Planning and Optimization

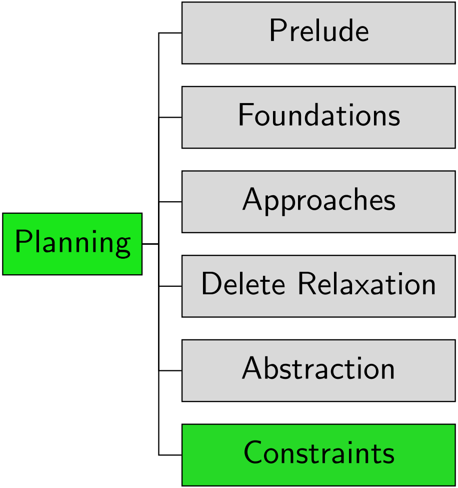
F1. Constraints: Introduction

Malte Helmert and Gabriele Röger

Universität Basel

November 26, 2025

Content of the Course



Constraint-based Heuristics

Coming Up with Heuristics in a Principled Way

General Procedure for Obtaining a Heuristic

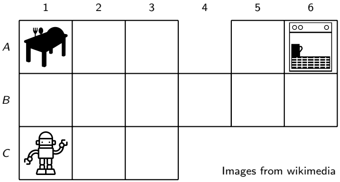
Solve a simplified version of the problem.

Major ideas for heuristics in the planning literature:

- delete relaxation
- abstraction
- critical paths
- landmarks
- network flows
- potential heuristic

Landmarks, network flows and potential heuristics are based on constraints that can be specified for a planning task.

Constraints: Example



FDR planning task $\langle V, I, O, \gamma \rangle$ with

- $V = \{robot\text{-}at, dishes\text{-}at\}$ with
  - $\mathrm{dom}(robot\text{-}at) = \{\mathrm{A1}, \dots, \mathrm{C3}, \mathrm{B4}, \mathrm{A5}, \dots, \mathrm{B6}\}$
  - $\mathrm{dom}(dishes\text{-}at) = \{\mathrm{Table}, \mathrm{Robot}, \mathrm{Dishwasher}\}$
- $I = \{robot\text{-}at \mapsto \mathrm{C1}, dishes\text{-}at \mapsto \mathrm{Table}\}$
- operators
  - move-x-y to move from cell x to adjacent cell y
  - pickup dishes, and
  - load dishes into the dishwasher.
- $\gamma = (robot\text{-}at = B6) \wedge (dishes\text{-}at = \mathrm{Dishwasher})$

Constraints

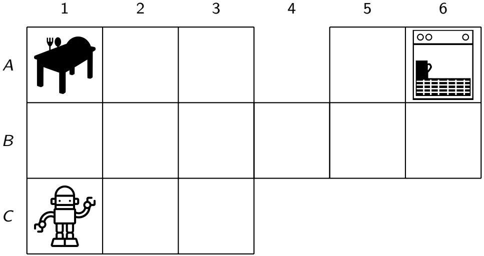
Some heuristics exploit constraints that describe something that holds in every solution of the task.

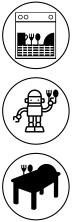
For instance, every solution is such that

- a variable takes a certain value in at least one visited state. (a fact landmark constraint)

Fact Landmarks: Example

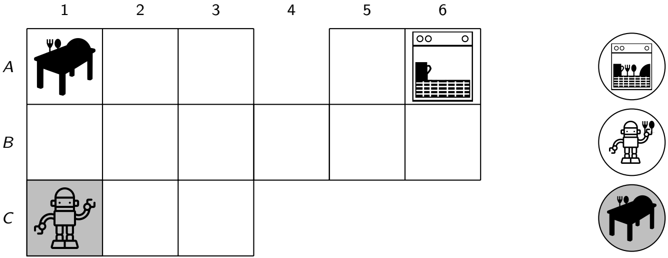
Which values do *robot-at* and *dishes-at* take in every solution?





