Planning and Optimization

B3. Formal Definition of Planning

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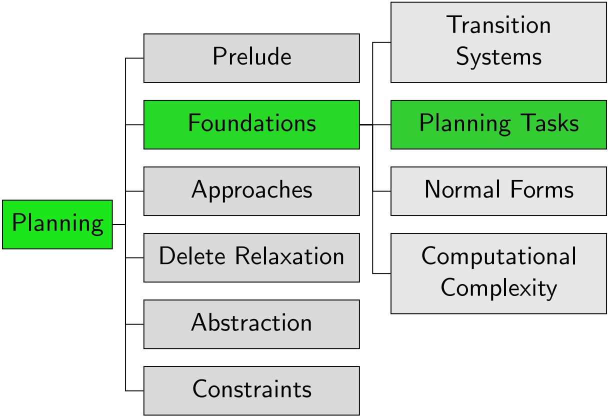
Content of the Course



Semantics of Effects and Operators

Semantics of Effects: Effect Conditions

Definition (Effect Condition for an Effect)

Let ℓ be an atomic effect, and let e be an effect.

The effect condition $effcond(\ell, e)$ under which ℓ triggers given the effect e is a propositional formula defined as follows:

- $effcond(\ell, \top) = \bot$
- $effcond(\ell, e) = \top$ for the atomic effect $e = \ell$
- $effcond(\ell, e) = \bot$ for all atomic effects $e = \ell' \neq \ell$
- $effcond(\ell, (e \wedge e')) = (effcond(\ell, e) \vee effcond(\ell, e'))$
- $effcond(\ell, (\chi \rhd e)) = (\chi \wedge effcond(\ell, e))$

Intuition: $effcond(\ell, e)$ represents the condition that must be true in the current state for the effect e to lead to the atomic effect ℓ

Effect Condition: Example (1)

Example

Consider the move operator m_1 from the running example:
$\mathit{eff}(m_1) = ((t_1 \rhd \neg t_1) \wedge (\neg t_1 \rhd t_1))$.

Under which conditions does it set t_1 to false?

$$\begin{aligned}
\mathit{effcond}(\neg t_1, \mathit{eff}(m_1)) &= \mathit{effcond}(\neg t_1, ((t_1 \rhd \neg t_1) \wedge (\neg t_1 \rhd t_1))) \\
&= \mathit{effcond}(\neg t_1, (t_1 \rhd \neg t_1)) \vee \\
&\quad\ \mathit{effcond}(\neg t_1, (\neg t_1 \rhd t_1)) \\
&= (t_1 \wedge \mathit{effcond}(\neg t_1, \neg t_1)) \vee \\
&\quad\ (\neg t_1 \wedge \mathit{effcond}(\neg t_1, t_1)) \\
&= (t_1 \wedge \top) \vee (\neg t_1 \wedge \bot) \\
&\equiv t_1 \vee \bot \\
&\equiv t_1
\end{aligned}$$

Effect Condition: Example (2)

Example

Consider the move operator m_1 from the running example:
$\mathit{eff}(m_1) = ((t_1 \rhd \neg t_1) \wedge (\neg t_1 \rhd t_1))$.

Under which conditions does it set i to true?

$$\begin{aligned}
\mathit{effcond}(i, \mathit{eff}(m_1)) &= \mathit{effcond}(i, ((t_1 \rhd \neg t_1) \wedge (\neg t_1 \rhd t_1))) \\
&= \mathit{effcond}(i, (t_1 \rhd \neg t_1)) \vee \\
&\ \mathit{effcond}(i, (\neg t_1 \rhd t_1)) \\
&= (t_1 \wedge \mathit{effcond}(i, \neg t_1)) \vee \\
&\ (\neg t_1 \wedge \mathit{effcond}(i, t_1)) \\
&= (t_1 \wedge \bot) \vee (\neg t_1 \wedge \bot) \\
&\equiv \bot \vee \bot \\
&\equiv \bot
\end{aligned}$$

Semantics of Effects: Applying an Effect

first attempt:

Definition (Applying Effects)

Let V be a set of propositional state variables.
Let s be a state over V, and let e be an effect over V.

The resulting state of applying e in s, written $s[\![e]\!]$, is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \\ s(v) & \text{otherwise} \end{cases}$$

What is the problem with this definition?

