

Planning and Optimization

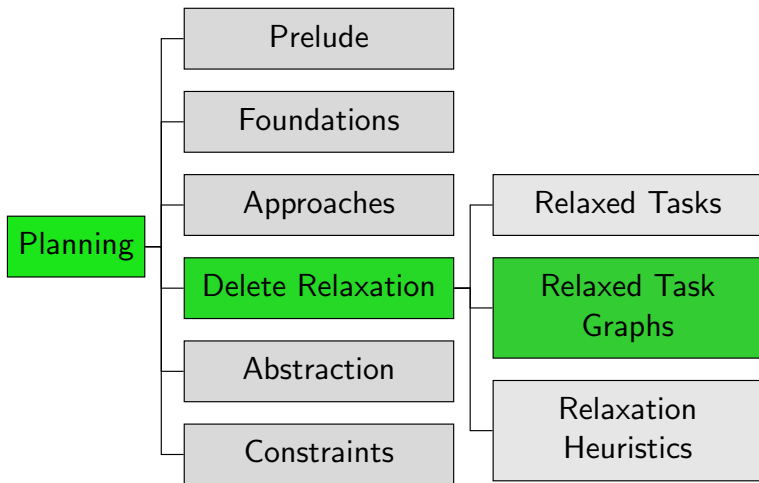
D5. Delete Relaxation: Relaxed Task Graphs

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Content of the Course



Relaxed Task Graphs

Relaxed Task Graphs

Let Π^+ be a relaxed planning task.

The **relaxed task graph** of Π^+ , in symbols $RTG(\Pi^+)$, is an AND/OR graph that encodes

- **which state variables** can become true in an applicable operator sequence for Π^+ ,
- **which operators** of Π^+ can be included in an applicable operator sequence for Π^+ ,
- if the **goal** of Π^+ can be reached,
- and **how** these things can be achieved.

We present its definition in stages.

Note: Throughout this chapter, we assume flat operators.

Running Example

As a running example, consider the relaxed planning task $\langle V, I, \{o_1, o_2, o_3, o_4\}, \gamma \rangle$ with

$$V = \{a, b, c, d, e, f, g, h\}$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{T}, \\ e \mapsto \mathbf{F}, f \mapsto \mathbf{F}, g \mapsto \mathbf{F}, h \mapsto \mathbf{F}\}$$

$$o_1 = \langle c \vee (a \wedge b), c \wedge ((c \wedge d) \triangleright e), 1 \rangle$$

$$o_2 = \langle \top, f, 2 \rangle$$

$$o_3 = \langle f, g, 1 \rangle$$

$$o_4 = \langle f, h, 1 \rangle$$

$$\gamma = e \wedge (g \wedge h)$$

Construction

Components of Relaxed Task Graphs

A relaxed task graph has four kinds of components:

- **Variable nodes** represent the state variables.
- The **initial node** represent the initial state.
- **Operator subgraphs** represent the preconditions and effects of operators.
- The **goal subgraph** represents the goal.

The idea is to construct the graph in such a way that all nodes representing **reachable** aspects of the task are **forced true**.

Variable Nodes

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task.

- For each $v \in V$, $RTG(\Pi^+)$ contains an OR node n_v .
These nodes are called **variable nodes**.

Variable Nodes: Example

$$V = \{a, b, c, d, e, f, g, h\}$$



a

b

c

d

e

f

g

h

Initial Node

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task.

- $RTG(\Pi^+)$ contains an AND node n_I .
This node is called the **initial node**.
- For all $v \in V$ with $I(v) = \mathbf{T}$, $RTG(\Pi^+)$ has an arc from n_v to n_I . These arcs are called **initial state arcs**.
- The initial node has no successor nodes.

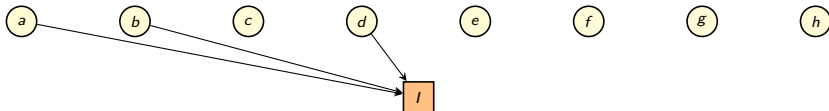
Initial Node and Initial State Arcs: Example

$$V = \{a, b, c, d, e, f, g, h\}$$

A yellow circle with a black border containing the letter 'a'.A yellow circle with a black border containing the letter 'b'.A yellow circle with a black border containing the letter 'c'.A yellow circle with a black border containing the letter 'd'.A yellow circle with a black border containing the letter 'e'.A yellow circle with a black border containing the letter 'f'.A yellow circle with a black border containing the letter 'g'.A yellow circle with a black border containing the letter 'h'.

Initial Node and Initial State Arcs: Example

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{T}, e \mapsto \mathbf{F}, f \mapsto \mathbf{F}, g \mapsto \mathbf{F}, h \mapsto \mathbf{F}\}$$



Operator Subgraphs

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task.

For each operator $o^+ \in O^+$, $RTG(\Pi^+)$ contains an **operator subgraph** with the following parts:

- for each formula φ that occurs as a subformula of the precondition or of some effect condition of o^+ , a **formula node** n_φ (details follow)
- for each conditional effect $(\chi \triangleright v)$ that occurs in the effect of o^+ , an **effect node** $n_{o^+}^\chi$ (details follow); unconditional effects are treated as $(\top \triangleright v)$

Formula Nodes

Formula nodes n_φ are defined as follows:

- If $\varphi = v$ for some state variable v , n_φ is the variable node n_v (so no new node is introduced).
- If $\varphi = \top$, n_φ is an AND node without outgoing arcs.
- If $\varphi = \perp$, n_φ is an OR node without outgoing arcs.
- If $\varphi = (\varphi_1 \wedge \varphi_2)$, n_φ is an AND node with outgoing arcs to n_{φ_1} and n_{φ_2} .
- If $\varphi = (\varphi_1 \vee \varphi_2)$, n_φ is an OR node with outgoing arcs to n_{φ_1} and n_{φ_2} .

Note: identically named nodes are identical,
so if the same formula occurs multiple times in the task,
the **same** node is reused.

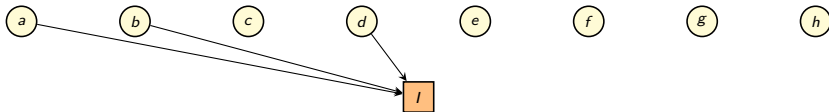
Effect Nodes

Effect nodes n_{o+}^{χ} are defined as follows:

- n_{o+}^{χ} is an AND node
- It has an outgoing arc to the formula nodes $n_{pre(o+)}^{\chi}$ (**precondition arcs**) and n_{χ}^{χ} (**effect condition arcs**).
- Exception: if $\chi = \top$, there is no effect condition arc. (This makes our pictures cleaner.)
- For every conditional effect $(\chi \triangleright v)$ in the operator, there is an arc from variable node n_v to n_{o+}^{χ} (**effect arcs**).

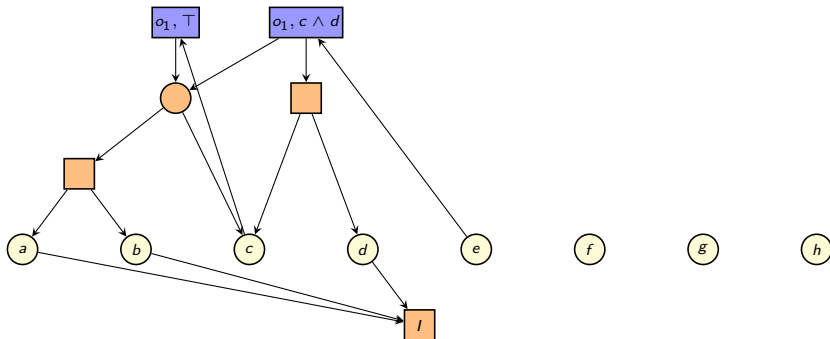
Note: identically named nodes are identical,
so if the same effect condition occurs multiple times
in the same operator, this only induces **one** node.

