Planning and Optimization

D3. Delete Relaxation: Finding Relaxed Plans

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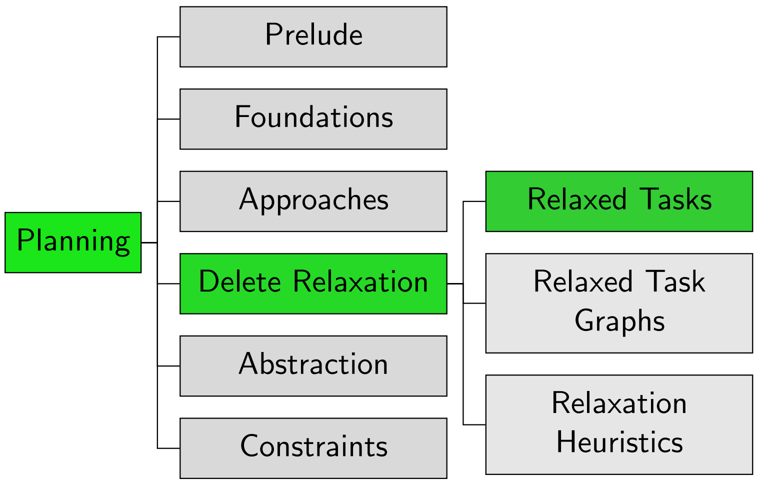
Content of the Course



D3.1 Greedy Algorithm

The Story So Far

- A general way to come up with heuristics is to solve a simplified version of the real problem.
- delete relaxation: given a task in positive normal form, discard all delete effects
  - relaxation lemma: solutions for a state s also work for any dominating state s'
  - monotonicity lemma: $s[\![o]\!]$ dominates s

Greedy Algorithm for Relaxed Planning Tasks

The relaxation and monotonicity lemmas suggest the following algorithm for solving relaxed planning tasks:

Greedy Planning Algorithm for $\langle V, I, O^+, \gamma\rangle$

$s := I$
$\pi^+ := \langle\rangle$
loop forever:
  if $s \models \gamma$:
    return π^+
  else if there is an operator $o^+ \in O^+$ applicable in s
      with $s[\![o^+]\!] \neq s$:
    Append such an operator o^+ to π^+.
    $s := s[\![o^+]\!]$
  else:
    return unsolvable

Correctness of the Greedy Algorithm

The algorithm is sound:

- If it returns a plan, this is indeed a correct solution.
- If it returns "unsolvable", the task is indeed unsolvable
  - Upon termination, there clearly is no relaxed plan from s.
  - By iterated application of the monotonicity lemma, s dominates I.
  - By the relaxation lemma, there is no solution from I.

What about completeness (termination) and runtime?

- Each iteration of the loop adds at least one atom to $on(s)$.
- This guarantees termination after at most $|V|$ iterations.
- Thus, the algorithm can clearly be implemented to run in polynomial time.
  - A good implementation runs in $O(\|\Pi\|)$.

Using the Greedy Algorithm as a Heuristic

We can apply the greedy algorithm within heuristic search for a general (non-relaxed) planning task:

- When evaluating a state s in progression search, solve relaxation of planning task with initial state s.
- When evaluating a subgoal φ in regression search, solve relaxation of planning task with goal φ.
- Set $h(s)$ to the cost of the generated relaxed plan.
  - in general not well-defined: different choices of o^+ in the algorithm lead to different $h(s)$

Is this admissible/safe/goal-aware/consistent?

Properties of the Greedy Algorithm as a Heuristic

Is this an admissible heuristic?

- Yes if the relaxed plans are optimal (due to the plan preservation corollary).
- However, usually they are not, because the greedy algorithm can make poor choices of which operators to apply.

How hard is it to find optimal relaxed plans?

D3.2 Optimal Relaxed Plans

Optimal Relaxation Heuristic

Definition (h^+ heuristic)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task in positive normal form with states S.

The optimal delete relaxation heuristic h^+ for Π is the function $h : S \to \mathbb{R}_0^+ \cup \{\infty\}$ where $h(s)$ is the cost of an optimal relaxed plan for s, i.e., of an optimal plan for $\Pi_s^+ = \langle V, s, O^+, \gamma \rangle$.

(can analogously define a heuristic for regression)

admissible/safe/goal-aware/consistent?

The Set Cover Problem

Can we compute h^+ efficiently?

This question is related to the following problem:

Problem (Set Cover)

Given: *a finite set U, a collection of subsets $C = \{C_1, \dots, C_n\}$ with $C_i \subseteq U$ for all $i \in \{1, \dots, n\}$, and a natural number K.*

Question: *Is there a set cover of size at most K, i.e., a subcollection $S = \{S_1, \dots, S_m\} \subseteq C$ with $S_1 \cup \dots \cup S_m = U$ and $m \leq K$?*

The following is a classical result from complexity theory:

Theorem (Karp 1972)

The set cover problem is NP-complete.

Complexity of Optimal Relaxed Planning (1)

Theorem (Complexity of Optimal Relaxed Planning)

The BCPlanEx problem restricted to delete-relaxed planning tasks is NP-complete.

Proof.

For membership in NP, guess a plan and verify.

It is sufficient to check plans of length at most $|V|$ where V is the set of state variables, so this can be done in nondeterministic polynomial time.

For hardness, we reduce from the set cover problem. ...

Complexity of Optimal Relaxed Planning (2)

Proof (continued).

Given a set cover instance $\langle U, C, K \rangle$, we generate the following relaxed planning task $\Pi^+ = \langle V, I, O^+, \gamma \rangle$:

- $V = U$
- $I = \{v \mapsto \mathbf{F} \mid v \in V\}$
- $O^+ = \{\langle \top, \bigwedge_{v \in C_i} v, 1 \rangle \mid C_i \in C\}$
- $\gamma = \bigwedge_{v \in U} v$

If S is a set cover, the corresponding operators form a plan. Conversely, each plan induces a set cover by taking the subsets corresponding to the operators. There exists a plan of cost at most K iff there exists a set cover of size K.

Moreover, Π^+ can be generated from the set cover instance in polynomial time, so this is a polynomial reduction. □

D3.3 Summary

Summary

- Because of their monotonicity property, delete-relaxed tasks can be solved in polynomial time by a greedy algorithm.
- However, the solution quality of this algorithm is poor.
- For an informative heuristic, we would ideally want to find optimal relaxed plans.
- The solution cost of an optimal relaxed plan is the estimate of the h^+ heuristic.
- However, the bounded-cost plan existence problem for relaxed planning tasks is NP-complete.