Planning and Optimization
B5. Positive Normal Form and STRIPS

Malte Helmert and Gabriele Röger

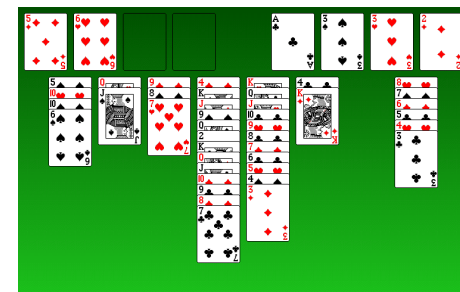Universität Basel

September 29, 2025

B5.1 Motivation

B5.2 Positive Normal Form

B5.3 STRIPS

B5.4 Summary

B5.1 Motivation

Example: Freecell

Example (Good and Bad Effects)

If we move K$\diamondsuit$ to a free tableau position,
the good effect is that 4$\clubsuit$ is now accessible.
The bad effect is that we lose one free tableau position.

What is a Good or Bad Effect?

Question: Which operator effects are good, and which are bad?

Difficult to answer in general, because it depends on context:

- ▶ Locking our door is good if we want to keep burglars out.
- ▶ Locking our door is bad if we want to enter.

We now consider a reformulation of propositional planning tasks that makes the distinction between good and bad effects obvious.

B5.2 Positive Normal Form

Content of the Course
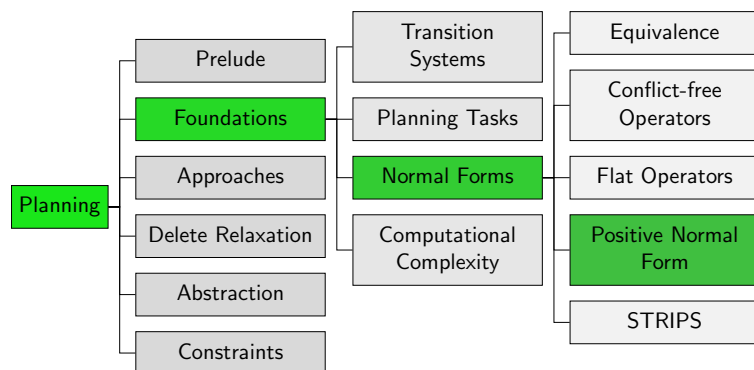
Positive Formulas, Operators and Tasks

Definition (Positive Formula)
A logical formula φ is positive if no negation symbols appear in φ.

Note: This includes the negation symbols implied by $\rightarrow$ and $\leftrightarrow$.

Definition (Positive Operator)
An operator o is positive if $pre(o)$ and all effect conditions in $eff(o)$ are positive.

Definition (Positive Propositional Planning Task)
A propositional planning task $\langle V, I, O, \gamma \rangle$ is positive if all operators in O and the goal γ are positive.

Positive Normal Form

> **Definition (Positive Normal Form)**
>
> A propositional planning task is in positive normal form
> if it is positive and all operator effects are flat.

Positive Normal Form: Example

> **Example (Transformation to Positive Normal Form)**
>
> $V = \{home, uni, lecture, bike, bike\text{-}locked\}$
> $I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$
> $\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}\}$
> $O = \{\langle home \wedge bike \wedge \neg bike\text{-}locked, \neg home \wedge uni \rangle,$
> $\quad \langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \rangle,$
> $\quad \langle bike \wedge \neg bike\text{-}locked, bike\text{-}locked \rangle,$
> $\quad \langle uni, lecture \wedge ((bike \wedge \neg bike\text{-}locked) \rhd \neg bike) \rangle\}$
> $\gamma = lecture \wedge bike$

Positive Normal Form: Example

> **Example (Transformation to Positive Normal Form)**
>
> $V = \{home, uni, lecture, bike, bike\text{-}locked\}$
> $I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$
> $\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}\}$
> $O = \{\langle home \wedge bike \wedge \neg bike\text{-}locked, \neg home \wedge uni \rangle,$
> $\quad \langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \rangle,$
> $\quad \langle bike \wedge \neg bike\text{-}locked, bike\text{-}locked \rangle,$
> $\quad \langle uni, lecture \wedge ((bike \wedge \neg bike\text{-}locked) \rhd \neg bike) \rangle\}$
> $\gamma = lecture \wedge bike$

Identify state variable v occurring negatively in conditions.

