Planning and Optimization

C3. Progression and Regression Search

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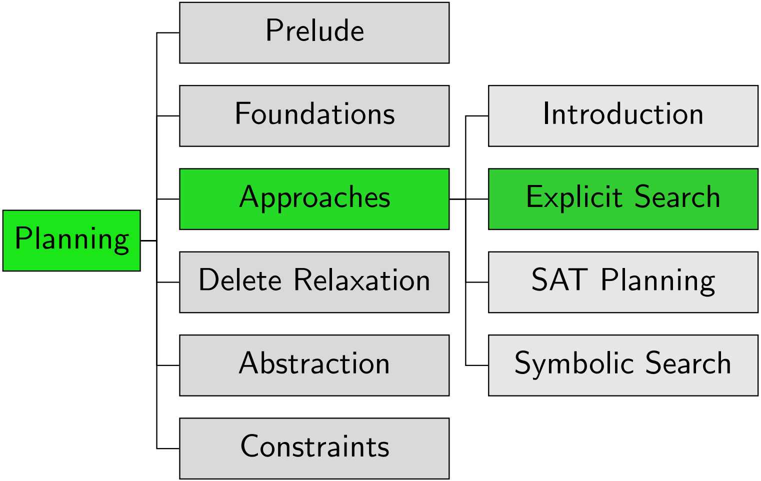
Content of the Course



C3.1 Introduction

Search Direction

Search direction

- one dimension for classifying search algorithms
- forward search from initial state to goal based on progression
- backward search from goal to initial state based on regression
- bidirectional search

In this chapter we look into progression and regression planning.

Reminder: Interface for Heuristic Search Algorithms

Abstract Interface Needed for Heuristic Search Algorithms

- init() $\rightsquigarrow$ returns initial state
- is_goal(s) $\rightsquigarrow$ tests if s is a goal state
- succ(s) $\rightsquigarrow$ returns all pairs $\langle a, s' \rangle$ with $s \xrightarrow{a} s'$
- cost(a) $\rightsquigarrow$ returns cost of action a
- h(s) $\rightsquigarrow$ returns heuristic value for state s

C3.2 Progression

Planning by Forward Search: Progression

Progression: Computing the successor state $s[\![o]\!]$ of a state s with respect to an operator o.

Progression planners find solutions by forward search:

- start from initial state
- iteratively pick a previously generated state and progress it through an operator, generating a new state
- solution found when a goal state generated

pro: very easy and efficient to implement

Search Space for Progression

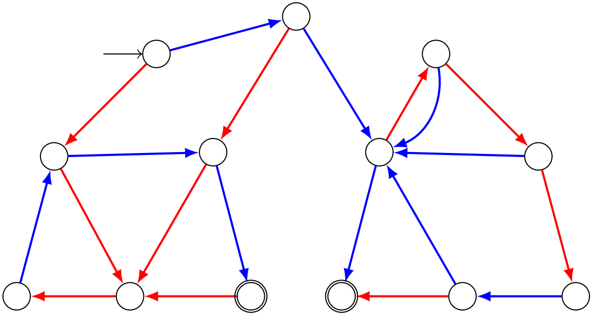
Search Space for Progression

search space for progression in a planning task $\Pi = \langle V, I, O, \gamma \rangle$
(search states are world states s of Π;
actions of search space are operators $o \in O$)

- init() $\rightsquigarrow$ returns I
- is_goal(s) $\rightsquigarrow$ tests if $s \models \gamma$
- succ(s) $\rightsquigarrow$ returns all pairs $\langle o, s[\![o]\!] \rangle$ where $o \in O$ and o is applicable in s
- cost(o) $\rightsquigarrow$ returns $cost(o)$ as defined in Π
- h(s) $\rightsquigarrow$ estimates cost from s to γ ($\rightsquigarrow$ Parts D–F)

