

Planning and Optimization

D5. Delete Relaxation: Relaxed Task Graphs

Malte Helmert and Gabriele Röger

Universität Basel

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D5.1 Relaxed Task Graphs

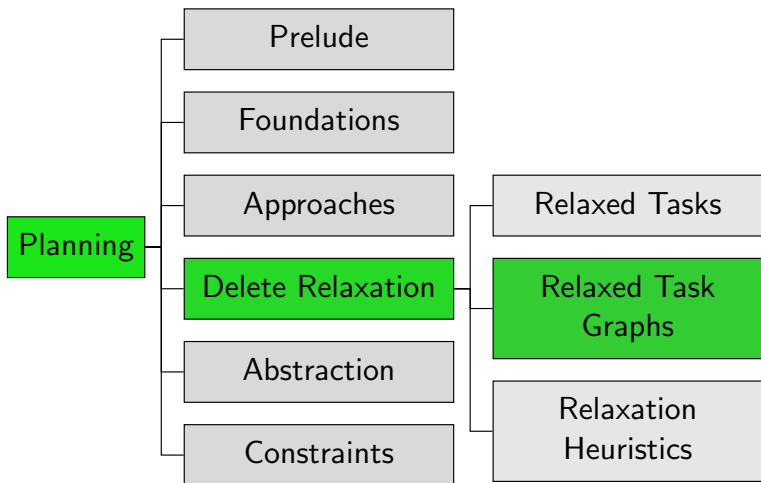
D5.2 Construction

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Content of the Course



D5.1 Relaxed Task Graphs

Relaxed Task Graphs

Let Π^+ be a relaxed planning task.

The **relaxed task graph** of Π^+ , in symbols $RTG(\Pi^+)$, is an AND/OR graph that encodes

- ▶ **which state variables** can become true in an applicable operator sequence for Π^+ ,
- ▶ **which operators** of Π^+ can be included in an applicable operator sequence for Π^+ ,
- ▶ if the **goal** of Π^+ can be reached,
- ▶ and **how** these things can be achieved.

We present its definition in stages.

Note: Throughout this chapter, we assume flat operators.

Running Example

As a running example, consider the relaxed planning task $\langle V, I, \{o_1, o_2, o_3, o_4\}, \gamma \rangle$ with

$$V = \{a, b, c, d, e, f, g, h\}$$

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{T}, \\ e \mapsto \mathbf{F}, f \mapsto \mathbf{F}, g \mapsto \mathbf{F}, h \mapsto \mathbf{F}\}$$

$$o_1 = \langle c \vee (a \wedge b), c \wedge ((c \wedge d) \triangleright e), 1 \rangle$$

$$o_2 = \langle \mathbf{T}, f, 2 \rangle$$

$$o_3 = \langle f, g, 1 \rangle$$

$$o_4 = \langle f, h, 1 \rangle$$

$$\gamma = e \wedge (g \wedge h)$$

D5.2 Construction

Components of Relaxed Task Graphs

A relaxed task graph has four kinds of components:

- ▶ **Variable nodes** represent the state variables.
- ▶ The **initial node** represent the initial state.
- ▶ **Operator subgraphs** represent the preconditions and effects of operators.
- ▶ The **goal subgraph** represents the goal.

The idea is to construct the graph in such a way that all nodes representing **reachable** aspects of the task are **forced true**.

Variable Nodes

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task.

- ▶ For each $v \in V$, $RTG(\Pi^+)$ contains an OR node n_v .
These nodes are called **variable nodes**.

Variable Nodes: Example

$$V = \{a, b, c, d, e, f, g, h\}$$



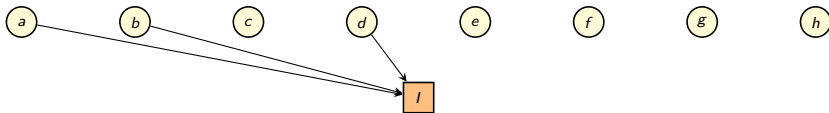
Initial Node

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task.

- ▶ $RTG(\Pi^+)$ contains an AND node n_I .
This node is called the **initial node**.
- ▶ For all $v \in V$ with $I(v) = \mathbf{T}$, $RTG(\Pi^+)$ has an arc from n_v to n_I . These arcs are called **initial state arcs**.
- ▶ The initial node has no successor nodes.

