

Planning and Optimization

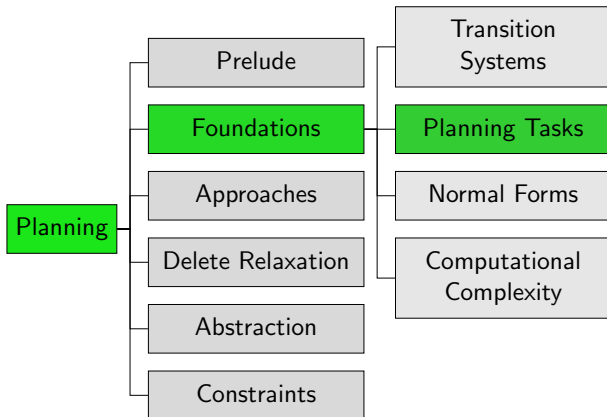
B2. Introduction to Planning Tasks

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Content of the Course



Introduction

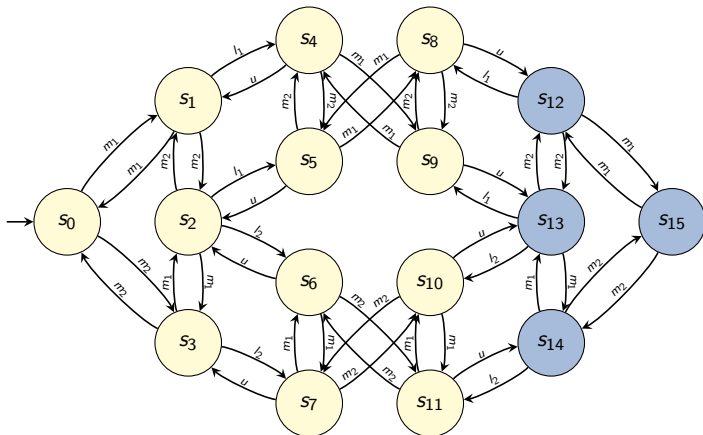
The State Explosion Problem

- We saw in blocks world:
 n blocks $\rightsquigarrow$ number of states **exponential** in n
- same is true everywhere we look
- known as the **state explosion problem**

To represent transitions systems compactly,
need to tame these exponentially growing aspects:

- states
- goal states
- transitions

Running Example: Transition System



$$c(m_1) = 5, c(m_2) = 5, c(l_1) = 1, c(l_2) = 1, c(u) = 1$$

State Variables

Compact Descriptions of Transition Systems

How to specify huge transition systems
without enumerating the states?

- represent different aspects of the world
in terms of different (propositional) **state variables**
- individual state variables are atomic propositions
 $\rightsquigarrow$ a state is an **interpretation of state variables**
- n state variables induce 2^n states
 $\rightsquigarrow$ **exponentially more compact** than “flat” representations

Example: n^2 variables suffice for blocks world with n blocks

Blocks World State with Propositional Variables

Example

$$s(A\text{-on-}B) = \mathbf{F}$$

$$s(A\text{-on-}C) = \mathbf{F}$$

$$s(A\text{-on-table}) = \mathbf{T}$$

$$s(B\text{-on-}A) = \mathbf{T}$$

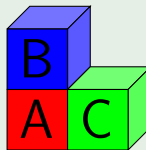
$$s(B\text{-on-}C) = \mathbf{F}$$

$$s(B\text{-on-table}) = \mathbf{F}$$

$$s(C\text{-on-}A) = \mathbf{F}$$

$$s(C\text{-on-}B) = \mathbf{F}$$

$$s(C\text{-on-table}) = \mathbf{T}$$



↪ 9 variables for 3 blocks

Propositional State Variables

Definition (Propositional State Variable)

A **propositional state variable** is a symbol X .