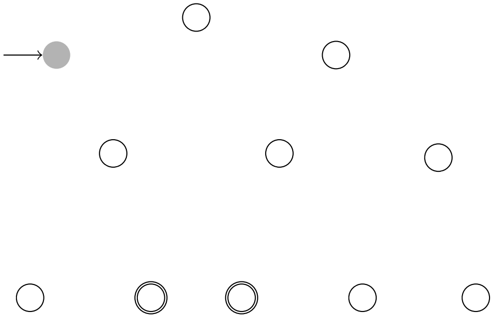
Progression Planning Example

Example of a progression search



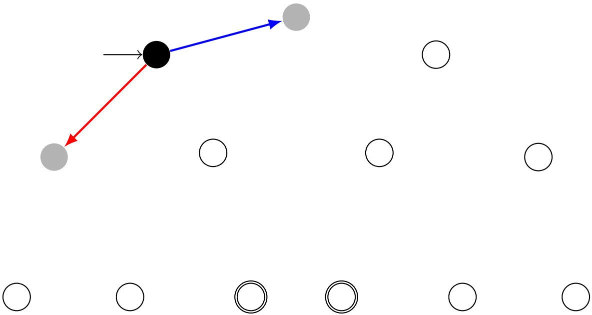
Progression Planning Example

Example of a progression search



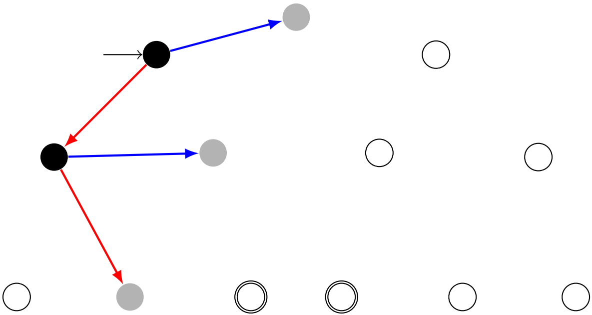
Progression Planning Example

Example of a progression search



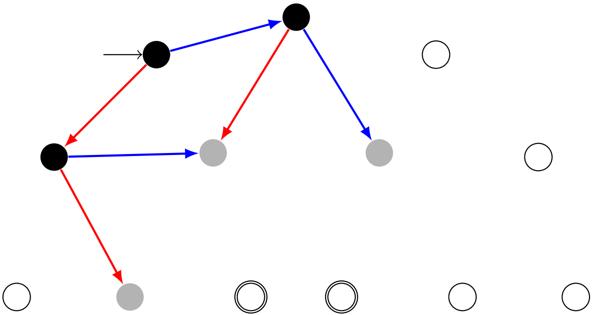
Progression Planning Example

Example of a progression search



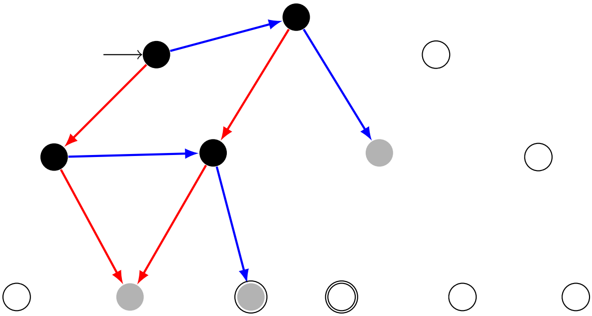
Progression Planning Example

Example of a progression search



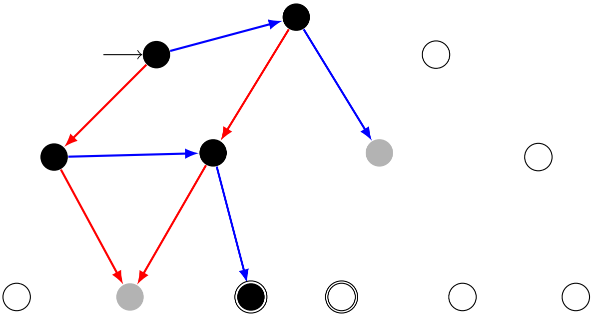
Progression Planning Example

Example of a progression search



Progression Planning Example

Example of a progression search



C3.3 Regression

Forward Search vs. Backward Search

Searching planning tasks in forward vs. backward direction is not symmetric:

- forward search starts from a single initial state;
  backward search starts from a set of goal states
- when applying an operator o in a state s in forward direction, there is a unique successor state s';
  if we just applied operator o and ended up in state s', there can be several possible predecessor states s

$\rightsquigarrow$ in most natural representation for backward search in planning, each search state corresponds to a set of world states

Planning by Backward Search: Regression

Regression: Computing the possible predecessor states $regr(S', o)$ of a set of states S' ("subgoal") given the last operator o that was applied.