Fact Landmarks: Example

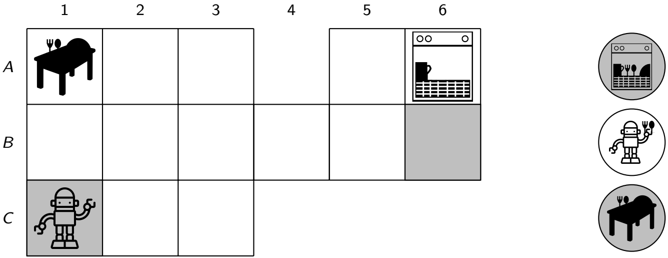
Which values do *robot-at* and *dishes-at* take in every solution?



- `robot-at` = C1, `dishes-at` = Table (initial state)

Fact Landmarks: Example

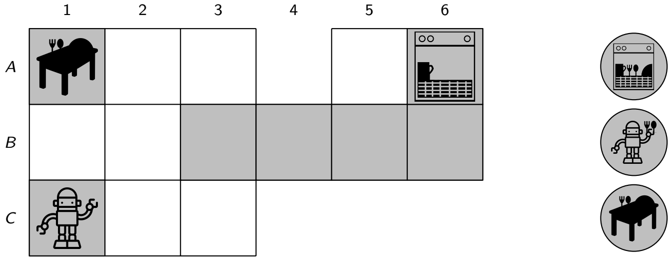
Which values do *robot-at* and *dishes-at* take in every solution?



- `robot-at` = C1, `dishes-at` = Table (initial state)
- `robot-at` = B6, `dishes-at` = Dishwasher (goal state)

Fact Landmarks: Example

Which values do *robot-at* and *dishes-at* take in every solution?



- `robot-at` = C1, `dishes-at` = Table (initial state)
- `robot-at` = B6, `dishes-at` = Dishwasher (goal state)
- `robot-at` = A1, `robot-at` = B3, `robot-at` = B4, `robot-at` = B5, `robot-at` = A6, `dishes-at` = Robot

Constraints

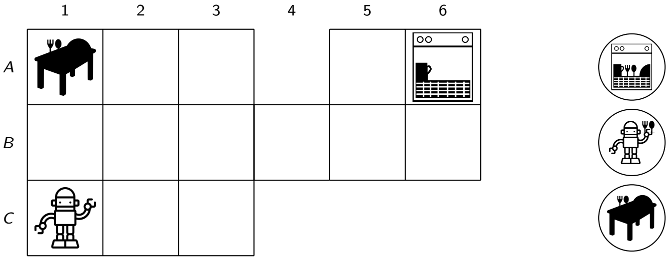
Some heuristics exploit constraints that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some value in at least one visited state. (a fact landmark constraint)
- an action must be applied. (an action landmark constraint)

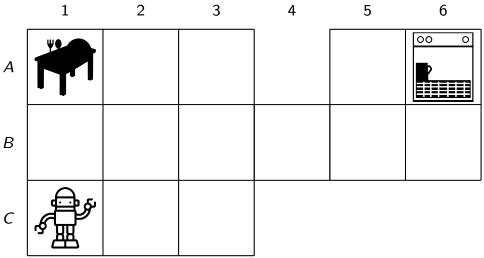
Action Landmarks: Example

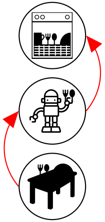
Which actions must be applied in every solution?



Action Landmarks: Example

Which actions must be applied in every solution?

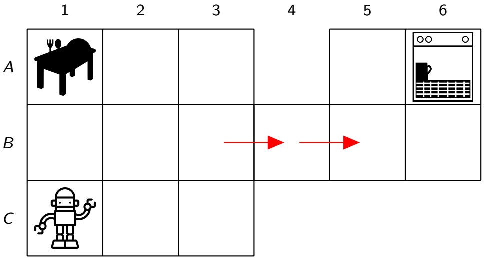


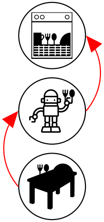


- pickup
- load

Action Landmarks: Example

Which actions must be applied in every solution?