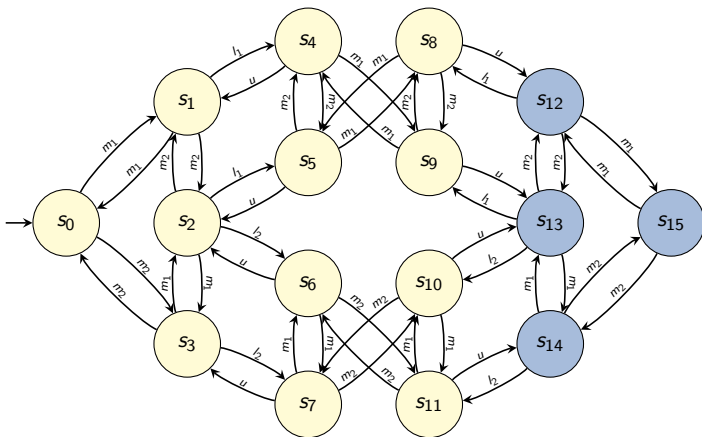
Let V be a finite set of propositional state variables.

A **state** s over V is an interpretation of V , i.e., a truth assignment $s : V \rightarrow \{\mathbf{T}, \mathbf{F}\}$.

Running Example: Compact State Descriptions

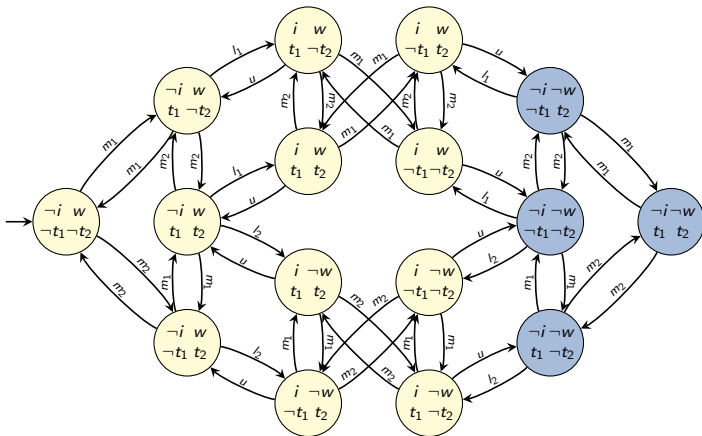
- In the running example, we describe 16 states with 4 propositional state variables ($2^4 = 16$).

Running Example: Opaque States



Running Example: Using State Variables

state variables $V = \{i, w, t_1, t_2\}$



states shown by true literals

example: $\{i \mapsto \mathbf{T}, w \mapsto \mathbf{F}, t_1 \mapsto \mathbf{T}, t_2 \mapsto \mathbf{F}\} \rightsquigarrow i \neg w t_1 \neg t_2$

Running Example: Intuition

Intuition: delivery task with 2 trucks, 1 package, locations L and R
transition labels:

- m_1/m_2 : move first/second truck
- l_1/l_2 : load package into first/second truck
- u : unload package from a truck

state variables:

- t_1 true if first truck is at location L (else at R)
- t_2 true if second truck is at location L (else at R)
- i true if package is inside a truck
- w encodes where exactly the package is:
 - if i is true, w true if package in first truck
 - if i is false, w true if package at location L

State Formulas

Representing Sets of States

How do we compactly represent sets of states,
for example the set of goal states?

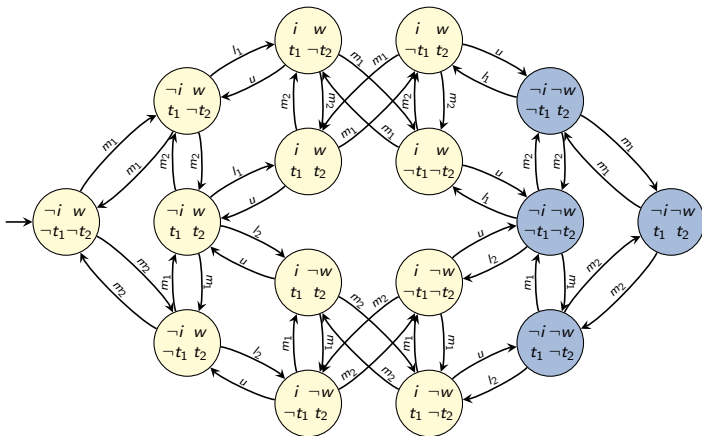
Idea: **formula** φ over the state variables represents the **models** of φ .