$\leadsto$ formal definition in next chapter

Regression planners find solutions by backward search:

- start from set of goal states
- iteratively pick a previously generated subgoal (state set) and regress it through an operator, generating a new subgoal
- solution found when a generated subgoal includes initial state

pro: can handle many states simultaneously
con: basic operations complicated and expensive

Search Space Representation in Regression Planners

identify state sets with logical formulas (again):

- each search state corresponds to a set of world states ("subgoal")
- each search state is represented by a logical formula: φ represents $\{s \in S \mid s \models \varphi\}$
- many basic search operations like detecting duplicates are NP-complete or coNP-complete

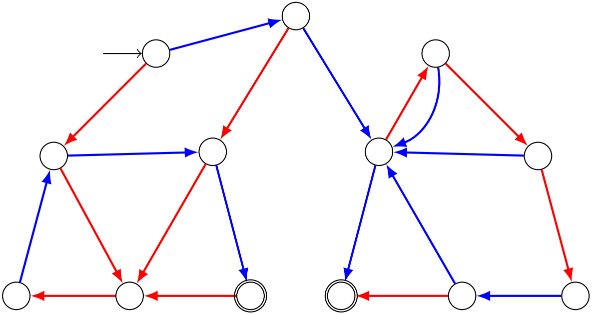
Search Space for Regression

Search Space for Regression

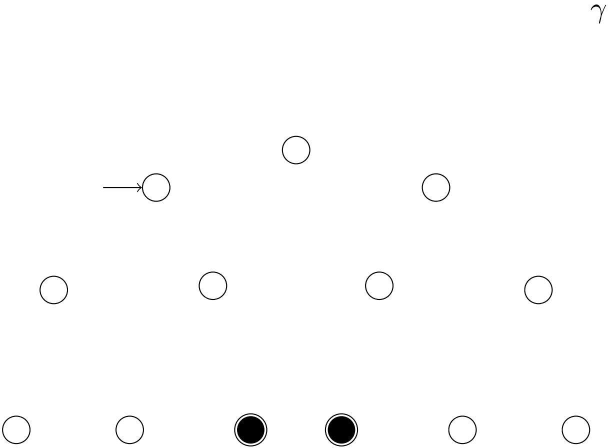
search space for regression in a planning task $\Pi = \langle V, I, O, \gamma \rangle$
(search states are formulas φ describing sets of world states;
actions of search space are operators $o \in O$)

- init() $\rightsquigarrow$ returns γ
- is_goal(φ) $\rightsquigarrow$ tests if $I \models \varphi$
- succ(φ) $\rightsquigarrow$ returns all pairs $\langle o, \mathit{regr}(\varphi, o) \rangle$ where $o \in O$ and $\mathit{regr}(\varphi, o)$ is defined
- cost(o) $\rightsquigarrow$ returns $\mathit{cost}(o)$ as defined in Π
- h(φ) $\rightsquigarrow$ estimates cost from I to φ ($\rightsquigarrow$ Parts D–F)

Regression Planning Example (Depth-first Search)

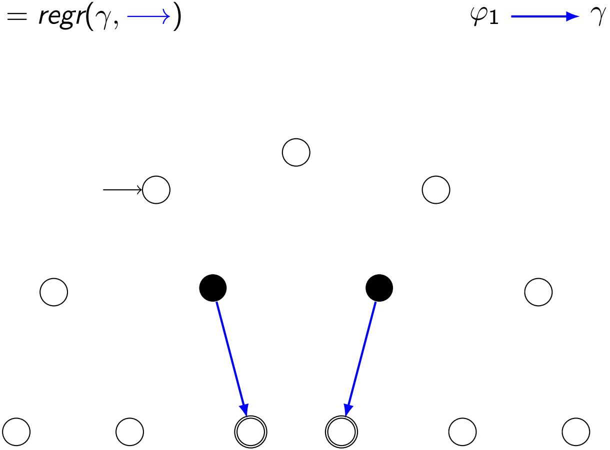


Regression Planning Example (Depth-first Search)