Operator Subgraphs: Example



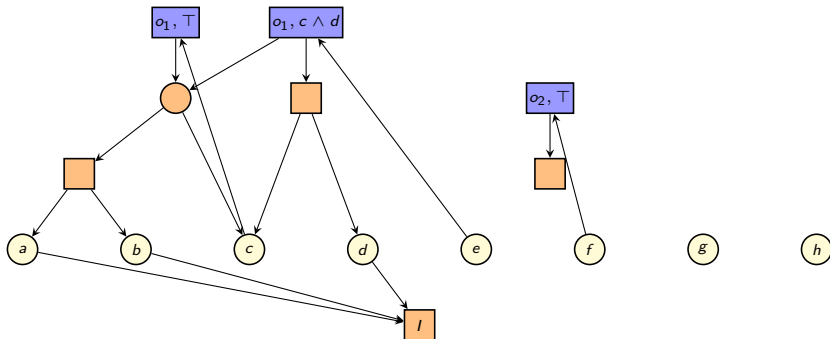
Operator Subgraphs: Example

$$o_1 = \langle c \vee (a \wedge b), c \wedge ((c \wedge d) \triangleright e), 1 \rangle$$



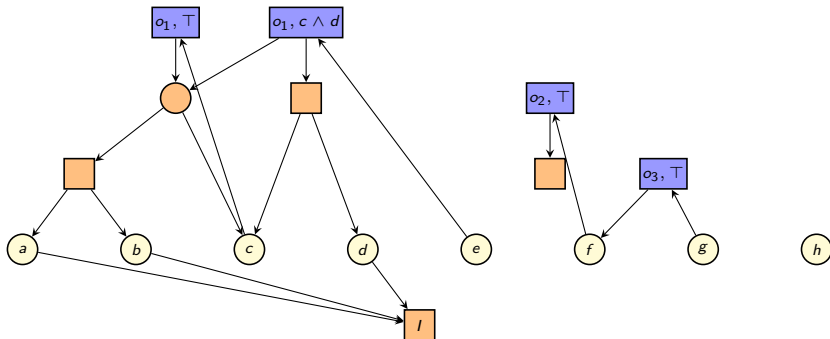
Operator Subgraphs: Example

$$o_2 = \langle \top, f, 2 \rangle$$



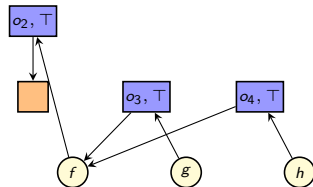
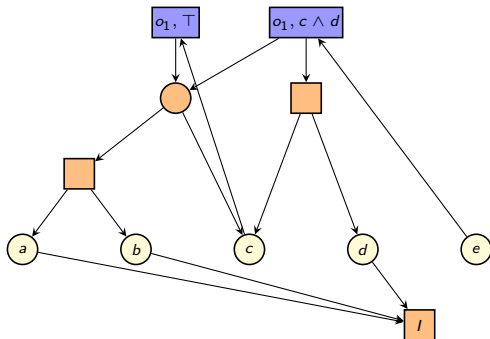
Operator Subgraphs: Example

$$o_3 = \langle f, g, 1 \rangle$$



Operator Subgraphs: Example

$$o_4 = \langle f, h, 1 \rangle$$

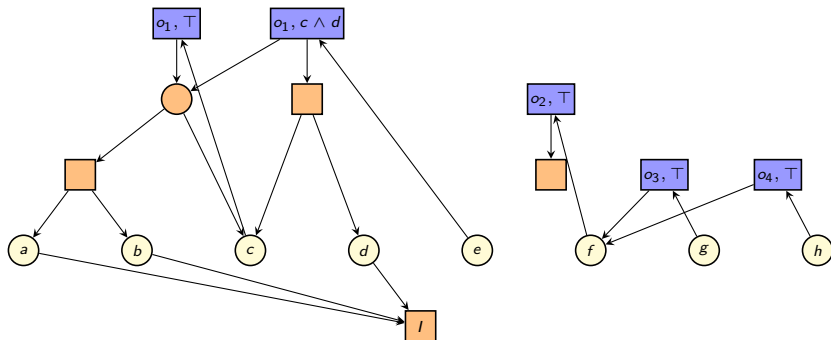


Goal Subgraph

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task.

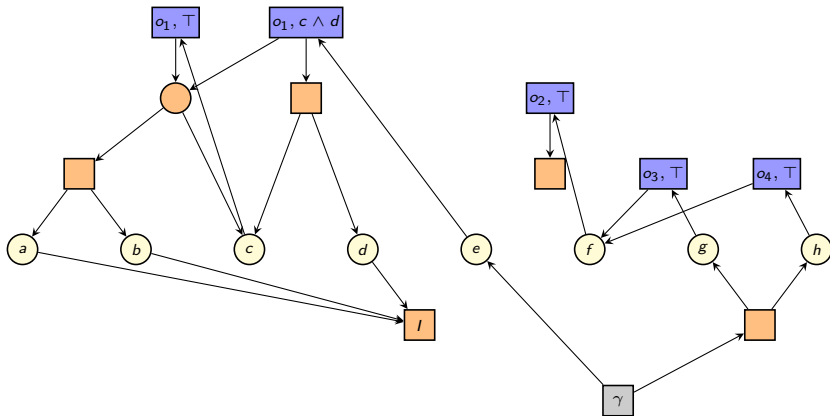
$RTG(\Pi^+)$ contains a **goal subgraph**, consisting of formula nodes for the goal γ and its subformulas, constructed in the same way as formula nodes for preconditions and effect conditions.

Goal Subgraph and Final Relaxed Task Graph: Example



Goal Subgraph and Final Relaxed Task Graph: Example

$$\gamma = e \wedge (g \wedge h)$$



Reachability Analysis

How Can We Use Relaxed Task Graphs?

- We are now done with the definition of relaxed task graphs.
- Now we want to **use** them to derive information about planning tasks.
- In the following chapter, we will use them to compute heuristics for delete-relaxed planning tasks.
- Here, we start with something simpler: **reachability analysis**.

Forced True Nodes and Reachability

Theorem (Forced True Nodes vs. Reachability)

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task, and let N_T be the forced true nodes of $RTG(\Pi^+)$.

For all **formulas** over state variables φ that occur in the definition of Π^+ :

φ is true in some reachable state of Π^+ iff $n_\varphi \in N_T$.

(We omit the proof.)

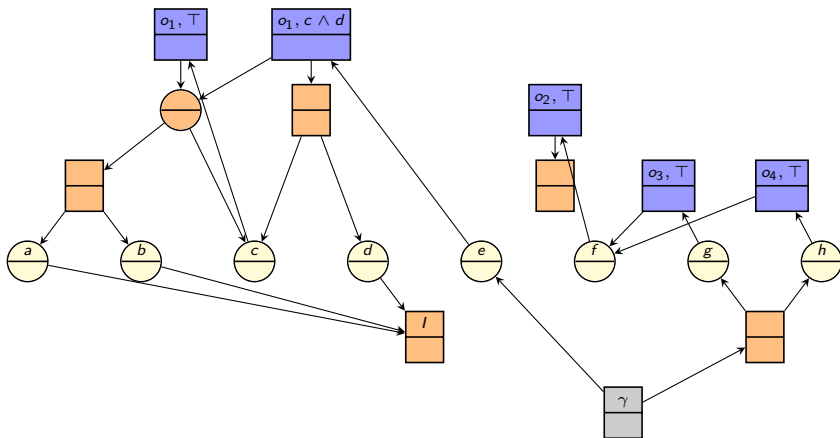
Forced True Nodes and Reachability: Consequences