Definition (State Formula)

Let V be a finite set of propositional state variables.

A **formula** over V is a propositional logic formula using V as the set of atomic propositions.

Running Example: Representing Goal States



goal formula $\gamma = \neg i \wedge \neg w$ represents goal states S_*

Operators and Effects

Operators Representing Transitions

How do we compactly represent **transitions**?

- most complex aspect of a planning task
- central concept: **operators**

Idea: one operator o for each transition label ℓ , describing

- **in which states** s a transition $s \xrightarrow{\ell} s'$ exists (precondition)
- how state s' **differs** from state s (effect)
- what the cost of ℓ is

Syntax of Operators

Definition (Operator)

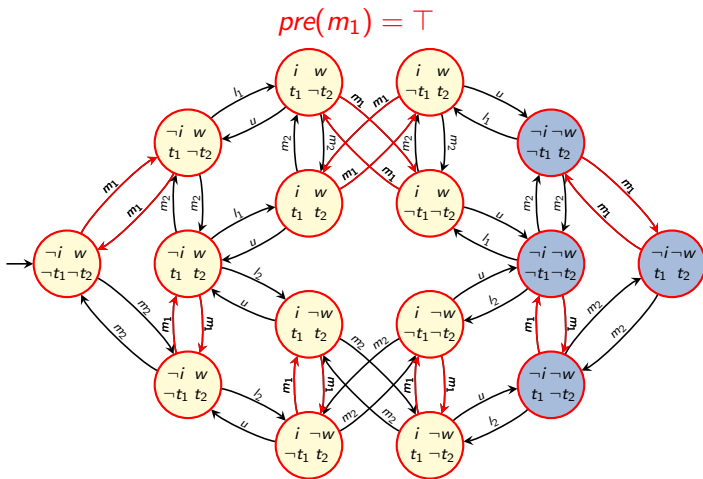
An **operator** o over state variables V is an object with three properties:

- a **precondition** $pre(o)$, a formula over V
- an **effect** $eff(o)$ over V , defined later in this chapter
- a **cost** $cost(o) \in \mathbb{R}_0^+$

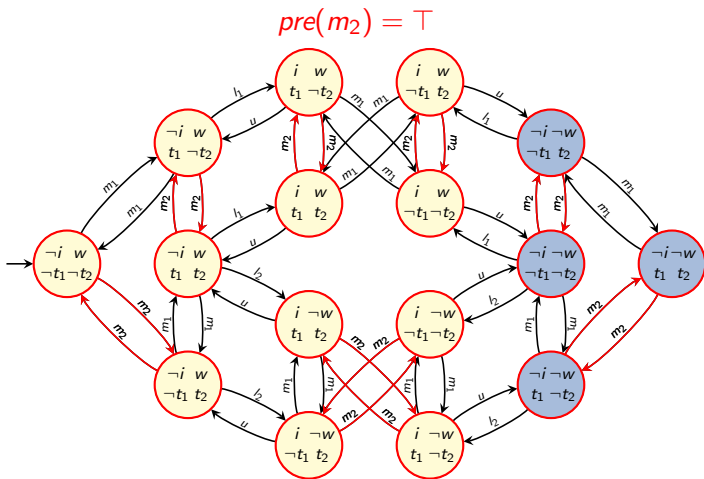
Notes:

- Operators are also called **actions**.
- Operators are often written as triples $\langle pre(o), eff(o), cost(o) \rangle$.
- This can be abbreviated to pairs $\langle pre(o), eff(o) \rangle$ when the cost of the operator is irrelevant.

