Planning and Optimization

E12. Merge-and-Shrink: Merge Strategies & Outlook

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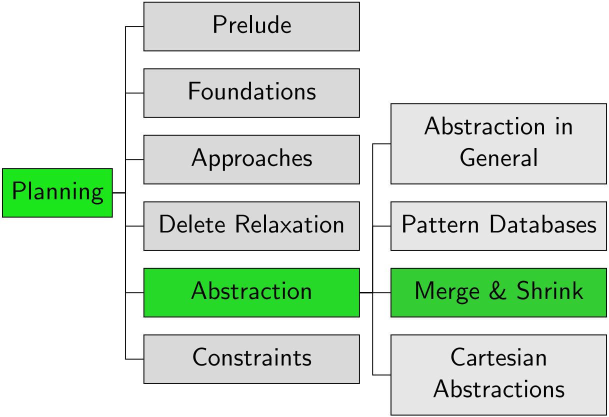
November 19, 2025

Content of the Course



Merge Strategies

Reminder: Generic Algorithm Template

Generic Merge & Shrink Algorithm for planning task Π

$F := F(\Pi)$
while $|F| > 1$:
  select $type \in \{\text{merge}, \text{shrink}\}$
  if $type = \text{merge}$:
    select $\mathcal{T}_1, \mathcal{T}_2 \in F$
    $F := (F \setminus \{\mathcal{T}_1, \mathcal{T}_2\}) \cup \{\mathcal{T}_1 \otimes \mathcal{T}_2\}$
  if $type = \text{shrink}$:
    select $\mathcal{T} \in F$
    choose an abstraction mapping β on $\mathcal{T}$
    $F := (F \setminus \{\mathcal{T}\}) \cup \{\mathcal{T}^{\beta}\}$
return the remaining factor $\mathcal{T}^{\alpha}$ in F

Remaining Question:

- Which abstractions to select for merging? $\rightsquigarrow$ merge strategy

Linear vs. Non-linear Merge Strategies

Linear Merge Strategy

In each iteration after the first, choose the abstraction computed in the previous iteration as $\mathcal{T}_1$.

Rationale: only maintains one “complex” abstraction at a time

- Fully defined by an ordering of atomic projections/variables.
- Each merge-and-shrink heuristic computed with a non-linear merge strategy can also be computed with a linear merge strategy.
- However, linear merging can require a super-polynomial blow-up of the final representation size.
- Recent research turned from linear to non-linear strategies, also because better label reduction techniques (later in this chapter) enabled a more efficient computation.

Classes of Merge Strategies

We can distinguish two major types of merge strategies:

- precomputed merge strategies fix a unique merge order up-front.
  One-time effort but cannot react to other transformations applied to the factors.
- stateless merge strategies only consider the current FTS and decide what factors to merge.
  Typically computing a score for each pair of factors and naturally non-linear; easy to implement but cannot capture dependencies between more than two factors.

Hybrid strategies combine ideas from precomputed and stateless strategies.

Example Linear Precomputed Merge Strategy

Idea: Use similar causal graph criteria as for growing patterns.

Example: Strategy of h_{HHH}

h_{HHH}: Ordering of atomic projections

- Start with a goal variable.
- Add variables that appear in preconditions of operators affecting previous variables.
- If that is not possible, add a goal variable.

Rationale: increases h quickly

Example Non-linear Precomputed Merge Strategy

Idea: Build clusters of variables with strong interactions and first merge variables within each cluster.

Example: MIASM ("maximum intermediate abstraction size minimizing merging strategy")

MIASM strategy

- Measure interaction by ratio of unnecessary states in the merged system (= states not traversed by any abstract plan).
- Best-first search to identify interesting variable sets.
- Disjoint variable sets chosen by a greedy algorithm for maximum weighted set packing.

Rationale: increase power of pruning (later in this chapter)

Example Non-linear Stateless Merge Strategy

Idea: Preferrably merge transition systems that must synchronize on labels that occur close to a goal state.

