cost}(a), h(s', \mathbb{L}(s')) \rangle)$ 

```

$\mathbb{L}(s) := \langle \mathcal{L}_I, \emptyset \rangle$ and $\text{distances}(s) := \infty$ if read before set.

Merging Landmark States

Merging combines the information from two landmark states.

`merge_landmark_states(\mathbb{L}, \mathbb{L}')`

```

if  $\mathbb{L} = \perp$  or  $\mathbb{L}' = \perp$  then return  $\perp$ ;
 $\langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle := \mathbb{L}$ 
 $\langle \mathcal{L}'_{\text{past}}, \mathcal{L}'_{\text{fut}} \rangle := \mathbb{L}'$ 
return  $\langle \mathcal{L}_{\text{past}} \cap \mathcal{L}'_{\text{past}}, \mathcal{L}_{\text{fut}} \cup \mathcal{L}'_{\text{fut}} \rangle$ 

```

Theorem

If \mathbb{L} and \mathbb{L}' are valid in a state s then also `merge_landmark_states(\mathbb{L}, \mathbb{L}')` is valid in s .

Context: Progression for a Transition

```

 $\mathbb{L}(\text{init}), \mathcal{L}_I, \mathcal{O}_I := \text{compute\_landmark\_info}(\text{init}())$ 
if  $h(\text{init}(), \mathbb{L}(\text{init})) < \infty$  then
   $\text{open.insert}(\langle \text{init}(), 0, h(\text{init}(), \mathbb{L}(\text{init})) \rangle)$ 
while  $\text{open} \neq \emptyset$  do
   $\langle s, g, v \rangle = \text{open.pop}()$ 
  if  $v < h(s, \mathbb{L}(s))$  then
     $\text{open.insert}(\langle s, g, h(s, \mathbb{L}(s)) \rangle)$ 
  else if  $g < \text{distances}(s)$  then
     $\text{distances}(s) := g$ 
    if  $\text{is\_goal}(s)$  then return  $\text{extract\_plan}(s)$ ;
    foreach  $\langle a, s' \rangle \in \text{succ}(s)$  do
       $\mathbb{L}' := \text{progress\_landmark\_state}(\mathbb{L}(s), \langle s, a, s' \rangle)$ 
       $\mathbb{L}(s') := \text{merge\_landmark\_states}(\mathbb{L}(s'), \mathbb{L}')$ 
      if  $\mathbb{L}(s') \neq \perp$  and  $h(s', \mathbb{L}(s')) < \infty$  then
         $\text{open.insert}(\langle s', g + \text{cost}(a), h(s', \mathbb{L}(s')) \rangle)$ 

```

$\mathbb{L}(s) := \langle \mathcal{L}_I, \emptyset \rangle$ and $\text{distances}(s) := \infty$ if read before set.

Progressing Landmark States

- ▶ If we expand a state s with transition $\langle s, o, s' \rangle$, we use **progression** to determine a landmark state for s' from the one we know for s .
- ▶ We will only introduce progression methods that preserve the validity of landmark states.
- ▶ Since every progression method gives a valid landmark state, we can merge results from different methods into a valid landmark state.

Basic Progression

Definition (Basic Progression)

Basic progression maps landmark state $\langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle$ and transition $\langle s, o, s' \rangle$ to landmark state $\langle \mathcal{L}_{\text{past}} \cup \mathcal{L}_{\text{add}}, \mathcal{L}_{\text{fut}} \setminus \mathcal{L}_{\text{add}} \rangle$, where $\mathcal{L}_{\text{add}} = \{ \varphi \in \mathcal{L}_I \mid s \not\models \varphi \text{ and } s' \models \varphi \}$.

“Extend the past with all landmarks added in s' and remove them from the future.”

Goal Progression

Definition (Goal Progression)

Let γ be the goal of the task.

Goal progression maps landmark state $\langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle$ and transition $\langle s, o, s' \rangle$ to landmark state $\langle \mathcal{L}_I, \mathcal{L}_{\text{goal}} \rangle$, where $\mathcal{L}_{\text{goal}} = \{ \varphi \in \mathcal{L}_I \mid \gamma \models \varphi \text{ and } s' \not\models \varphi \}$.

“All landmarks that must be true in the goal but are false in s' must be achieved in the future.”

Weak Ordering Progression

$\varphi \rightarrow_w \psi$: “ φ must be true some time before ψ is last added.”

Definition (Weak Ordering Progression)

The weak ordering progression maps landmark state $\langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle$ and transition $\langle s, o, s' \rangle$ to landmark state $\langle \mathcal{L}_I, \{ \psi \mid \exists \varphi \rightarrow_w \psi : \varphi \notin \mathcal{L}_{\text{past}} \} \rangle$.

“Landmark ψ must be added in the future because we haven't done something that must be done before ψ is last added.”

Greedy-necessary Ordering Progression