- pickup
- load
- move-B3-B4
- move-B4-B5

Constraints

Some heuristics exploit constraints that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some value in at least one visited state.
  (a fact landmark constraint)
- an action must be applied.
  (an action landmark constraint)

Constraints

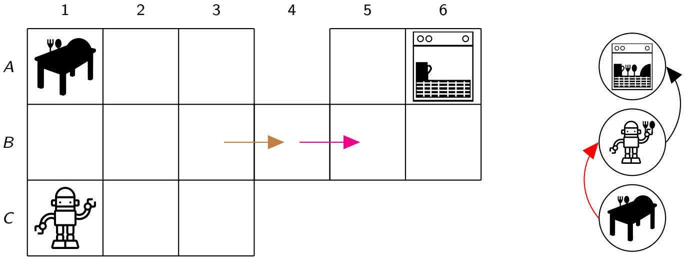
Some heuristics exploit constraints that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some value in at least one visited state.
(a fact landmark constraint)
- at least one action from a set of actions must be applied.
(a disjunctive action landmark constraint)

Disjunctive Action Landmarks: Example

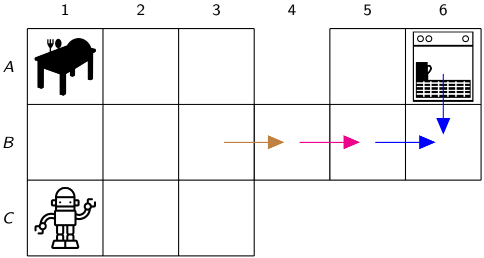
Which set of actions is such that at least one must be applied?

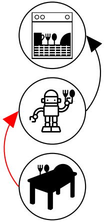


- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}

Disjunctive Action Landmarks: Example

Which set of actions is such that at least one must be applied?

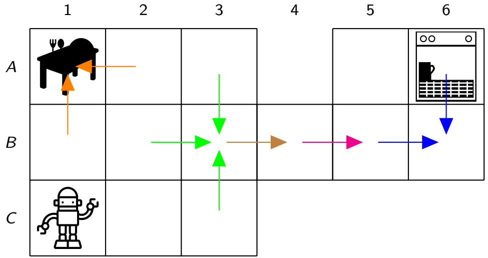


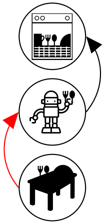


- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}

Disjunctive Action Landmarks: Example

Which set of actions is such that at least one must be applied?





- {pickup}
- {load}
- {move-B3-B4}
- {move-B4-B5}
- {move-A6-B6, move-B5-B6}
- {move-A3-B3, move-B2-B3, move-C3-B3}
- {move-B1-A1, move-A2-A1}
- ...

Constraints

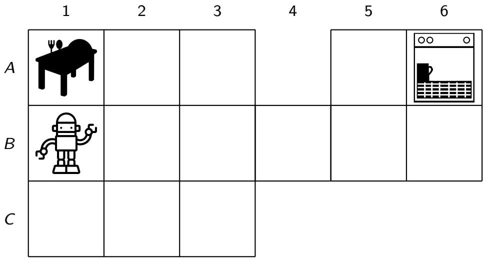
Some heuristics exploit constraints that describe something that holds in every solution of the task.

For instance, every solution is such that

- a variable takes some value in at least one visited state. (a fact landmark constraint)
- at least one action from a set of actions must be applied. (a disjunctive action landmark constraint)
- fact consumption and production is “balanced”. (a network flow constraint)

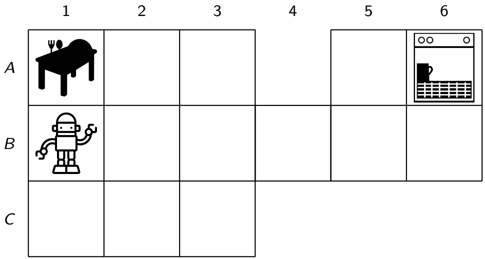
Network Flow: Example

Consider the fact robot-at $= B1$.
How often are actions used that enter this cell?



Network Flow: Example

Consider the fact robot-at $= B1$.
How often are actions used that enter this cell?



Answer: as often as actions that leave this cell

If Count_o denotes how often operator o is applied, we have:

$$\mathrm{Count}_{\text{move-A1-B1}} + \mathrm{Count}_{\text{move-B2-B1}} + \mathrm{Count}_{\text{move-C1-B1}} =$$
$$\mathrm{Count}_{\text{move-B1-A1}} + \mathrm{Count}_{\text{move-B1-B2}} + \mathrm{Count}_{\text{move-B1-C1}}$$

Multiple Heuristics

Combining Admissible Heuristics Admissibly

Major ideas to combine heuristics admissibly:

- maximize
- canoncial heuristic (for abstractions)
- minimum hitting set (for landmarks)
- cost partitioning
- operator counting

Often computed as solution to a (integer) linear program.

