Planning and Optimization

C6. SAT Planning: Parallel Encoding

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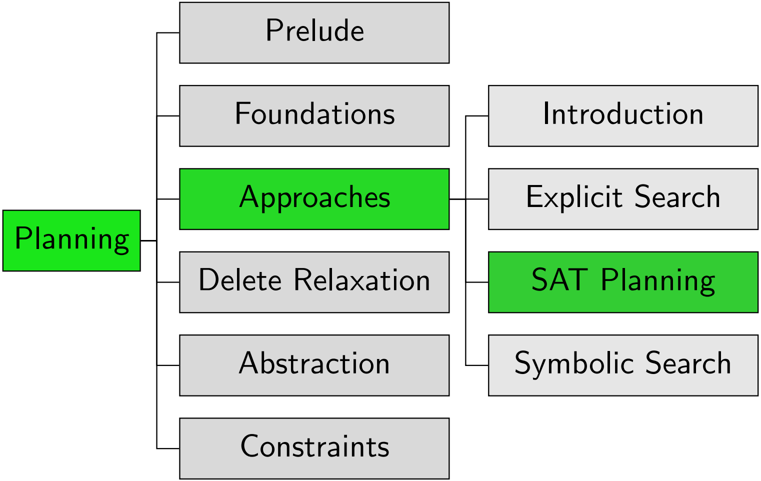
Content of the Course



C6.1 Introduction

Efficiency of SAT Planning

- All other things being equal, the most important aspect for efficient SAT solving is the number of propositional variables in the input formula.
- For sufficiently difficult inputs, runtime scales exponentially in the number of variables.

$\rightsquigarrow$ Can we make SAT planning more efficient by using fewer variables?

Number of Variables

Reminder:

- given propositional planning task $\Pi = \langle V, I, O, \gamma \rangle$
- given horizon $T \in \mathbb{N}_0$

Variables of the SAT Formula

- propositional variables v^i for all $v \in V$, $0 \leq i \leq T$ encode state after i steps of the plan
- propositional variables o^i for all $o \in O$, $1 \leq i \leq T$ encode operator(s) applied in i-th step of the plan

$\rightsquigarrow$ $|V| \cdot (T + 1) + |O| \cdot T$ variables

$\rightsquigarrow$ SAT solving runtime usually exponential in T

Parallel Plans and Commutativity

Can we get away with shorter horizons?

Idea:

- allow parallel plans in the SAT encoding:
  multiple operators can be applied in the same step
  if they do not interfere

Definition (commutative, interfere)

Let $O' = \{o_1, \dots, o_n\}$ be a set of operators applicable in state s.

We say that O' is commutative in s if

- for all permutations π of O', $s[\![\pi]\!]$ is defined, and
- for all permutations π, π' of O', $s[\![\pi]\!] = s[\![\pi']\!]$.

We say that the set O' interferes in s if it is not commutative in s.

Parallel Plan Extraction

- If we can guarantee commutativity, we can allow multiple operators at the same time in the SAT encoding.
- A parallel plan (with multiple o^i used for the same i) extracted from the SAT formula can then be converted into a "regular" plan by ordering the operators within each time step arbitrarily.

Challenges for Parallel SAT Encodings

Two challenges remain:

- our current SAT encoding does not allow concurrent operators
- how do we ensure that concurrent operators are commutative?

C6.2 Adapting the SAT Encoding

Reminder: Sequential SAT Encoding (1)

Sequential SAT Encoding (1)

initial state clauses:

- v^0 for all $v \in V$ with $I(v) = \mathbf{T}$
- $\neg v^0$ for all $v \in V$ with $I(v) = \mathbf{F}$

goal clauses:

- γ^T

operator selection clauses:

- $o_1^i \lor \dots \lor o_n^i$ for all $1 \leq i \leq T$

operator exclusion clauses:

- $\neg o_j^i \lor \neg o_k^i$ for all $1 \leq i \leq T$, $1 \leq j < k \leq n$

$\rightsquigarrow$ operator exclusion clauses must be adapted

Reminder: Sequential SAT Encoding (2)

Sequential SAT Encoding (2)

precondition clauses:

- $\neg o^i \lor \mathit{pre}(o)^{i-1}$ for all $1 \leq i \leq T$, $o \in O$

positive and negative effect clauses:

- $\neg o^i \lor \neg\alpha^{i-1} \lor v^i$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$
- $\neg o^i \lor \alpha^{i-1} \lor \neg\delta^{i-1} \lor \neg v^i$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$

positive and negative frame clauses:

- $\neg o^i \lor \neg v^{i-1} \lor \delta^{i-1} \lor v^i$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$
- $\neg o^i \lor \alpha^{i-1} \lor v^{i-1} \lor \neg v^i$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$

where $\alpha = \mathit{effcond}(v, \mathit{eff}(o))$, $\delta = \mathit{effcond}(\neg v, \mathit{eff}(o))$.