Corollary

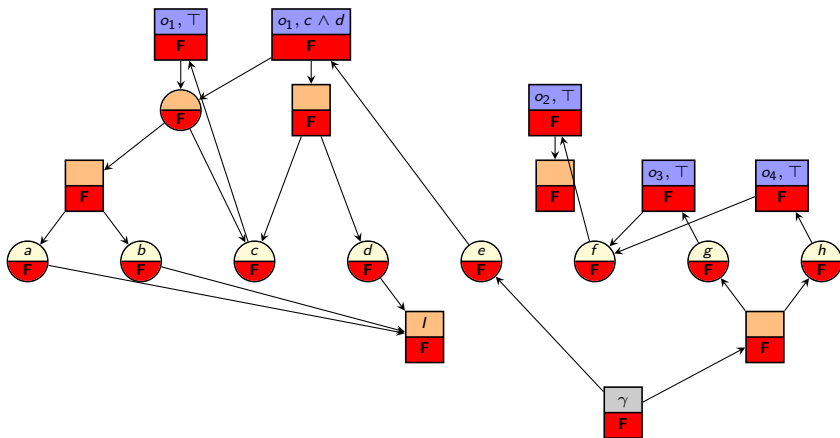
Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task, and let $N_{\mathbf{T}}$ be the forced true nodes of $\text{RTG}(\Pi^+)$. Then:

- A **state variable** $v \in V$ is true in at least one reachable state iff $n_v \in N_{\mathbf{T}}$.
- An **operator** $o^+ \in O^+$ is part of at least one applicable operator sequence iff $n_{\text{pre}(o^+)} \in N_{\mathbf{T}}$.
- The relaxed task is **solvable** iff $n_{\gamma} \in N_{\mathbf{T}}$.