Combining Heuristics Admissibly: Example

Example

Consider an FDR planning task $\langle V, I, \{o_1, o_2, o_3, o_4\}, \gamma\rangle$ with $V = \{v_1, v_2, v_3\}$ with $dom(v_1) = \{A, B\}$ and $\mathrm{dom}(v_2) = \mathrm{dom}(v_3) = \{A, B, C\}$, $I = \{v_1 \mapsto A, v_2 \mapsto A, v_3 \mapsto A\}$,

$$\begin{aligned} o_1 &= \langle v_1 = \mathrm{A}, v_1 := \mathrm{B}, 1\rangle \\ o_2 &= \langle v_2 = \mathrm{A} \land v_3 = \mathrm{A}, v_2 := \mathrm{B} \land v_3 := \mathrm{B}, 1\rangle \\ o_3 &= \langle v_2 = \mathrm{B}, v_2 := \mathrm{C}, 1\rangle \\ o_4 &= \langle v_3 = \mathrm{B}, v_3 := \mathrm{C}, 1\rangle \end{aligned}$$

and $\gamma = (v_1 = \mathrm{B}) \land (v_2 = \mathrm{C}) \land (v_3 = \mathrm{C})$.

Let $\mathcal{C}$ be the pattern collection that contains all atomic projections. What is the canonical heuristic function $h^{\mathcal{C}}$?

Combining Heuristics Admissibly: Example

Example

Consider an FDR planning task $\langle V, I, \{o_1, o_2, o_3, o_4\}, \gamma \rangle$ with $V = \{v_1, v_2, v_3\}$ with $dom(v_1) = \{A, B\}$ and $\mathrm{dom}(v_2) = \mathrm{dom}(v_3) = \{A, B, C\}$, $I = \{v_1 \mapsto A, v_2 \mapsto A, v_3 \mapsto A\}$,

$$\begin{aligned} o_1 &= \langle v_1 = \mathrm{A}, v_1 := \mathrm{B}, 1 \rangle \\ o_2 &= \langle v_2 = \mathrm{A} \wedge v_3 = \mathrm{A}, v_2 := \mathrm{B} \wedge v_3 := \mathrm{B}, 1 \rangle \\ o_3 &= \langle v_2 = \mathrm{B}, v_2 := \mathrm{C}, 1 \rangle \\ o_4 &= \langle v_3 = \mathrm{B}, v_3 := \mathrm{C}, 1 \rangle \end{aligned}$$

and $\gamma = (v_1 = \mathrm{B}) \wedge (v_2 = \mathrm{C}) \wedge (v_3 = \mathrm{C})$.

Let $\mathcal{C}$ be the pattern collection that contains all atomic projections. What is the canonical heuristic function $h^{\mathcal{C}}$?

Answer: Let $h_i := h^{v_i}$. Then $h^{\mathcal{C}} = \max\{h_1 + h_2, h_1 + h_3\}$.

Reminder: Orthogonality and Additivity

Why can we add h_1 and h_2 (h_1 and h_3) admissibly?

Theorem (Additivity for Orthogonal Abstractions)

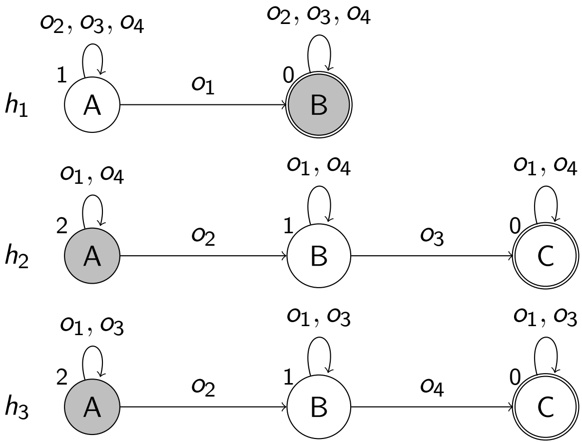
Let $h^{\alpha_1}, \dots, h^{\alpha_n}$ be abstraction heuristics of the same transition system such that α_i and α_j are orthogonal for all $i \neq j$.