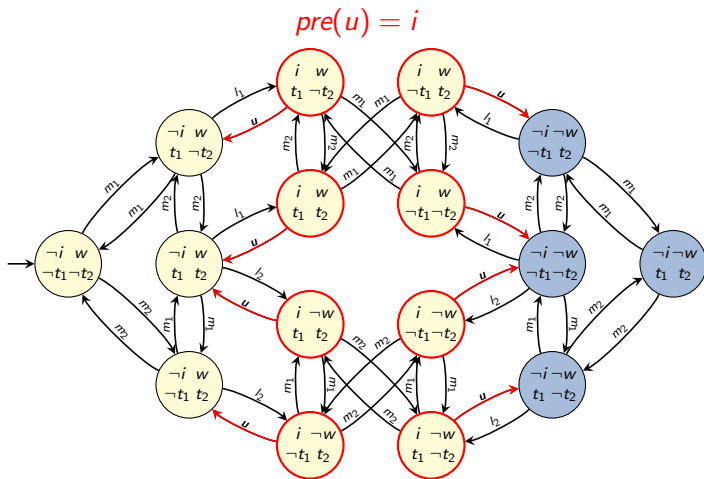
Running Example: Operator Preconditions



Running Example: Operator Preconditions

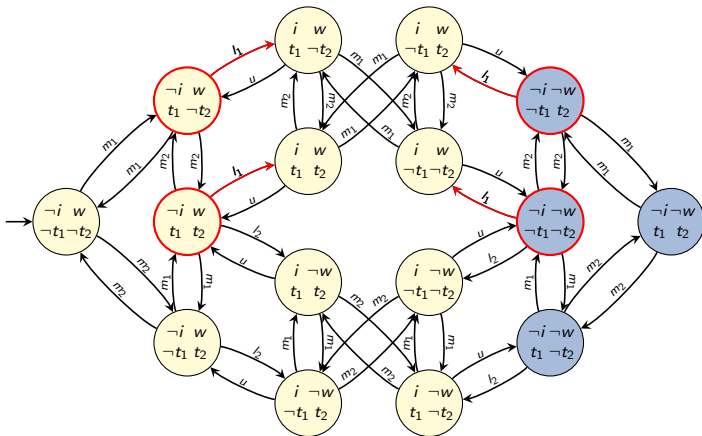


Running Example: Operator Preconditions



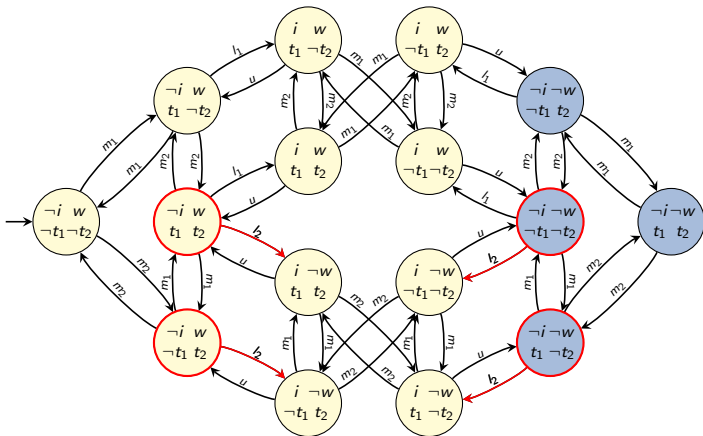
Running Example: Operator Preconditions

$$pre(l_1) = \neg i \wedge (w \leftrightarrow t_1)$$



Running Example: Operator Preconditions

$$pre(l_2) = \neg i \wedge (w \leftrightarrow t_2)$$



Syntax of Effects

Definition (Effect)

Effects over propositional state variables V are inductively defined as follows:

- $\top$ is an effect (**empty effect**).
- If $v \in V$ is a propositional state variable, then v and $\neg v$ are effects (**atomic effect**).
- If e and e' are effects, then $(e \wedge e')$ is an effect (**conjunctive effect**).
- If χ is a formula over V and e is an effect, then $(\chi \triangleright e)$ is an effect (**conditional effect**).