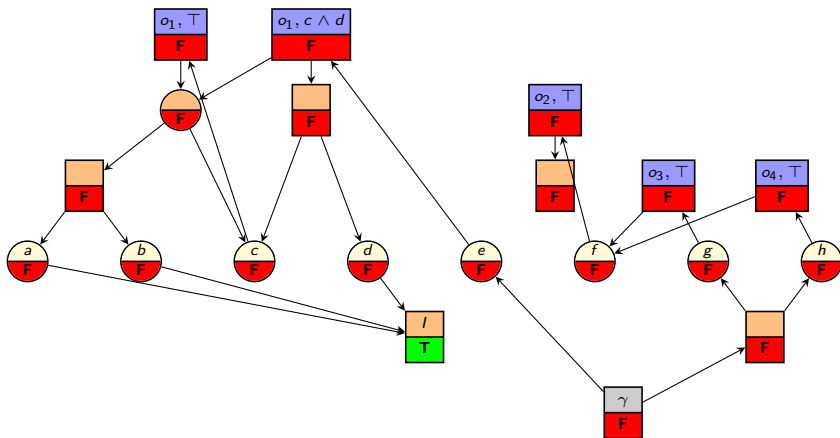
Reachability Analysis: Example



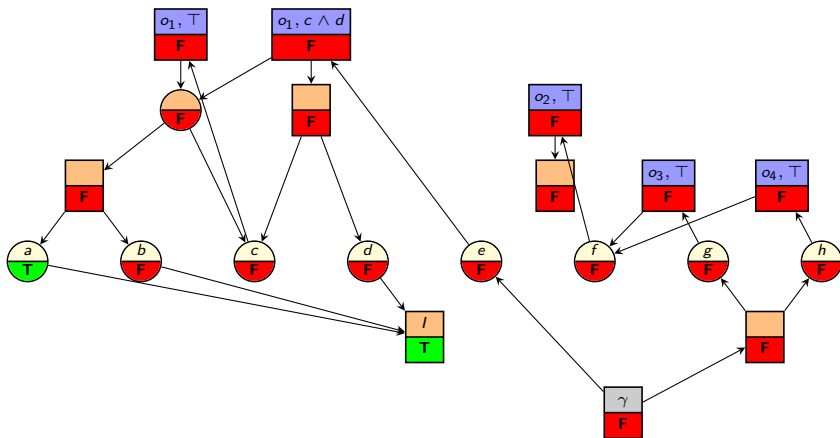
Reachability Analysis: Example



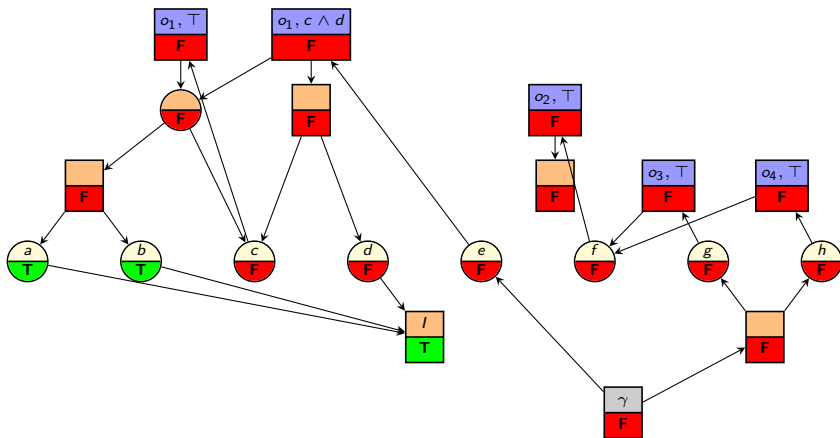
Reachability Analysis: Example



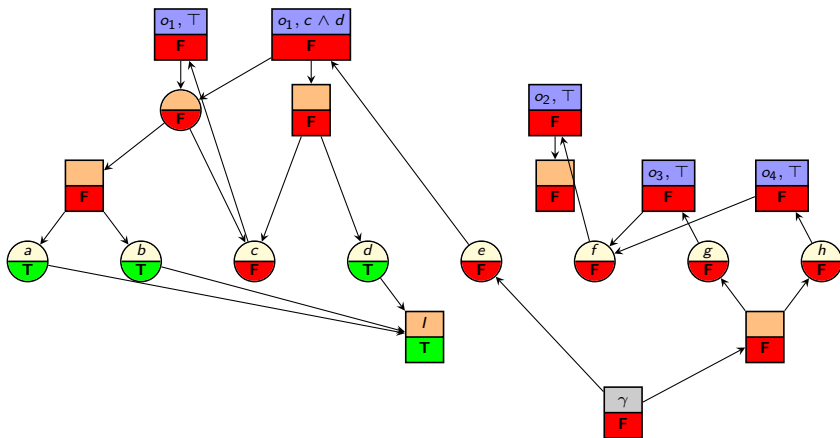
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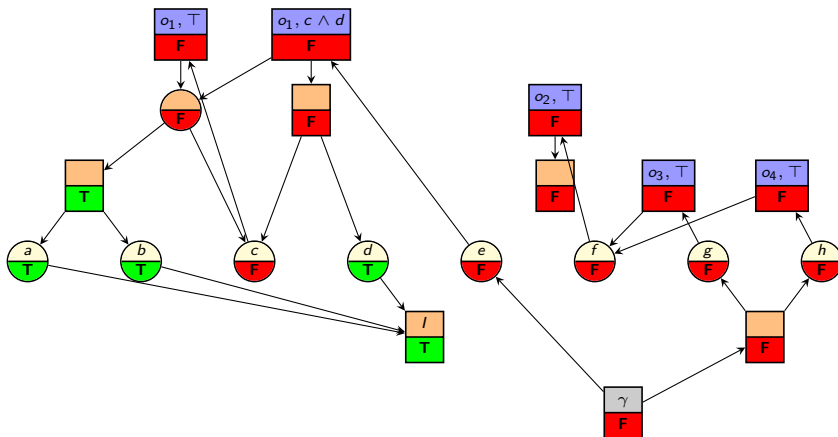
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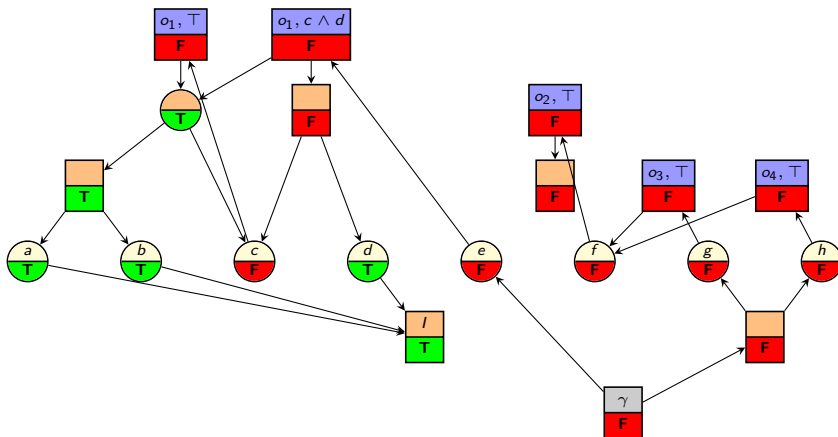
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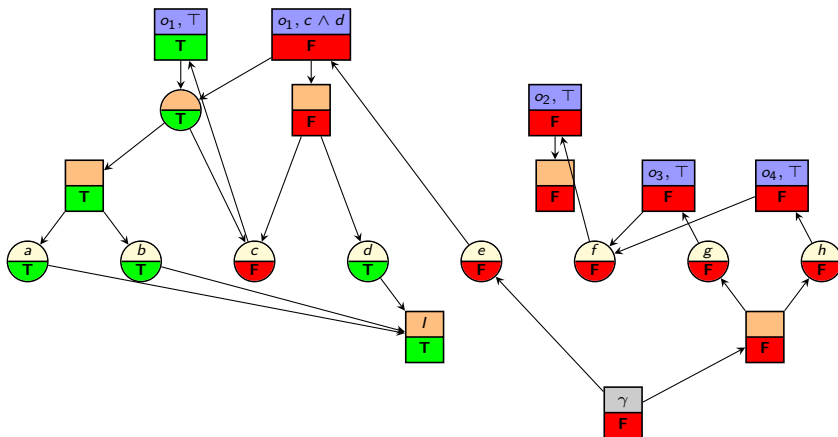
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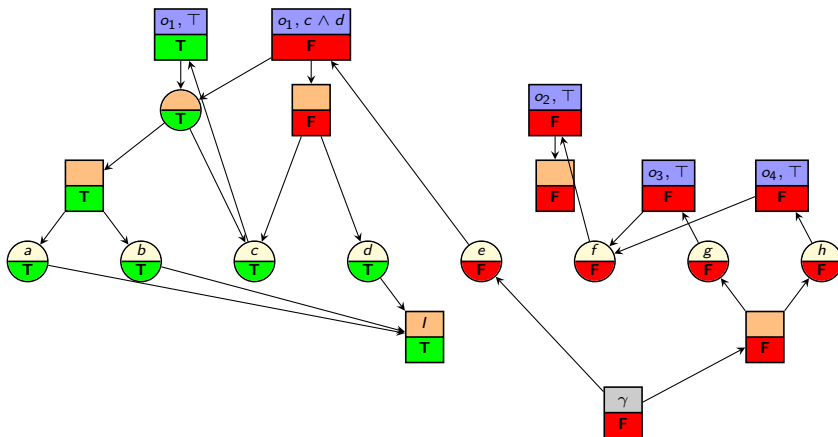
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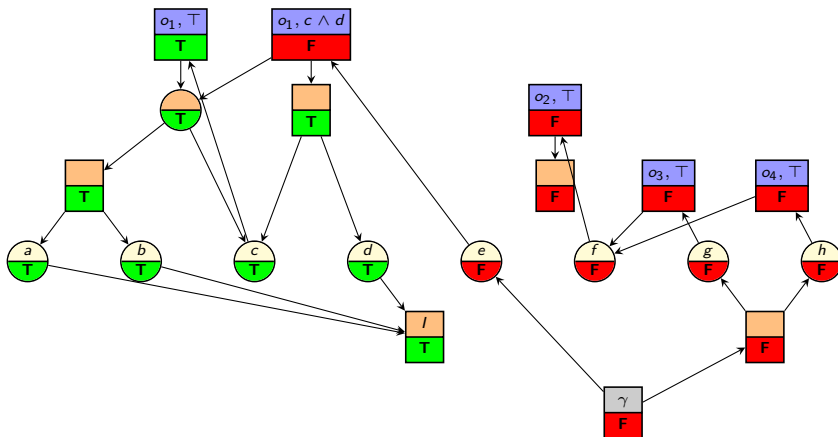
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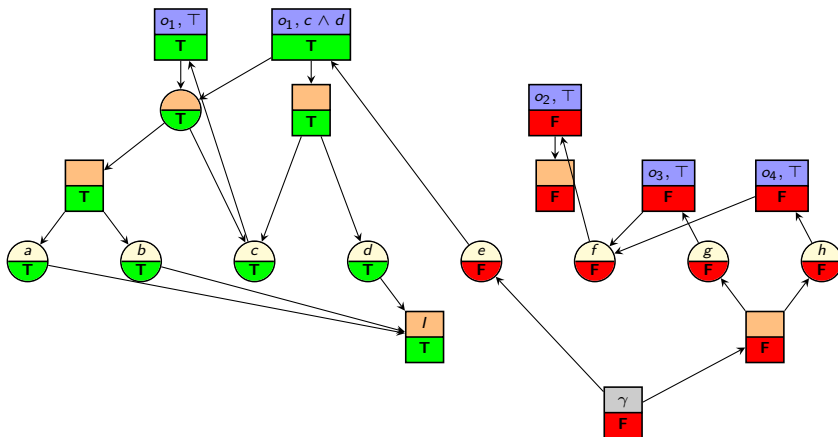
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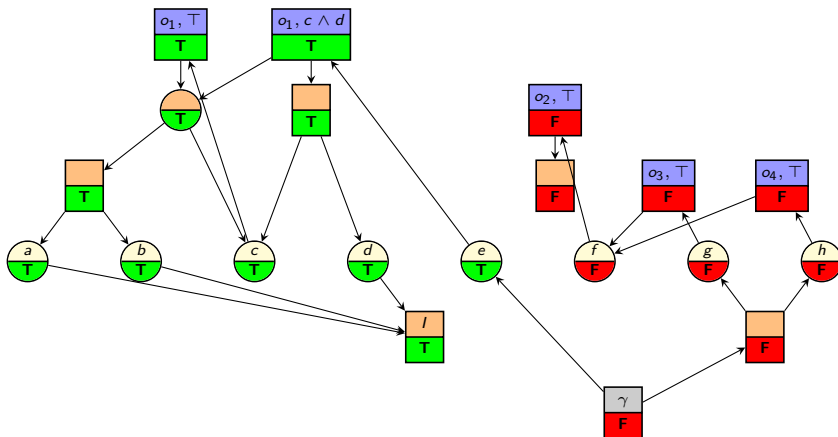
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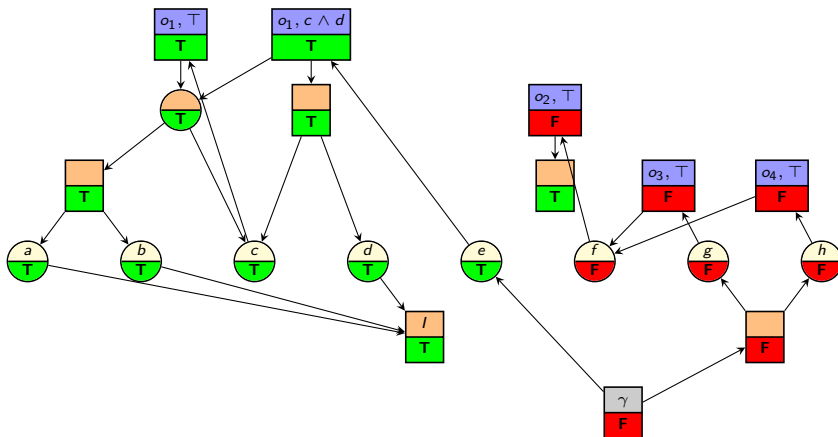
Reachability Analysis: Example