Regression Planning Example (Depth-first Search)

$\varphi_1 = regr(\gamma, \longrightarrow)$

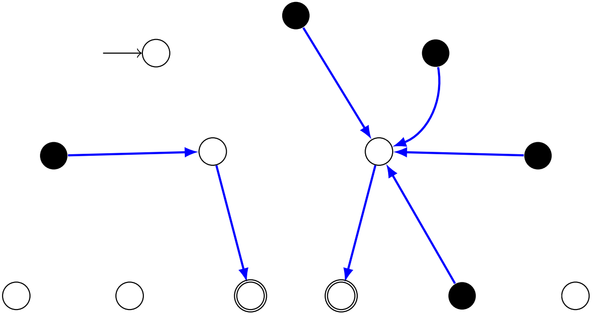


Regression Planning Example (Depth-first Search)

$\varphi_1 = regr(\gamma, \longrightarrow)$

$\varphi_2 = regr(\varphi_1, \longrightarrow)$

$\varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$



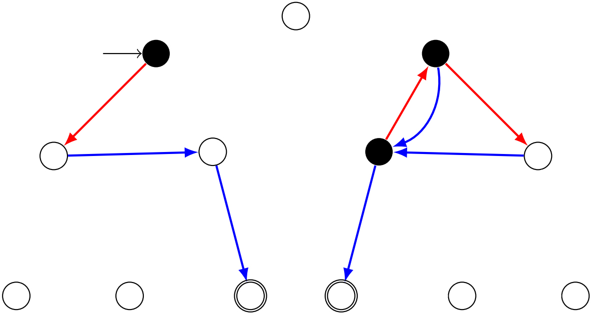
Regression Planning Example (Depth-first Search)

$\varphi_1 = regr(\gamma, \longrightarrow)$

$\varphi_2 = regr(\varphi_1, \longrightarrow)$

$\varphi_3 = regr(\varphi_2, \longrightarrow), I \models \varphi_3$

$\varphi_3 \longrightarrow \varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$



C3.4 Regression for STRIPS Tasks

Regression for STRIPS Planning Tasks

Regression for STRIPS planning tasks is much simpler than the general case:

- Consider subgoal φ that is conjunction of atoms $a_1 \land \dots \land a_n$ (e.g., the original goal γ of the planning task).
- First step: Choose an operator o that deletes no a_i.
- Second step: Remove any atoms added by o from φ.
- Third step: Conjoin $pre(o)$ to φ.
- ⇝ Outcome of this is regression of φ w.r.t. o. It is again a conjunction of atoms.

optimization: only consider operators adding at least one a_i

STRIPS Regression

Definition (STRIPS Regression)

Let $\varphi = \varphi_1 \land \dots \land \varphi_n$ be a conjunction of atoms, and let o be a STRIPS operator which adds the atoms $a_1, \dots, a_k$ and deletes the atoms $d_1, \dots, d_l$.

The STRIPS regression of φ with respect to o is

$$sregr(\varphi, o) := \begin{cases} \bot & \text{if } \varphi_i = d_j \text{ for some } i, j \\ pre(o) \land \bigwedge(\{\varphi_1, \dots, \varphi_n\} \setminus \{a_1, \dots, a_k\}) & \text{else} \end{cases}$$

Note: $sregr(\varphi, o)$ is again a conjunction of atoms, or $\bot$.

Does this Capture the Idea of Regression?

For our definition to capture the concept of regression,
it must have the following property:

Regression Property

For all sets of states described by a conjunction of atoms φ,
all states s and all STRIPS operators o,

$$s \models sregr(\varphi, o) \quad \text{iff} \quad s[\![o]\!] \models \varphi.$$

This is indeed true. We do not prove it now because we prove
this property for general regression (not just STRIPS) later.

C3.5 Summary

Summary

- Progression search proceeds forward from the initial state.
- In progression search, the search space is identical to the state space of the planning task.
- Regression search proceeds backwards from the goal.
- Each search state corresponds to a set of world states, for example represented by a formula.
- Regression is simple for STRIPS operators.
- The theory for general regression is more complex. This is the topic of the following chapter.