Initial Node and Initial State Arcs: Example

$$I = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{F}, d \mapsto \mathbf{T}, e \mapsto \mathbf{F}, f \mapsto \mathbf{F}, g \mapsto \mathbf{F}, h \mapsto \mathbf{F}\}$$



Operator Subgraphs

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task.

For each operator $o^+ \in O^+$, $RTG(\Pi^+)$ contains an **operator subgraph** with the following parts:

- ▶ for each formula φ that occurs as a subformula of the precondition or of some effect condition of o^+ , a **formula node** n_φ (details follow)
- ▶ for each conditional effect $(\chi \triangleright v)$ that occurs in the effect of o^+ , an **effect node** $n_{o^+}^\chi$ (details follow); unconditional effects are treated as $(\top \triangleright v)$

Formula Nodes

Formula nodes n_φ are defined as follows:

- ▶ If $\varphi = v$ for some state variable v , n_φ is the variable node n_v (so no new node is introduced).
- ▶ If $\varphi = \top$, n_φ is an AND node without outgoing arcs.
- ▶ If $\varphi = \perp$, n_φ is an OR node without outgoing arcs.
- ▶ If $\varphi = (\varphi_1 \wedge \varphi_2)$, n_φ is an AND node with outgoing arcs to n_{φ_1} and n_{φ_2} .
- ▶ If $\varphi = (\varphi_1 \vee \varphi_2)$, n_φ is an OR node with outgoing arcs to n_{φ_1} and n_{φ_2} .

Note: identically named nodes are identical, so if the same formula occurs multiple times in the task, the **same** node is reused.

Effect Nodes

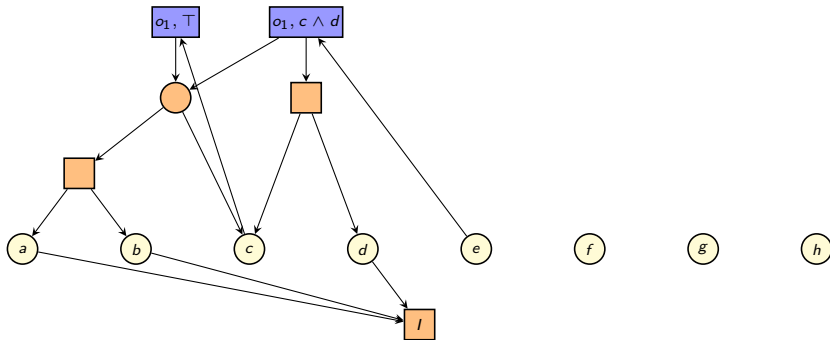
Effect nodes n_{o+}^{χ} are defined as follows:

- ▶ n_{o+}^{χ} is an AND node
- ▶ It has an outgoing arc to the formula nodes $n_{pre(o+)}$ (**precondition arcs**) and n_{χ} (**effect condition arcs**).
- ▶ Exception: if $\chi = \top$, there is no effect condition arc. (This makes our pictures cleaner.)
- ▶ For every conditional effect $(\chi \triangleright v)$ in the operator, there is an arc from variable node n_v to n_{o+}^{χ} (**effect arcs**).

Note: identically named nodes are identical, so if the same effect condition occurs multiple times in the same operator, this only induces **one** node.

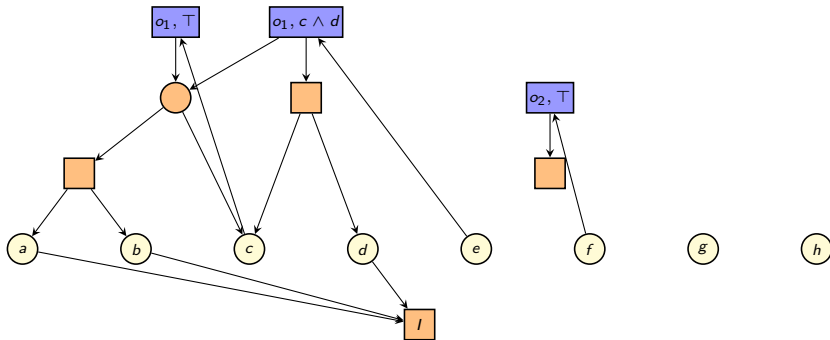
Operator Subgraphs: Example

$$o_1 = \langle c \vee (a \wedge b), c \wedge ((c \wedge d) \triangleright e), 1 \rangle$$