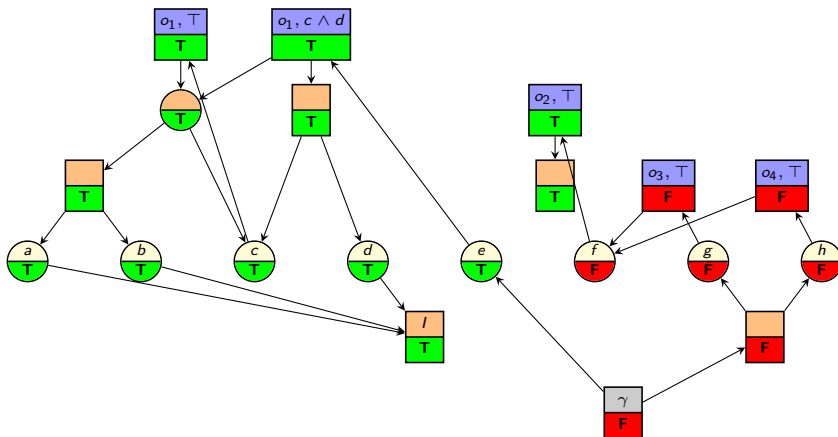
Reachability Analysis: Example



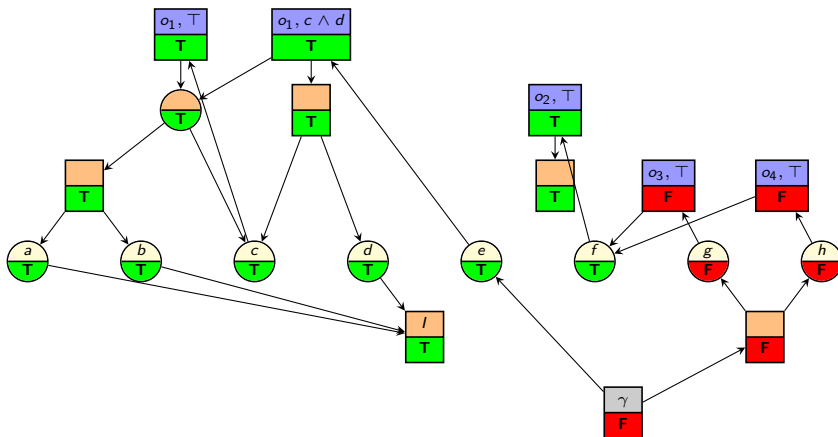
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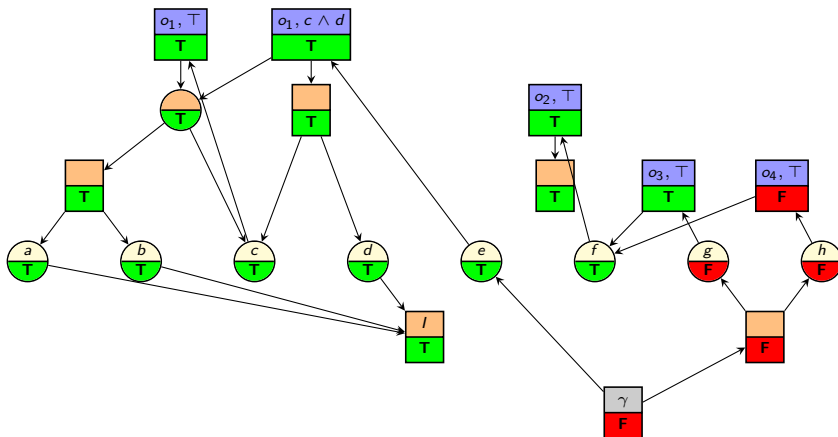
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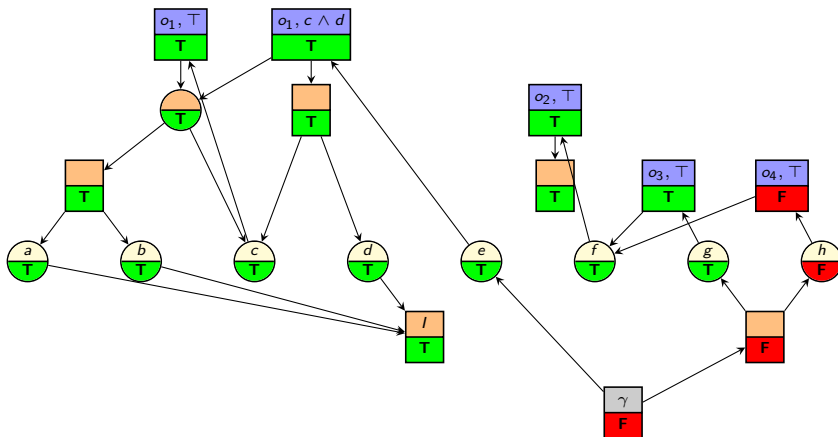
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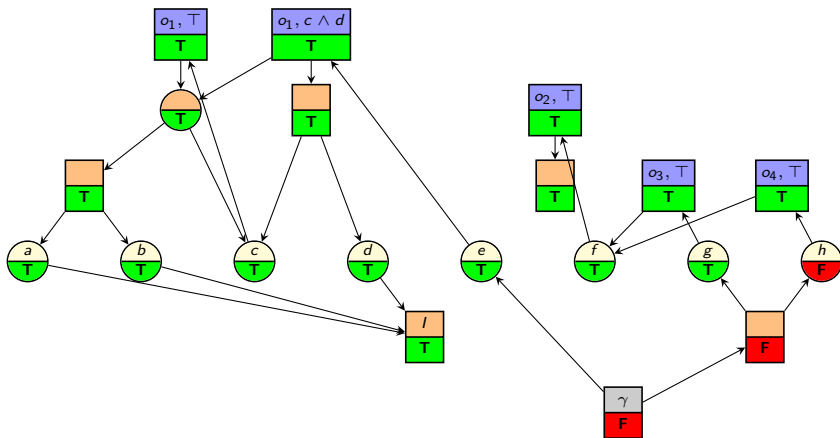
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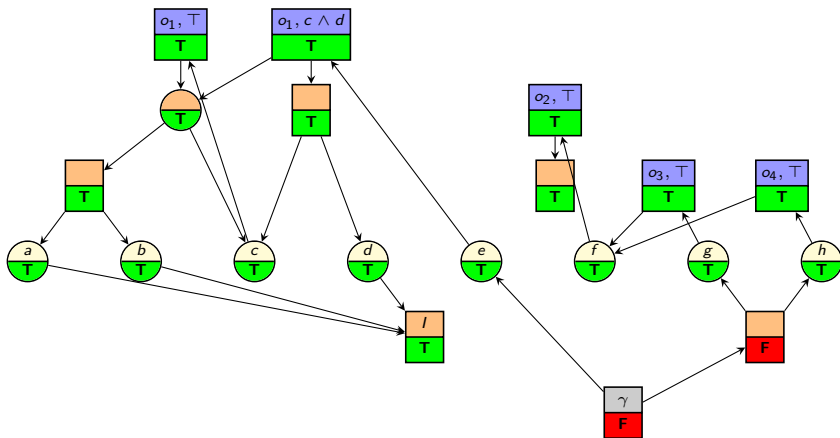
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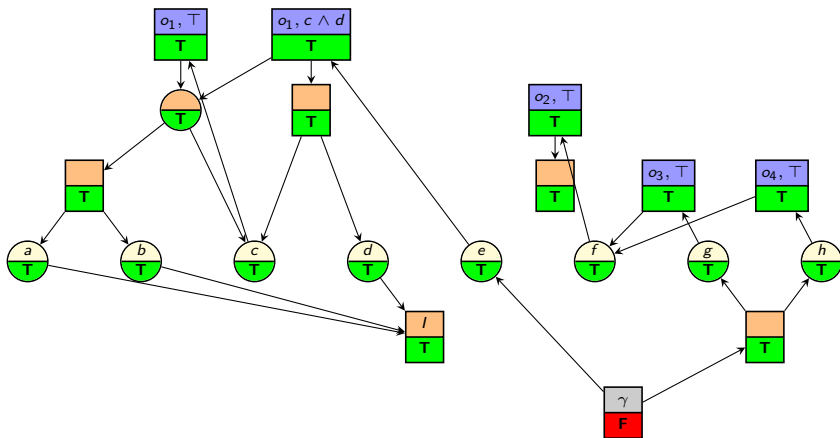
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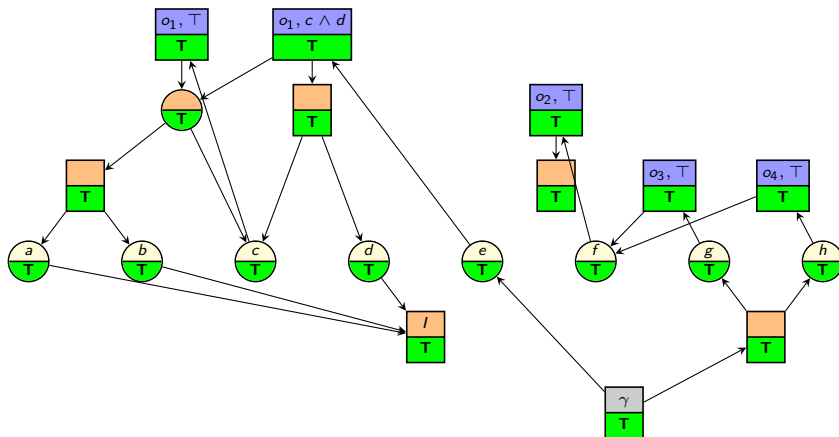
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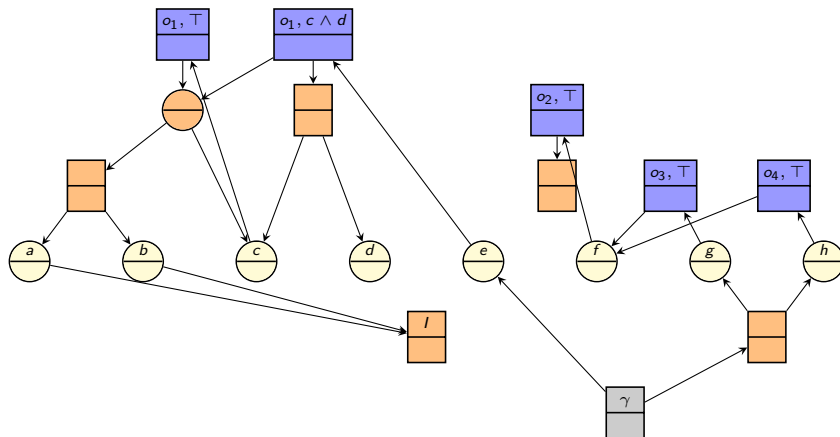
Reachability Analysis: Example