Positive Normal Form: Example

> **Example (Transformation to Positive Normal Form)**
>
> $V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$
> $I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$
> $\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$
> $O = \{\langle home \wedge bike \wedge \neg bike\text{-}locked, \neg home \wedge uni \rangle,$
> $\quad \langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \rangle,$
> $\quad \langle bike \wedge \neg bike\text{-}locked, bike\text{-}locked \rangle,$
> $\quad \langle uni, lecture \wedge ((bike \wedge \neg bike\text{-}locked) \rhd \neg bike) \rangle\}$
> $\gamma = lecture \wedge bike$

Introduce new variable $\hat{v}$ with complementary initial value.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$

$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$
$\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$

$O = \{\langle home \wedge bike \wedge \neg bike\text{-}locked, \neg home \wedge uni\rangle,$
$\quad \langle bike \wedge bike\text{-}locked, \color{red}{\neg bike\text{-}locked}\rangle,$
$\quad \langle bike \wedge \neg bike\text{-}locked, \color{red}{bike\text{-}locked}\rangle,$
$\quad \langle uni, lecture \wedge ((bike \wedge \neg bike\text{-}locked) \triangleright \neg bike))\rangle\}$

$\gamma = lecture \wedge bike$

Identify effects on variable v.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$

$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$
$\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$

$O = \{\langle home \wedge bike \wedge \neg bike\text{-}locked, \neg home \wedge uni\rangle,$
$\quad \langle bike \wedge bike\text{-}locked, \color{red}{\neg bike\text{-}locked \wedge bike\text{-}unlocked}\rangle,$
$\quad \langle bike \wedge \neg bike\text{-}locked, \color{red}{bike\text{-}locked \wedge \neg bike\text{-}unlocked}\rangle,$
$\quad \langle uni, lecture \wedge ((bike \wedge \neg bike\text{-}locked) \triangleright \neg bike))\rangle\}$

$\gamma = lecture \wedge bike$

Introduce complementary effects for $\hat{v}$.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$

$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$
$\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$

$O = \{\langle home \wedge bike \wedge \color{red}{\neg bike\text{-}locked}, \neg home \wedge uni\rangle,$
$\quad \langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \wedge bike\text{-}unlocked\rangle,$
$\quad \langle bike \wedge \color{red}{\neg bike\text{-}locked}, bike\text{-}locked \wedge \neg bike\text{-}unlocked\rangle,$
$\quad \langle uni, lecture \wedge ((bike \wedge \color{red}{\neg bike\text{-}locked}) \triangleright \neg bike))\rangle\}$

$\gamma = lecture \wedge bike$

Identify negative conditions for v.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$

$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$
$\quad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$

$O = \{\langle home \wedge bike \wedge \color{red}{bike\text{-}unlocked}, \neg home \wedge uni\rangle,$
$\quad \langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \wedge bike\text{-}unlocked\rangle,$
$\quad \langle bike \wedge \color{red}{bike\text{-}unlocked}, bike\text{-}locked \wedge \neg bike\text{-}unlocked\rangle,$
$\quad \langle uni, lecture \wedge ((bike \wedge \color{red}{bike\text{-}unlocked}) \triangleright \neg bike))\rangle\}$

$\gamma = lecture \wedge bike$

Replace by positive condition $\hat{v}$.

Positive Normal Form: Example

Example (Transformation to Positive Normal Form)

$V = \{home, uni, lecture, bike, bike\text{-}locked, bike\text{-}unlocked\}$

$I = \{home \mapsto \mathbf{T}, bike \mapsto \mathbf{T}, bike\text{-}locked \mapsto \mathbf{T},$
$\qquad uni \mapsto \mathbf{F}, lecture \mapsto \mathbf{F}, bike\text{-}unlocked \mapsto \mathbf{F}\}$

$O = \{\langle home \wedge bike \wedge bike\text{-}unlocked, \neg home \wedge uni\rangle,$
$\qquad \langle bike \wedge bike\text{-}locked, \neg bike\text{-}locked \wedge bike\text{-}unlocked\rangle,$
$\qquad \langle bike \wedge bike\text{-}unlocked, bike\text{-}locked \wedge \neg bike\text{-}unlocked\rangle,$
$\qquad \langle uni, lecture \wedge ((bike \wedge bike\text{-}unlocked) \rhd \neg bike)\rangle\}$

$\gamma = lecture \wedge bike$

Positive Normal Form: Existence

Theorem (Positive Normal Form)

For every propositional planning task Π, there is an equivalent propositional planning task Π' in positive normal form. Moreover, Π' can be computed from Π in polynomial time.

Note: Equivalence here means that the transition systems induced by Π and Π', restricted to the reachable states, are isomorphic.

We prove the theorem by describing a suitable algorithm.
(However, we do not prove its correctness or complexity.)

Positive Normal Form: Algorithm

Transformation of $\langle V, I, O, \gamma\rangle$ to Positive Normal Form

Replace all operators with equivalent conflict-free operators.
Convert all conditions to negation normal form (NNF).
while any condition contains a negative literal $\neg v$:
    Let v be a variable which occurs negatively in a condition.
    $V := V \cup \{\hat{v}\}$ for some new propositional state variable $\hat{v}$
    $I(\hat{v}) := \begin{cases} \mathbf{F} & \text{if } I(v) = \mathbf{T} \\ \mathbf{T} & \text{if } I(v) = \mathbf{F} \end{cases}$
    Replace the effect v by $(v \wedge \neg\hat{v})$ in all operators $o \in O$.
    Replace the effect $\neg v$ by $(\neg v \wedge \hat{v})$ in all operators $o \in O$.
    Replace $\neg v$ by $\hat{v}$ in all conditions.
Convert all operators $o \in O$ to flat operators.