We may omit parentheses when this does not cause ambiguity.

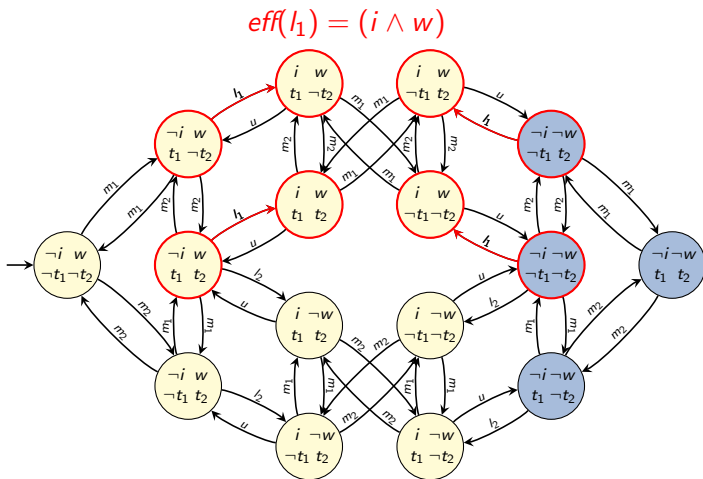
Example: we will later see that $((e \wedge e') \wedge e'')$ behaves identically to $(e \wedge (e' \wedge e''))$ and will write this as $e \wedge e' \wedge e''$.

Effects: Intuition

Intuition for effects:

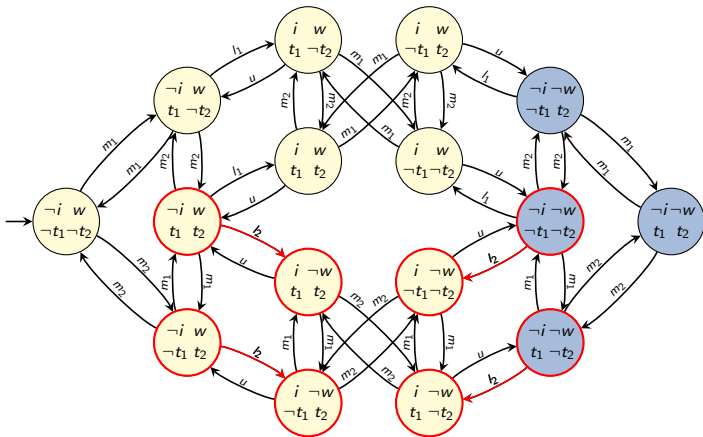
- The **empty effect** $\top$ changes nothing.
- **Atomic effects** can be understood as assignments that update the value of a state variable.
 - v means " $v := \mathbf{T}$ "
 - $\neg v$ means " $v := \mathbf{F}$ "
- A **conjunctive effect** $e = (e' \wedge e'')$ means that both subeffects e and e' take place simultaneously.
- A **conditional effect** $e = (\chi \triangleright e')$ means that subeffect e' takes place iff χ is true in the state where e takes place.