Semantics of Effects: Applying an Effect

correct definition:

Definition (Applying Effects)

Let V be a set of propositional state variables.
Let s be a state over V, and let e be an effect over V.

The resulting state of applying e in s, written $s[\![e]\!]$, is the state s' defined as follows for all $v \in V$:

$$s'(v) = \begin{cases} \mathbf{T} & \text{if } s \models \mathit{effcond}(v, e) \\ \mathbf{F} & \text{if } s \models \mathit{effcond}(\neg v, e) \land \neg \mathit{effcond}(v, e) \\ s(v) & \text{otherwise} \end{cases}$$

Add-after-Delete Semantics

Note:

- The definition implies that if a variable is simultaneously "added" (set to **T**) and "deleted" (set to **F**), the value **T** takes precedence.
- This is called add-after-delete semantics.
- This detail of effect semantics is somewhat arbitrary, but has proven useful in applications.

Semantics of Operators

Definition (Applicable, Applying Operators, Resulting State)

Let V be a set of propositional state variables.
Let s be a state over V, and let o be an operator over V.

Operator o is applicable in s if $s \models pre(o)$.

If o is applicable in s, the resulting state of applying o in s, written $s[\![o]\!]$, is the state $s[\![eff(o)]\!]$.

Planning Tasks

Planning Tasks

Definition (Planning Task)

A (propositional) planning task is a 4-tuple $\Pi = \langle V, I, O, \gamma \rangle$ where

- V is a finite set of propositional state variables,
- I is an interpretation of V called the initial state,
- O is a finite set of operators over V, and
- γ is a formula over V called the goal.

Running Example: Planning Task

Example

From the previous chapter, we see that the running example can be represented by the task $\Pi = \langle V, I, O, \gamma \rangle$ with

- $V = \{i, w, t_1, t_2\}$
- $I = \{i \mapsto \mathbf{F}, w \mapsto \mathbf{T}, t_1 \mapsto \mathbf{F}, t_2 \mapsto \mathbf{F}\}$
- $O = \{m_1, m_2, l_1, l_2, u\}$ where
  - $m_1 = \langle \top, ((t_1 \rhd \neg t_1) \land (\neg t_1 \rhd t_1)), 5 \rangle$
  - $m_2 = \langle \top, ((t_2 \rhd \neg t_2) \land (\neg t_2 \rhd t_2)), 5 \rangle$
  - $l_1 = \langle \neg i \land (w \leftrightarrow t_1), (i \land w), 1 \rangle$
  - $l_2 = \langle \neg i \land (w \leftrightarrow t_2), (i \land \neg w), 1 \rangle$
  - $u = \langle i, \neg i \land (w \rhd ((t_1 \rhd w) \land (\neg t_1 \rhd \neg w))) \land (\neg w \rhd ((t_2 \rhd w) \land (\neg t_2 \rhd \neg w))), 1 \rangle$
- $\gamma = \neg i \land \neg w$

Mapping Planning Tasks to Transition Systems

Definition (Transition System Induced by a Planning Task)

The planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces
the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$, where

- S is the set of all states over V,
- L is the set of operators O,
- $c(o) = cost(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

Planning Tasks: Terminology

- Terminology for transitions systems is also applied to the planning tasks Π that induce them.
- For example, when we speak of the states of Π, we mean the states of $\mathcal{T}(\Pi)$.
- A sequence of operators that forms a solution of $\mathcal{T}(\Pi)$ is called a plan of Π.

Satisficing and Optimal Planning

By planning, we mean the following two algorithmic problems:

Definition (Satisficing Planning)

Given: a planning task Π

Output: a plan for Π, or **unsolvable** if no plan for Π exists

Definition (Optimal Planning)

Given: a planning task Π

Output: a plan for Π with minimal cost among all plans for Π, or **unsolvable** if no plan for Π exists

Summary

Summary

- Planning tasks compactly represent transition systems and are suitable as inputs for planning algorithms.
- A planning task consists of a set of state variables and an initial state, operators and goal over these state variables.
- We gave formal definitions for these concepts.
- In satisficing planning, we must find a solution for a planning task (or show that no solution exists).
- In optimal planning, we must additionally guarantee that generated solutions are of minimal cost.