Here, all conditions refers to all operator preconditions,
operator effect conditions and the goal.

Why Positive Normal Form is Interesting

In the absence of conditional effects, positive normal form allows us to distinguish good and bad effects easily:

► Effects that make state variables true (add effects) are good.
► Effects that make state variables false (delete effects) are bad.

This is particularly useful for planning algorithms based on delete relaxation, which we will study in Part D.

(Why restriction "in the absence of conditional effects"?)

B5.3 STRIPS

Content of the Course
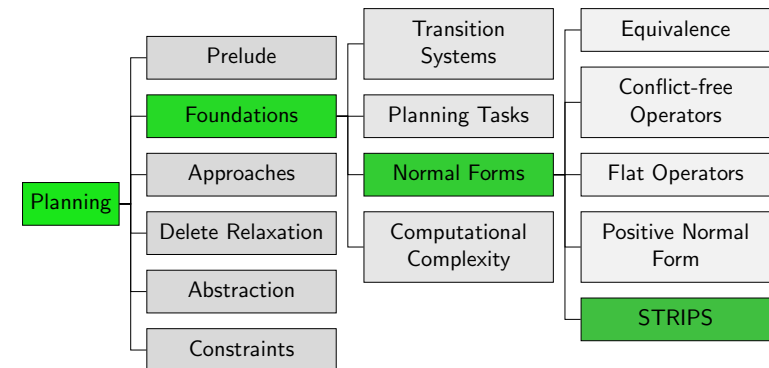
STRIPS Operators and Planning Tasks

Definition (STRIPS Operator)

An operator o of a prop. planning task is a STRIPS operator if

- $pre(o)$ is a conjunction of state variables, and
- $eff(o)$ is a conflict-free conjunction of atomic effects.

Definition (STRIPS Planning Task)

A propositional planning task $\langle V, I, O, \gamma \rangle$ is a STRIPS planning task if all operators $o \in O$ are STRIPS operators and γ is a conjunction of state variables.

Note:　STRIPS operators are conflict-free and flat.
　　　　STRIPS is a special case of positive normal form.

STRIPS Operators: Remarks

- Every STRIPS operator is of the form

$$\langle v_1 \wedge \cdots \wedge v_n, \quad \ell_1 \wedge \cdots \wedge \ell_m \rangle$$

where v_i are state variables and ℓ_j are atomic effects.
- Often, STRIPS operators o are described via three sets of state variables:
  - the preconditions (state variables occurring in $pre(o)$)
  - the add effects (state variables occurring positively in $eff(o)$)
  - the delete effects (state variables occurring negatively in $eff(o)$)
- Definitions of STRIPS in the literature often do not require conflict-freeness. But it is easy to achieve and makes many things simpler.
- There exists a variant called STRIPS with negation where negative literals are also allowed in conditions.

Why STRIPS is Interesting

- ▶ STRIPS is particularly simple, yet expressive enough
  to capture general planning tasks.
- ▶ In particular, STRIPS planning is no easier
  than planning in general (as we will see in Chapters B6–B7).
- ▶ Many algorithms in the planning literature
  are only presented for STRIPS planning tasks
  (generalization is often, but not always, obvious).

STRIPS
STanford Research Institute Problem Solver
(Fikes & Nilsson, 1971)

Transformation to STRIPS

- ▶ Not every operator is equivalent to a STRIPS operator.
- ▶ However, each operator can be transformed into
  a set of STRIPS operators whose "combination"
  is equivalent to the original operator. (How?)
- ▶ However, this transformation may exponentially increase
  the number of operators. There are planning tasks
  for which such a blow-up is unavoidable.
- ▶ There are polynomial transformations of propositional
  planning tasks to STRIPS, but these do not lead to
  isomorphic transition systems (auxiliary states are needed).
  (They are, however, equivalent in a weaker sense.)

B5.4 Summary

Summary

- ▶ A positive task helps distinguish good and bad effects.
  The notion of positive tasks only exists for propositional tasks.
- ▶ A positive task with flat operators is in positive normal form.
- ▶ STRIPS is even more restrictive than positive normal form,
  forbidding complex preconditions and conditional effects.
- ▶ Both forms are expressive enough to capture
  general propositional planning tasks.
- ▶ Transformation to positive normal form is possible
  with polynomial size increase.
- ▶ Isomorphic transformations of propositional planning tasks to
  STRIPS can increase the number of operators exponentially;
  non-isomorphic polynomial transformations exist.