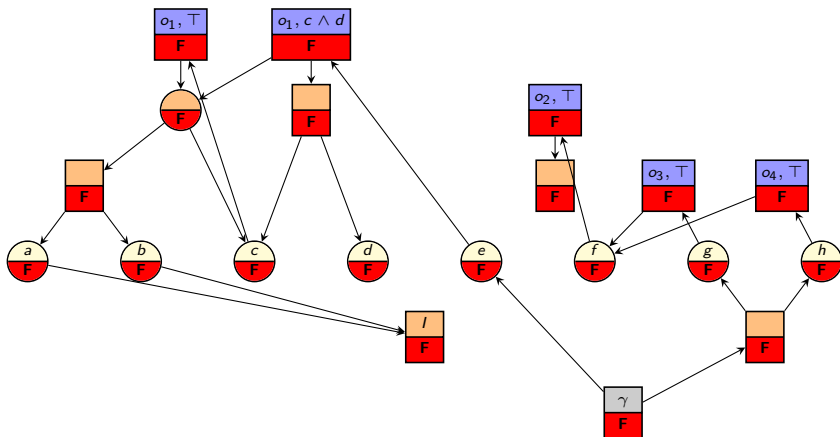
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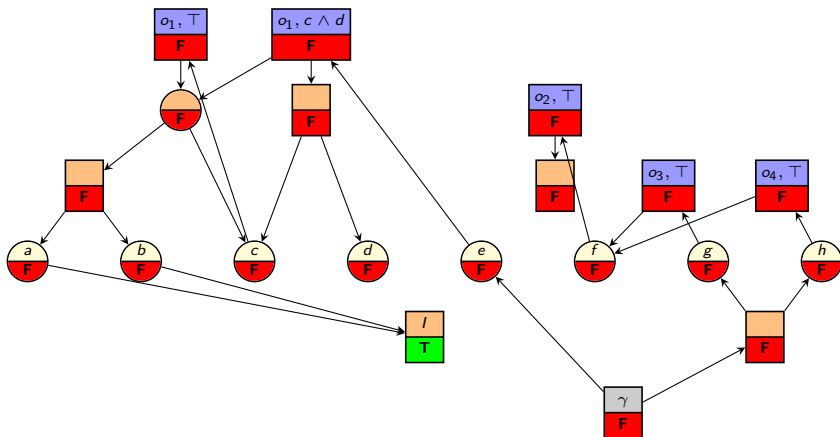
Reachability Analysis: Example with Different Initial State



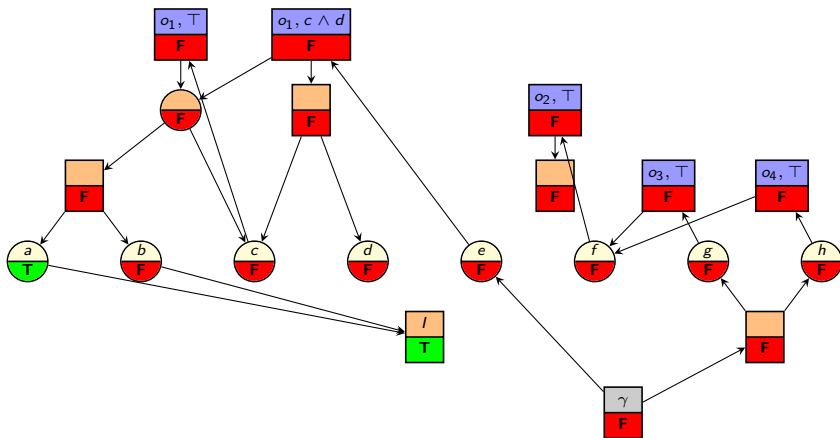
Reachability Analysis: Example with Different Initial State



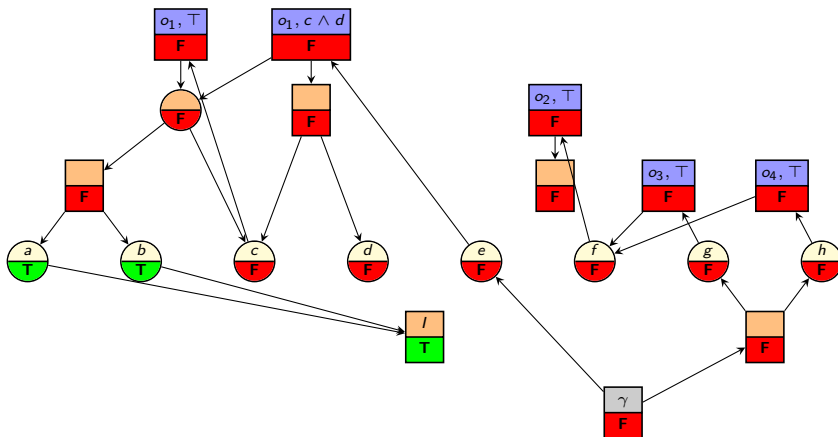
Reachability Analysis: Example with Different Initial State