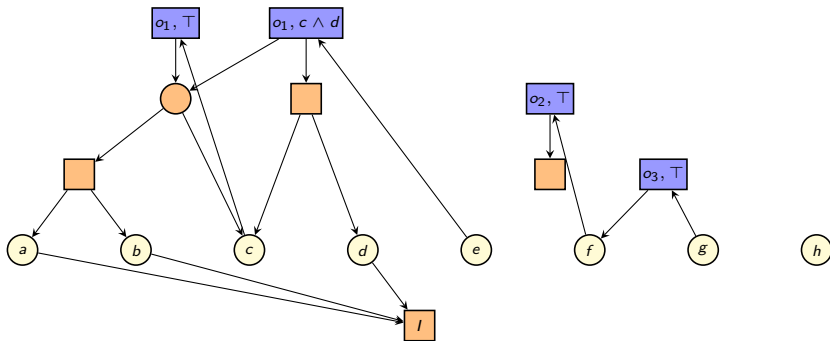
Operator Subgraphs: Example

$$o_2 = \langle \top, f, 2 \rangle$$



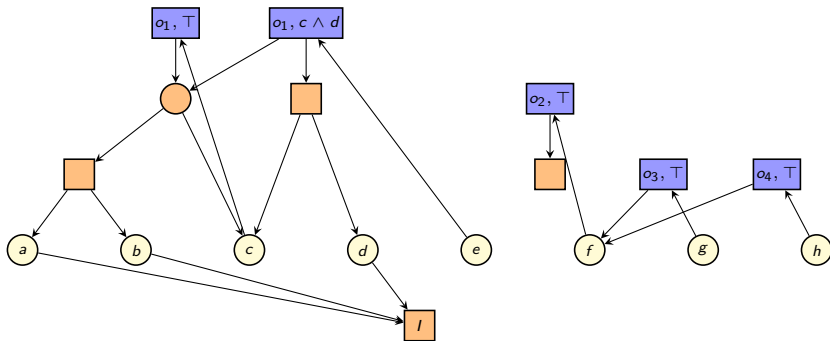
Operator Subgraphs: Example

$$o_3 = \langle f, g, 1 \rangle$$



Operator Subgraphs: Example

$$o_4 = \langle f, h, 1 \rangle$$

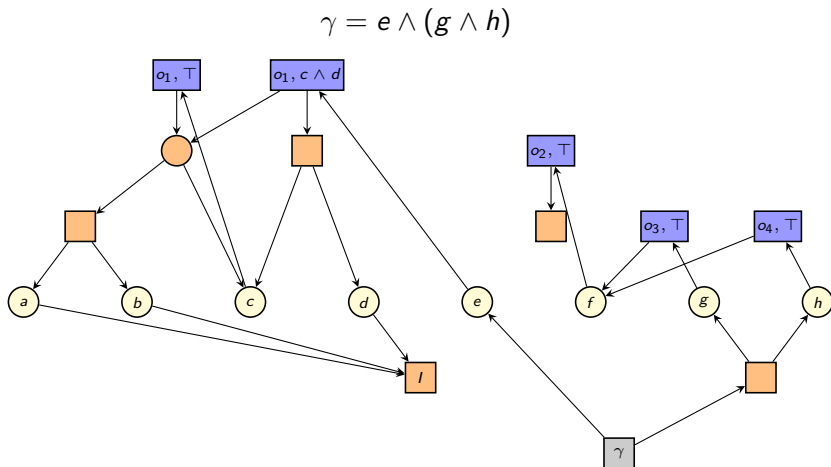


Goal Subgraph

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task.

$RTG(\Pi^+)$ contains a **goal subgraph**, consisting of formula nodes for the goal γ and its subformulas, constructed in the same way as formula nodes for preconditions and effect conditions.

Goal Subgraph and Final Relaxed Task Graph: Example



D5.3 Reachability Analysis

How Can We Use Relaxed Task Graphs?

- ▶ We are now done with the definition of relaxed task graphs.
- ▶ Now we want to **use** them to derive information about planning tasks.
- ▶ In the following chapter, we will use them to compute heuristics for delete-relaxed planning tasks.
- ▶ Here, we start with something simpler: **reachability analysis**.

Forced True Nodes and Reachability

Theorem (Forced True Nodes vs. Reachability)

Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task, and let $N_{\mathbf{T}}$ be the forced true nodes of $\text{RTG}(\Pi^+)$.

*For all **formulas** over state variables φ that occur in the definition of Π^+ :*

φ is true in some reachable state of Π^+ iff $n_{\varphi} \in N_{\mathbf{T}}$.