Then $\sum_{i=1}^{n} h^{\alpha_i}$ is a safe, goal-aware, admissible and consistent heuristic for Π.

The proof exploits that every concrete transition induces state-changing transition in at most one abstraction.

Combining Heuristics (In)admissibly: Example

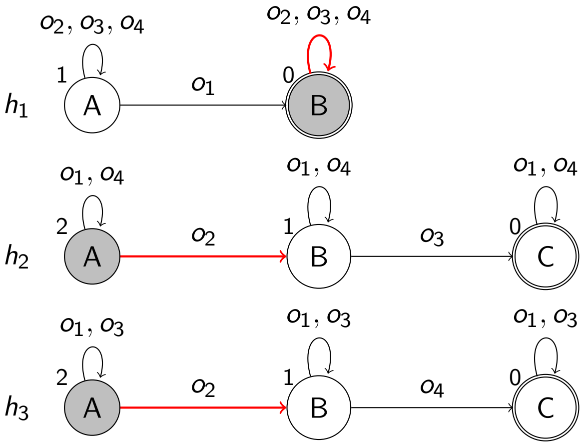
Let $h = h_1 + h_2 + h_3$.



$\langle o_2, o_3, o_4 \rangle$ is a plan for $s = \langle B, A, A \rangle$ but $h(s) = 4$.

Combining Heuristics (In)admissibly: Example

Let $h = h_1 + h_2 + h_3$.



$\langle o_2, o_3, o_4 \rangle$ is a plan for $s = \langle B, A, A \rangle$ but $h(s) = 4$.
Heuristics h_2 and h_3 both account for the single application of o_2.

Prevent Inadmissibility

The reason that h_2 and h_3 are not additive is because the cost of o_2 is considered in both.

Is there anything we can do about this?

Prevent Inadmissibility

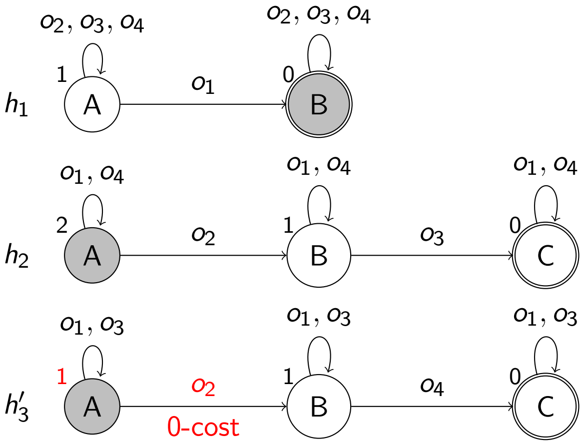
The reason that h_2 and h_3 are not additive is because the cost of o_2 is considered in both.

Is there anything we can do about this?

Solution: We can ignore the cost of o_2 in one heuristic by setting its cost to 0 (e.g., $cost_3(o_2) = 0$).

Combining Heuristics Admissibly: Example

Let $h' = h_1 + h_2 + h'_3$, where $h'_3 = h^{v_3}$ assuming $cost_3(o_2) = 0$.



$\langle o_2, o_3, o_4 \rangle$ is an optimal plan for $s = \langle B, A, A \rangle$ and $h'(s) = 3$ an admissible estimate.

Cost partitioning

Using the cost of every operator only in one heuristic is called a zero-one cost partitioning.

Cost partitioning

Using the cost of every operator only in one heuristic is called a zero-one cost partitioning.

More generally, heuristics are additive if all operator costs are distributed in a way that the sum of the individual costs is no larger than the cost of the operator.

This can also be expressed as a constraint,
the cost partitioning constraint:

$$\sum_{i=1}^{n} cost_i(o) \leq cost(o) \text{ for all } o \in O$$

(more details later)

Summary

Summary

- Landmarks and network flows are constraints that describe something that holds in every solution of the task.
- Heuristics can be combined admissibly if the cost partitioning constraint is satisfied.