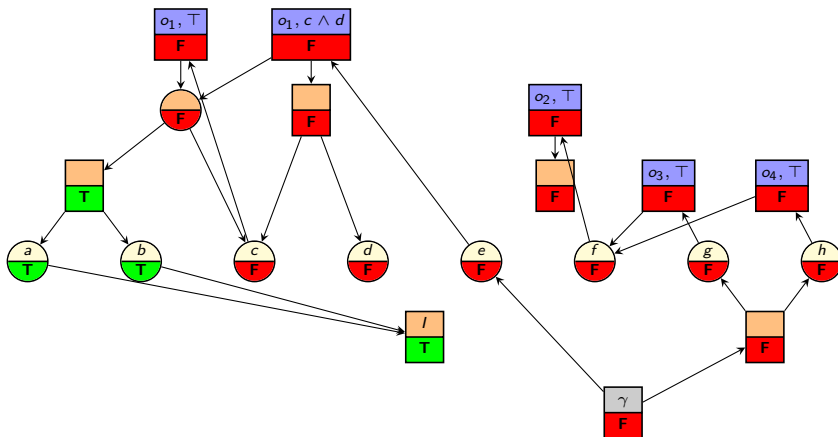
Reachability Analysis: Example with Different Initial State



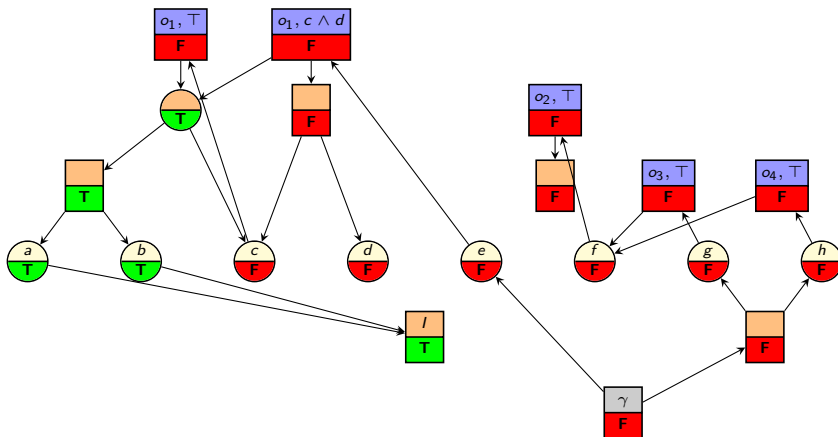
Reachability Analysis: Example with Different Initial State



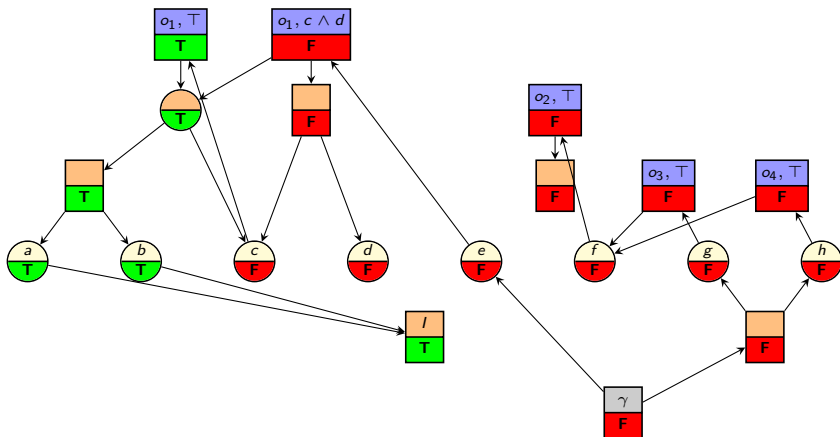
Reachability Analysis: Example with Different Initial State



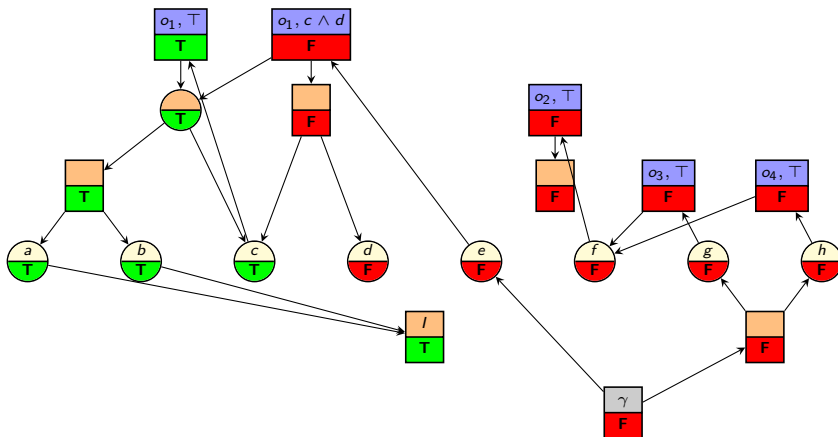
Reachability Analysis: Example with Different Initial State



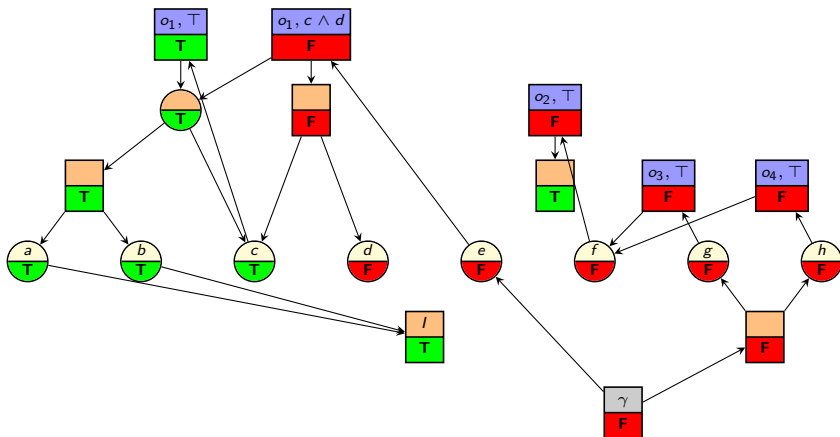
Reachability Analysis: Example with Different Initial State



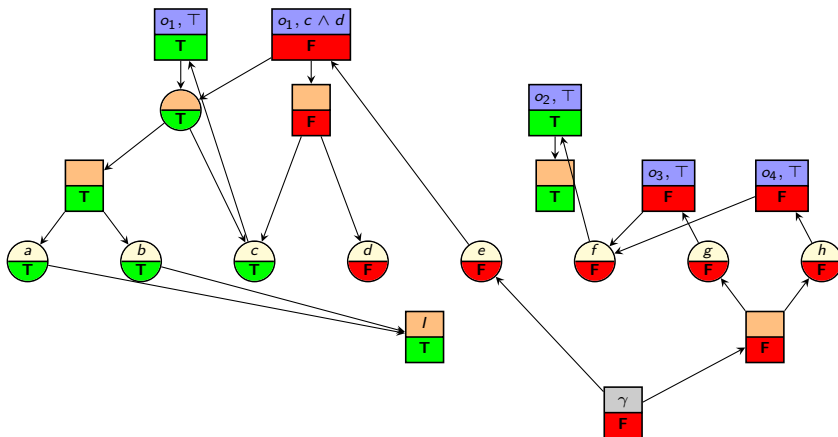
Reachability Analysis: Example with Different Initial State