$\rightsquigarrow$ rewrite clauses as implications

Sequential SAT Encoding (2) Rewritten as Implications

Sequential SAT Encoding (2) Rewritten

precondition clauses:

- $o^i \rightarrow pre(o)^{i-1}$ for all $1 \leq i \leq T$, $o \in O$

positive and negative effect clauses:

- $(o^i \wedge \alpha^{i-1}) \rightarrow v^i$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$
- $(o^i \wedge \delta^{i-1} \wedge \neg\alpha^{i-1}) \rightarrow \neg v^i$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$

positive and negative frame clauses:

- $(o^i \wedge v^{i-1} \wedge \neg v^i) \rightarrow \delta^{i-1}$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$
- $(o^i \wedge \neg v^{i-1} \wedge v^i) \rightarrow \alpha^{i-1}$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$

where $\alpha = effcond(v, eff(o))$, $\delta = effcond(\neg v, eff(o))$.

$\rightsquigarrow$ frame clauses must be adapted

Adapting the Operator Exclusion Clauses: Idea

Reminder: operator exclusion clauses $\neg o_j^i \vee \neg o_k^i$
for all $1 \leq i \leq T$, $1 \leq j < k \leq n$

- Ideally: replace with clauses that express "for all states s, the operators selected at time i are commutative in s"
- but: testing if a given set of operators interferes in any state is itself an NP-complete problem

$\rightsquigarrow$ use something less heavy: a sufficient condition for commutativity can be expressed at the level of pairs of operators

Conflicting Operators

- Intuitively, two operators conflict if
  - one can disable the precondition of the other,
  - one can override an effect of the other, or
  - one can enable or disable an effect condition of the other.
- If no two operators in a set O' conflict, then O' is commutative in all states.
- This is still difficult to test, so we restrict attention to the STRIPS case in the following.

Definition (Conflicting STRIPS Operator)

Operators o and o' of a STRIPS task Π conflict if

- o deletes a precondition of o' or vice versa, or
- o deletes an add effect of o' or vice versa.

Adapting the Operator Exclusion Clauses: Solution

Reminder: operator exclusion clauses $\neg o_j^i \lor \neg o_k^i$
for all $1 \leq i \leq T$, $1 \leq j < k \leq n$

Solution:

Parallel SAT Formula: Operator Exclusion Clauses

operator exclusion clauses:

- $\neg o_j^i \lor \neg o_k^i$ for all $1 \leq i \leq T$, $1 \leq j < k \leq n$
  such that o_j and o_k conflict

Adapting the Frame Clauses: Idea

Reminder: frame clauses

$(o^i \land v^{i-1} \land \neg v^i) \rightarrow \delta^{i-1}$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$

$(o^i \land \neg v^{i-1} \land v^i) \rightarrow \alpha^{i-1}$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$

What is the problem?

- These clauses express that if o is applied at time i and the value of v changes, then o caused the change.
- This is no longer true if we want to be able to apply two operators concurrently.

⇝ Instead, say "If the value of v changes, then some operator must have caused the change."

Adapting the Frame Clauses: Solution

Reminder: frame clauses

$(o^i \land v^{i-1} \land \neg v^i) \rightarrow \delta^{i-1}$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$

$(o^i \land \neg v^{i-1} \land v^i) \rightarrow \alpha^{i-1}$ for all $1 \leq i \leq T$, $o \in O$, $v \in V$

Solution:

Parallel SAT Formula: Frame Clauses

positive and negative frame clauses:

- $(v^{i-1} \land \neg v^i) \rightarrow ((o_1^i \land \delta_{o_1}^{i-1}) \lor \dots \lor (o_n^i \land \delta_{o_n}^{i-1}))$
  for all $1 \leq i \leq T$, $v \in V$
- $(\neg v^{i-1} \land v^i) \rightarrow ((o_1^i \land \alpha_{o_1}^{i-1}) \lor \dots \lor (o_n^i \land \alpha_{o_n}^{i-1}))$
  for all $1 \leq i \leq T$, $v \in V$

where $\alpha_o = \mathit{effcond}(v, \mathit{eff}(o))$, $\delta_o = \mathit{effcond}(\neg v, \mathit{eff}(o))$,
$O = \{o_1, \dots, o_n\}$.

For STRIPS, these are in clause form.

C6.3 Summary

Summary

- As a rule of thumb, SAT solvers generally perform better on formulas with fewer variables.
- Parallel encodings reduce the number of variables by shortening the horizon needed to solve a planning task.
- Parallel encodings replace the constraint that operators are not applied concurrently by the constraint that conflicting operators are not applied concurrently.
- To make parallelism possible, the frame clauses also need to be adapted.