Example: DFP (named after Dräger, Finkbeiner and Podelski)

DFP strategy

- $\mathit{labelrank}(\ell, \mathcal{T}) = \min\{h^*(t) \mid \langle s, \ell, t\rangle \text{ transition in } \mathcal{T}\}$
- $\mathit{score}(\mathcal{T}, \mathcal{T}') = \min\{\max\{\mathit{labelrank}(\ell, \mathcal{T}), \mathit{labelrank}(\ell, \mathcal{T}')\} \mid \ell \text{ label in } \mathcal{T} \text{ and } \mathcal{T}'\}$
- Select two transition systems with minimum score.

Rationale: abstraction fine-grained in the goal region, which is likely to be searched by A^*.

Example Hybrid Merge Strategy

Idea: first combine the variables within each strongly connected component of the causal graph.

Example: SCC framework

SCC strategy

- Compute strongly connected components of causal graph
- Secondary strategies for order in which
  - the SCCs are considered (e.g. topologic order),
  - the factors within an SCC are merged, and
  - the resulting product systems are merged.

Rationale: reflect strong interactions of variables well

State of the art: SCC+DFP or a stateless MIASM variant

Outlook: Label Reduction and Pruning

Further Transformations

State-of-the-art Merge & Shrink uses two further transformations:

- Label reduction
- Pruning

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- A label reduction $\langle \lambda, c' \rangle$ for a FTS F with label set L is given by a function $\lambda : L \to L'$, where L' is an arbitrary set of labels, and a label cost function c' on L' such that for all $\ell \in L$, $c'(\lambda(\ell)) \leq c(\ell)$.
  The label-reduced TSs have L' and c' for the labels and cost, and in each transition the original label ℓ is replaced with $\lambda(\ell)$.

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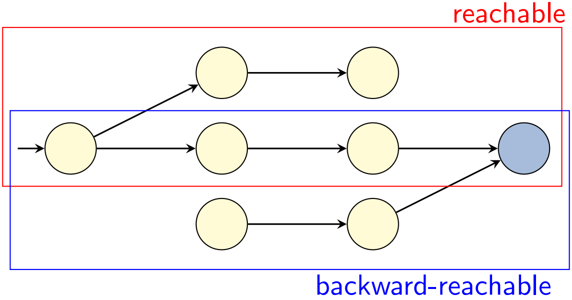
- Label reduction is a conservative transformation.
- There are also clear criteria when label reduction is exact.
- Reduces the time and memory requirement for merge and shrink steps and enables coarser bisimulation abstractions.

Alive States



- state s is reachable if we can reach it from the initial state
- state s is backward-reachable if we can reach the goal from s
- state s is alive if it is reachable and backward-reachable
  $\rightarrow$ only alive states can be traversed by a solution
- a state s is dead if it is not alive.

Pruning States (1)

- If in a factor, state s is dead/not backward-reachable then all states that “cover” s in a synchronized product are dead/not backward-reachable in the synchronized product.
- Removing such states and all adjacent transitions in a factor does not remove any solutions from the synchronized product.
- This pruning leads to states in the original state space for which the merge-and-shrink abstraction does not define an abstract state.
  $\rightarrow$ use heuristic estimate ∞

Pruning States (2)

- Keeping exactly all backward-reachable states we still obtain safe, consistent, goal-aware and admissible (with conservative transformations) or perfect heuristics (with exact transformations).
- Pruning unreachable, backward-reachable states can render the heuristic unsafe because pruned states lead to infinite estimates.
- However, all reachable states in the original state space will have admissible estimates, so we can use the heuristic like an admissible one in a forward state-space search such as A^*(but not in other contexts like such as orbit search).
  We usually prune all dead states to keep the factors small.

Summary

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- There is a wide range of merge strategies. We only covered some important ones.
- Label reduction is crucial for the performance of the merge-and-shrink algorithm, especially when using bisimilarity for shrinking.
- Pruning is used to keep the size of the factors small. It depends on the intended application how aggressive the pruning can be.

Literature

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