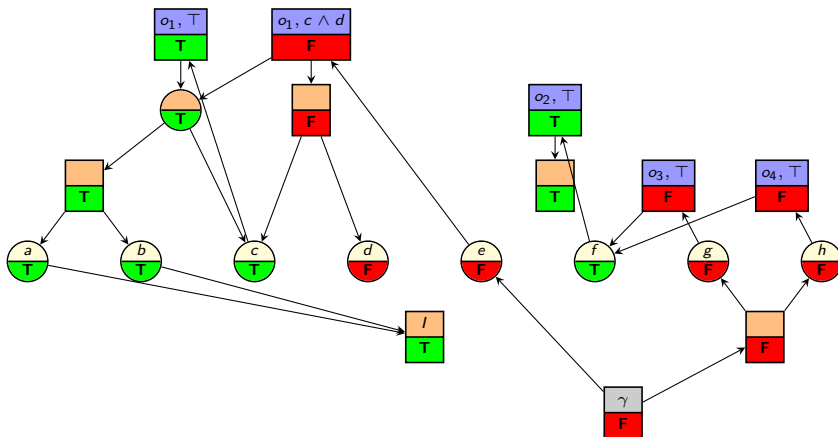
Reachability Analysis: Example with Different Initial State



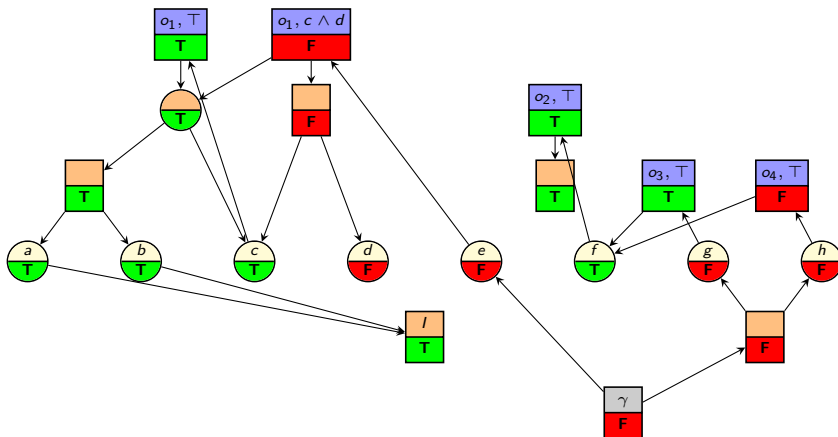
Reachability Analysis: Example with Different Initial State



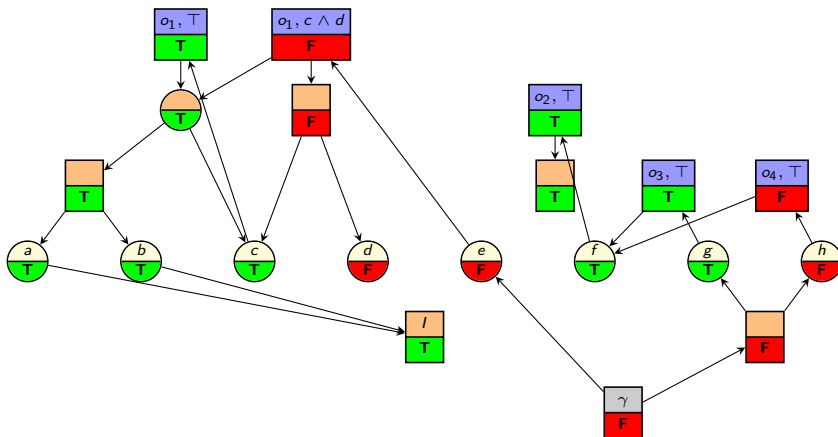
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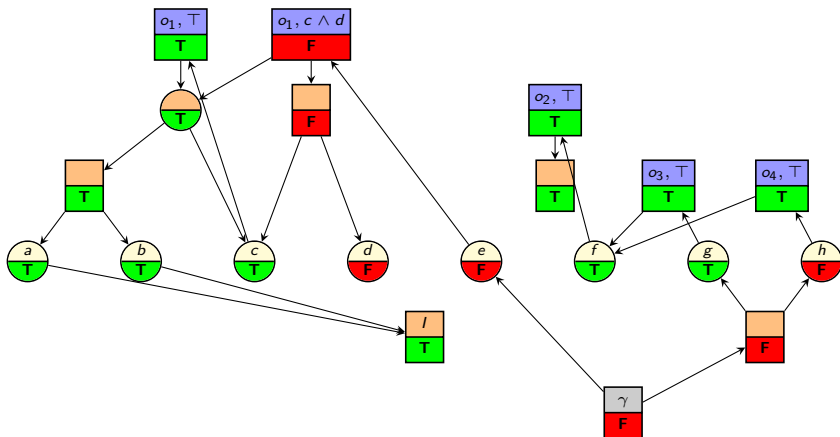
Reachability Analysis: Example with Different Initial State



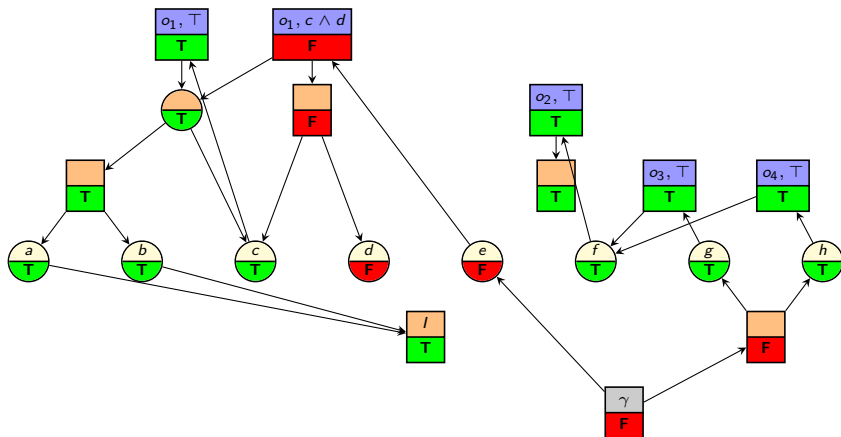
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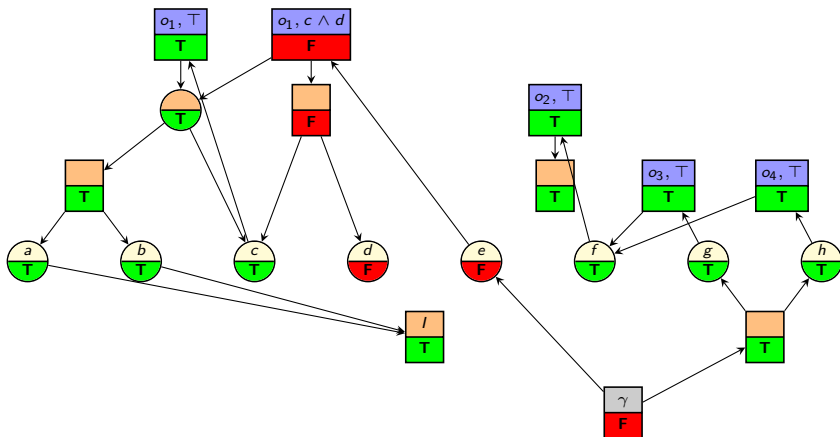
Reachability Analysis: Example with Different Initial State



Reachability Analysis: Example with Different Initial State



Reachability Analysis: Example with Different Initial State



Remarks

Relaxed Task Graphs in the Literature

Some remarks on the planning literature:

- Usually, only the **STRIPS** case is studied.
- ↪ definitions simpler: only **variable nodes** and **operator nodes**, no formula nodes or effect nodes
- Usually, so-called **relaxed planning graphs** (RPGs) are studied instead of RTGs.
- These are **temporally unrolled** versions of RTGs, i.e., they have multiple layers (“time steps”) and are acyclic.
- ↪ Foundations of Artificial Intelligence course FS 2025, Ch. F3–F4

Summary

Summary

- **Relaxed task graphs** (RTGs) represent (most of) the information of a relaxed planning task as an AND/OR graph.
- They consist of:
 - **variable nodes**
 - **an initial node**
 - **operator subgraphs** including **formula nodes** and **effect nodes**
 - a **goal subgraph** including **formula nodes**
- RTGs can be used to analyze **reachability** in relaxed tasks:
forced true nodes mean “reachable”,
other nodes mean “unreachable”.