Running Example: Operator Effects



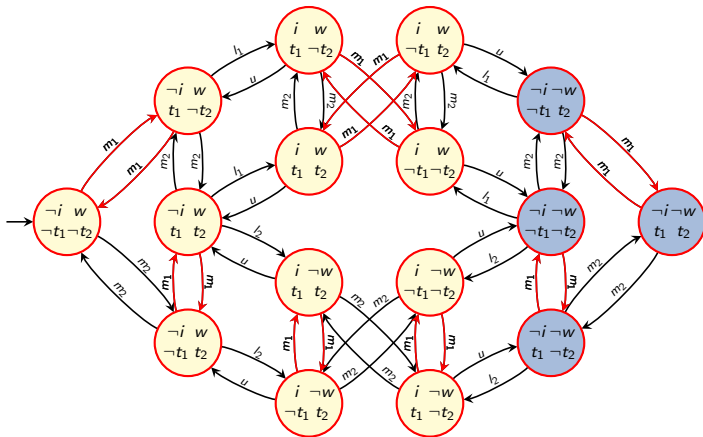
Running Example: Operator Effects

$$\text{eff}(l_2) = (i \wedge \neg w)$$



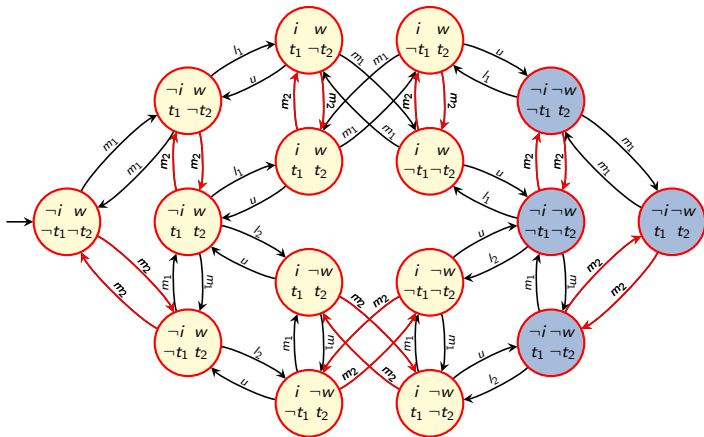
Running Example: Operator Effects

$$\text{eff}(m_1) = ((t_1 \triangleright \neg t_1) \wedge (\neg t_1 \triangleright t_1))$$



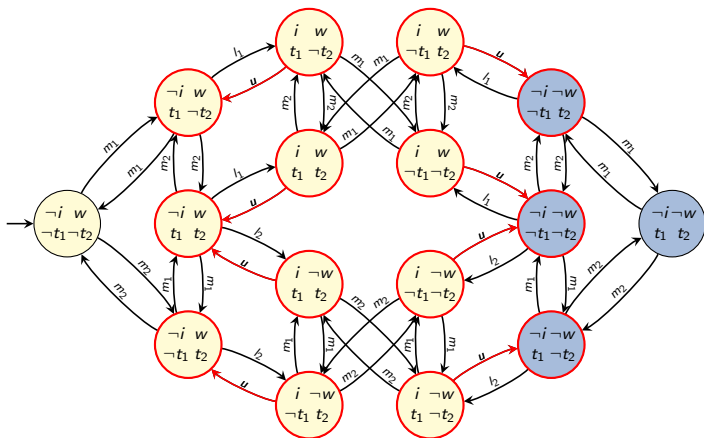
Running Example: Operator Effects

$$\text{eff}(m_2) = ((t_2 \triangleright \neg t_2) \wedge (\neg t_2 \triangleright t_2))$$



Running Example: Operator Effects

$$\begin{aligned} \text{eff}(u) = & \neg i \wedge (w \triangleright ((t_1 \triangleright w) \wedge (\neg t_1 \triangleright \neg w))) \\ & \wedge (\neg w \triangleright ((t_2 \triangleright w) \wedge (\neg t_2 \triangleright \neg w))) \end{aligned}$$



Summary

Summary

- Propositional **state variables** let us compactly describe properties of large transition systems.
- A **state** is an assignment to a set of state variables.
- Sets of states are represented as **formulas** over state variables.
- **Operators** describe **when** (precondition), **how** (effect) and at which **cost** the state of the world can be changed.
- **Effects** are structured objects including empty, atomic, conjunctive and conditional effects.