(We omit the proof.)

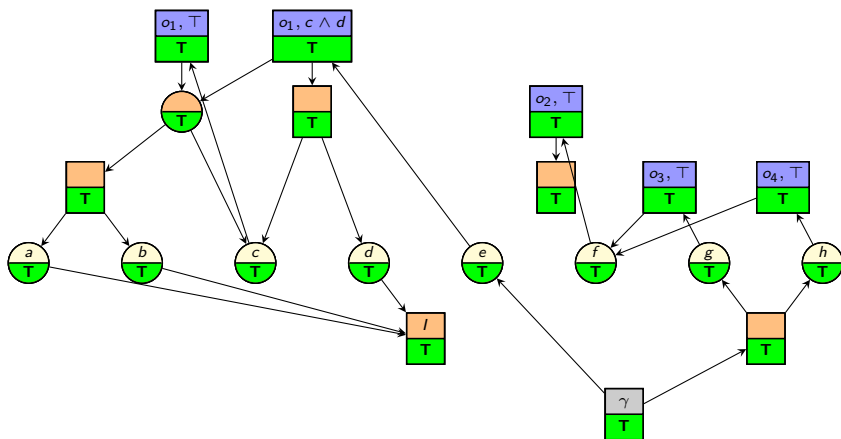
Forced True Nodes and Reachability: Consequences

Corollary

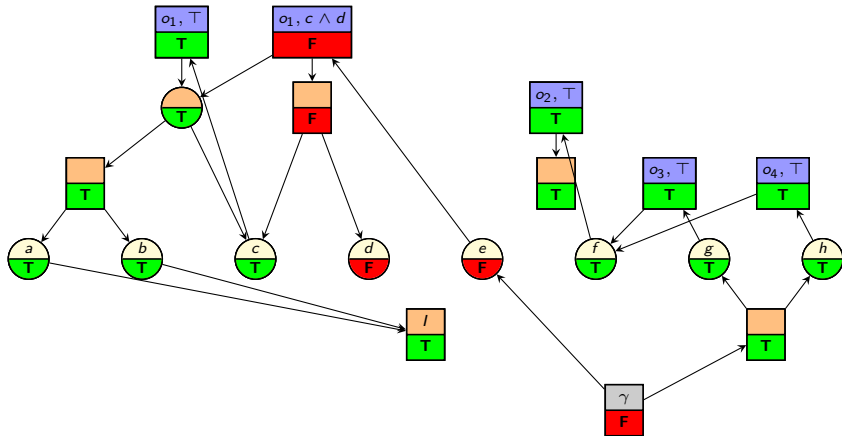
Let $\Pi^+ = \langle V, I, O^+, \gamma \rangle$ be a relaxed planning task, and let $N_{\mathbf{T}}$ be the forced true nodes of $RTG(\Pi^+)$. Then:

- ▶ A **state variable** $v \in V$ is true in at least one reachable state iff $n_v \in N_{\mathbf{T}}$.
- ▶ An **operator** $o^+ \in O^+$ is part of at least one applicable operator sequence iff $n_{pre(o^+)} \in N_{\mathbf{T}}$.
- ▶ The relaxed task is **solvable** iff $n_{\gamma} \in N_{\mathbf{T}}$.

Reachability Analysis: Example



Reachability Analysis: Example with Different Initial State



D5.4 Remarks

Relaxed Task Graphs in the Literature

Some remarks on the planning literature:

- ▶ Usually, only the **STRIPS** case is studied.
- ↪ definitions simpler: only **variable nodes** and **operator nodes**, no formula nodes or effect nodes
- ▶ Usually, so-called **relaxed planning graphs** (RPGs) are studied instead of RTGs.
- ▶ These are **temporally unrolled** versions of RTGs, i.e., they have multiple layers (“time steps”) and are acyclic.
- ↪ Foundations of Artificial Intelligence course FS 2025, Ch. F3–F4

D5.5 Summary

Summary

- ▶ **Relaxed task graphs** (RTGs) represent (most of) the information of a relaxed planning task as an AND/OR graph.
- ▶ They consist of:
 - ▶ **variable nodes**
 - ▶ **an initial node**
 - ▶ **operator subgraphs** including **formula nodes** and **effect nodes**
 - ▶ **a goal subgraph** including **formula nodes**
- ▶ RTGs can be used to analyze **reachability** in relaxed tasks: forced true nodes mean “reachable”, other nodes mean “unreachable”.