$\varphi \rightarrow_{\text{gn}} \psi$: “ φ must be true immediately before ψ is first added.”

Definition (Greedy-necessary Ordering Progression)

The greedy necessary ordering progression maps landmark state $\langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle$ and transition $\langle s, o, s' \rangle$ to landmark state

- ▶ \perp if there is a $\varphi \rightarrow_{\text{gn}} \psi \in \mathcal{O}_I$ with $\psi \notin \mathcal{L}_{\text{past}}$, $s \not\models \varphi$ and $s' \models \psi$, and
- ▶ $\langle \mathcal{L}_I, \{ \varphi \mid s' \not\models \varphi \text{ and } \exists \varphi \rightarrow_{\text{gn}} \psi \in \mathcal{O}_I : \psi \notin \mathcal{L}_{\text{past}}, s' \not\models \psi \} \rangle$ otherwise.

“Landmark ψ has not been true, yet, and φ must be true immediately before it becomes true. Since φ is currently false, we must make it true in the future (before making ψ true).”

Natural Ordering Progression

$\varphi \rightarrow \psi$: φ must be true some time strictly before ψ is first added.

Definition (Natural Ordering Progression)

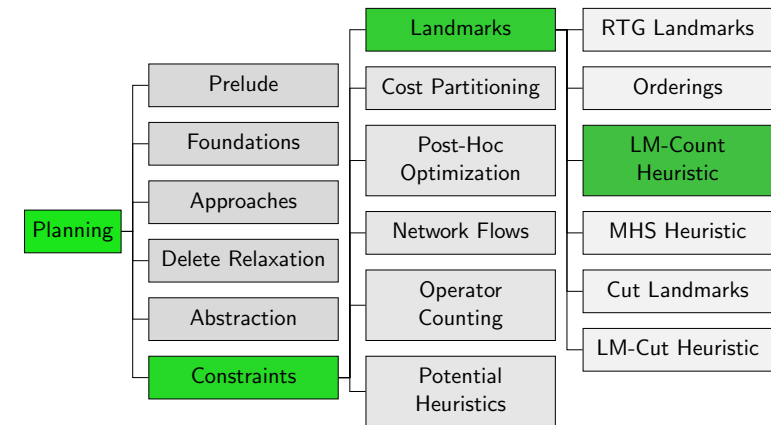
The natural ordering progression maps landmark state $\langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle$ and transition $\langle s, o, s' \rangle$ to landmark state

- ▶ \perp if there is a $\varphi \rightarrow \psi \in \mathcal{O}_I$ with $\varphi \notin \mathcal{L}_{\text{past}}$ and $s' \models \psi$, and
- ▶ $\langle \mathcal{L}_I, \emptyset \rangle$ otherwise.

Not (yet) useful: All known methods only find natural orderings that are true for every applicable operator sequence, so the interesting first case never happens in LM-BFS.

F3.3 Landmark-count Heuristic

Content of the Course



Landmark-count Heuristic

The landmark-count heuristic counts the landmarks that still have to be achieved.

Definition (LM-count Heuristic)

Let Π be a planning task, s be a state and $\mathbb{L} = \langle \mathcal{L}_{\text{past}}, \mathcal{L}_{\text{fut}} \rangle$ be a valid landmark state for s .

The **LM-count heuristic** for s and \mathbb{L} is

$$h^{\text{LM-count}}(s, \mathbb{L}) = \begin{cases} \infty & \text{if } \mathbb{L} = \perp, \\ |\mathcal{L}_{\text{fut}}| & \text{otherwise} \end{cases}$$

In the original work, \mathcal{L}_{fut} was determined without considering information from multiple paths and could not detect dead-ends.

LM-count Heuristic is Path-dependent

- ▶ LM-count heuristic gives estimates for landmark states, which depend on the considered paths.
- ▶ Search algorithms need estimates for states.
- ▶ \rightsquigarrow we use estimate from the **current** landmark state.
- ▶ \rightsquigarrow heuristic estimate for a state is **not well-defined**.

LM-count Heuristic is Inadmissible

Example

Consider STRIPS planning task $\Pi = \langle \{a, b\}, I, \{o\}, \{a, b\} \rangle$ with $I = \emptyset$, $o = \langle \emptyset, \{a, b\}, \emptyset, 1 \rangle$. Let $\mathcal{L} = \{a, b\}$ and $\mathcal{O} = \emptyset$.

Landmark state $\langle \emptyset, \mathcal{L} \rangle$ for the initial state is valid and the estimate is $h^{\text{LM-count}}(I, \langle \emptyset, \{a, b\} \rangle) = 2$ while $h^*(I) = 1$.

$\leadsto h^{\text{LM-count}}$ is **inadmissible**.

LM-count Heuristic: Comments

- ▶ LM-Count alone is not a particularly informative heuristic.
- ▶ On the positive side, it complements h^{FF} very well.
- ▶ For example, the LAMA planning system alternates between expanding a state with minimal h^{FF} and minimal $h^{\text{LM-count}}$ estimate.
- ▶ The LM-sum heuristic is a cost-aware variant of the heuristic that sums up the costs of the cheapest achiever (= operator that adds the fact landmark) of each landmark.
- ▶ There is an admissible variant of the heuristic based on operator cost partitioning.

F3.4 Summary

Summary

- ▶ We can propagate landmark sets over action applications.
- ▶ Landmark orderings can be useful for detecting when a landmark that has already been achieved should be further considered.
- ▶ We can combine the landmark information from several paths to the same state.
- ▶ The LM-count heuristic counts how many landmarks still need to be satisfied.
- ▶ The LM-count heuristic is inadmissible (but there is an admissible variant).